New perspectives for B-physics from the lattice

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I B-physics and lattice QCD

II B's on the lattice: the challenges

III New developments

- Non-perturbative HQET
- Results for $F_{\rm B_s}$
- Extrapolation in the quark mass of finite volume effects

IV Perspectives

I. B-physics and lattice QCD

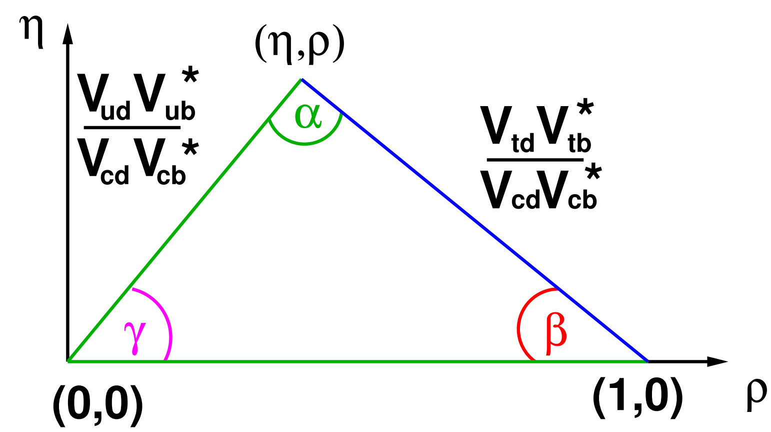
Relevant for

- the determination of the CKM-parameters
  - "fundamental" parameters of nature
  - CP puzzle
- the b-quark mass
- spectrum and lifetimes of b-hadrons
- non-perturbative tests of HQET

The CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

mass eigenstates $\neq$ weak eigenstates



$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- side from $\Delta m_{\rm d}$ $\qquad F^2_{\rm B_d} B_{\rm B_d}$

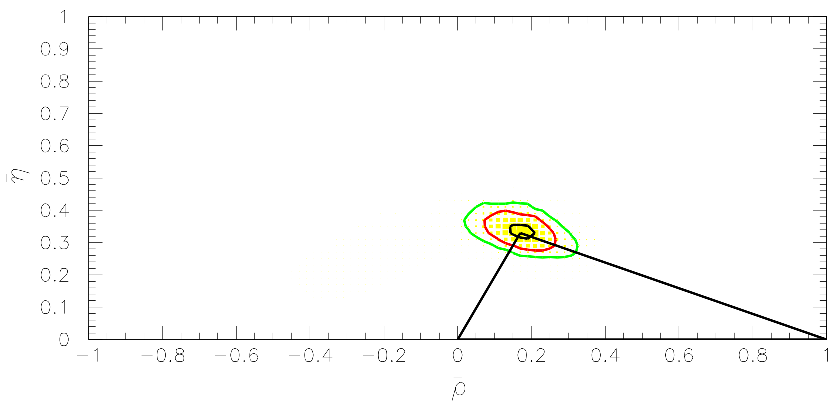
  $\Delta m_{\rm s}/\Delta m_{\rm d}$ $\qquad$ & $\qquad \xi = \frac{F_{\rm B_s}\sqrt{B_{\rm B_s}}}{F_{\rm B_d}\sqrt{B_{\rm B_d}}}$

- $\eta(1-\rho)$ from $\epsilon_{\rm K}$ $\qquad B_{\rm K}$
- angle γ from $B \to h^+h^-$
- $\sin 2\beta$ from $\rm J/\psi K_s$ decays

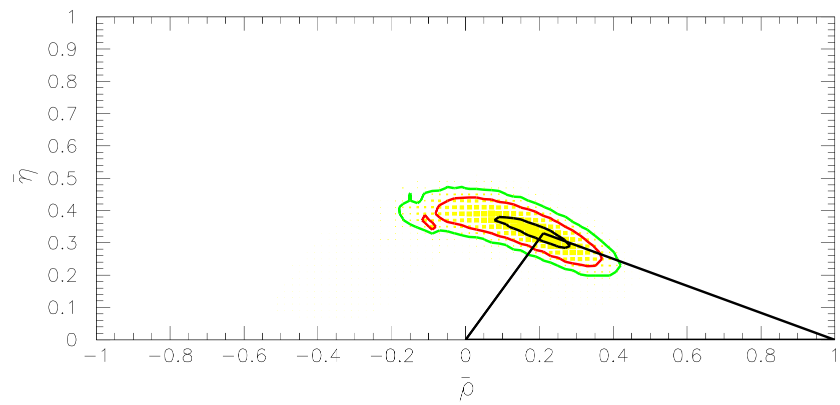
with $\langle \overline{M} | \mathcal{O}_{\Delta M=2} | M \rangle = \frac{4}{3} m^2_{\rm M} F^2_M B_M$ $\qquad \langle B_{\rm d} | \bar{b}\gamma_\mu\gamma_5 d | 0 \rangle = i p_\mu F_{\rm B_d}$

The CKM matrix

CKM-fit with lattice input
$(B_\mathrm{K}, F_{\mathrm{B_d}}, B_{\mathrm{B_d}}, \xi)$



CKM-fit
$B_\mathrm{K}, F_{\mathrm{B_d}}$ lattice input removed



Analysis: [M. Ciuchini et al., 2001; update by M. Ciuchini]

precision CKM-physics, e.g. check of unitarity
probably requires the determination of key parameters from "first principles"
(lattice QCD)

Key parameters from "first principles"

- Key parameters
  - e.g. $\xi, B_{\rm K}, F_{\rm B_d}, B_{\rm B_d}$
  - but also $M_{\rm b}$ (RGI quark mass) etc.

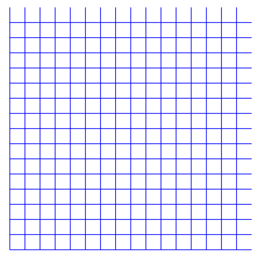
- first principles $$\mathcal{L}_{\rm QCD} = -\frac{1}{2g_0^2}\,\mathrm{tr}\,\{F_{\mu\nu}F_{\mu\nu}\} + \sum_f \overline{\psi}_f\{D + m_f\}\psi_f$$

$$\overbrace{\begin{bmatrix} F_\pi \\ m_\pi \\ m_{\rm K} \\ m_{\rm D} \\ m_{\rm B} \end{bmatrix}}^{\text{Experiment}} \quad \overset{\mathcal{L}_{\rm QCD}(g_0, m_f)}{\Longrightarrow} \quad \overbrace{\begin{bmatrix} \Lambda_{\rm QCD} \\ \hat{M} = (M_{\rm u} + M_{\rm d})/2 \\ M_{\rm s} \\ M_{\rm c} \\ M_{\rm b} \end{bmatrix}}^{\text{QCD parameters (RGI)}} \quad + \quad \overbrace{\begin{bmatrix} \xi \\ F_B \\ B_B \\ \cdots \end{bmatrix}}^{\textbf{Predictions}}$$

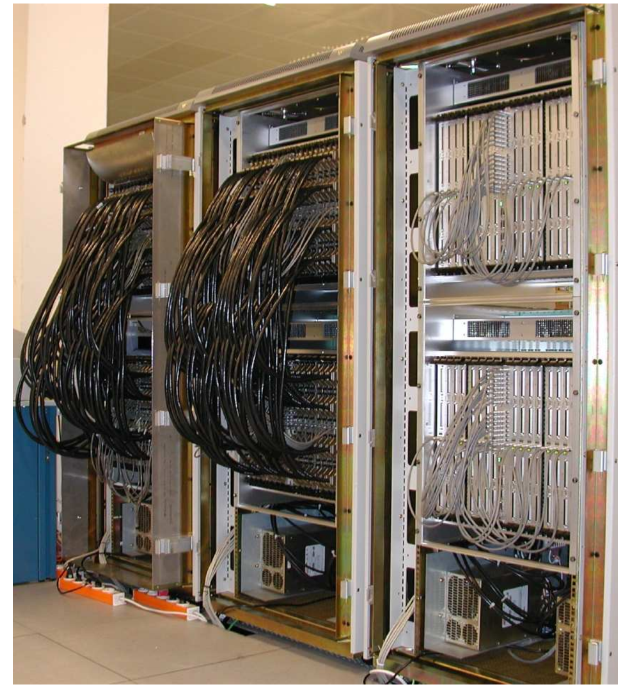
What does $\overset{\mathcal{L}_{\mathrm{QCD}}(g_0, m_f)}{\Longrightarrow}$ mean?

Discretization of $\mathcal{L}_{\mathrm{QCD}}$ with

- gauge invariance
- locality
- unitarity



$\Longrightarrow$



renormalization $\Downarrow$ continuum limit

low energy matrix elements

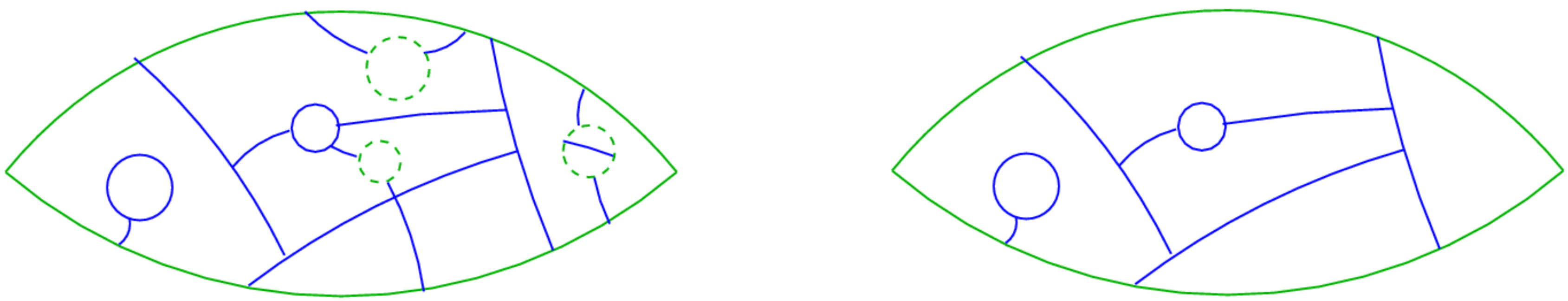
$$\pm \mathrm{O}\left(\frac{1}{\sqrt{\text{computer time}}}\right)$$

A meson correlation function in Feynman graphs (full QCD)

→ quenched approximation:

drop the determinant (neglect fluctuations of $\det(D + m_f)$)

← MODEL



Still: we often quench ...

... to practise (not first principles but excellent testing ground of methods & surprisingly accurate in tested cases)

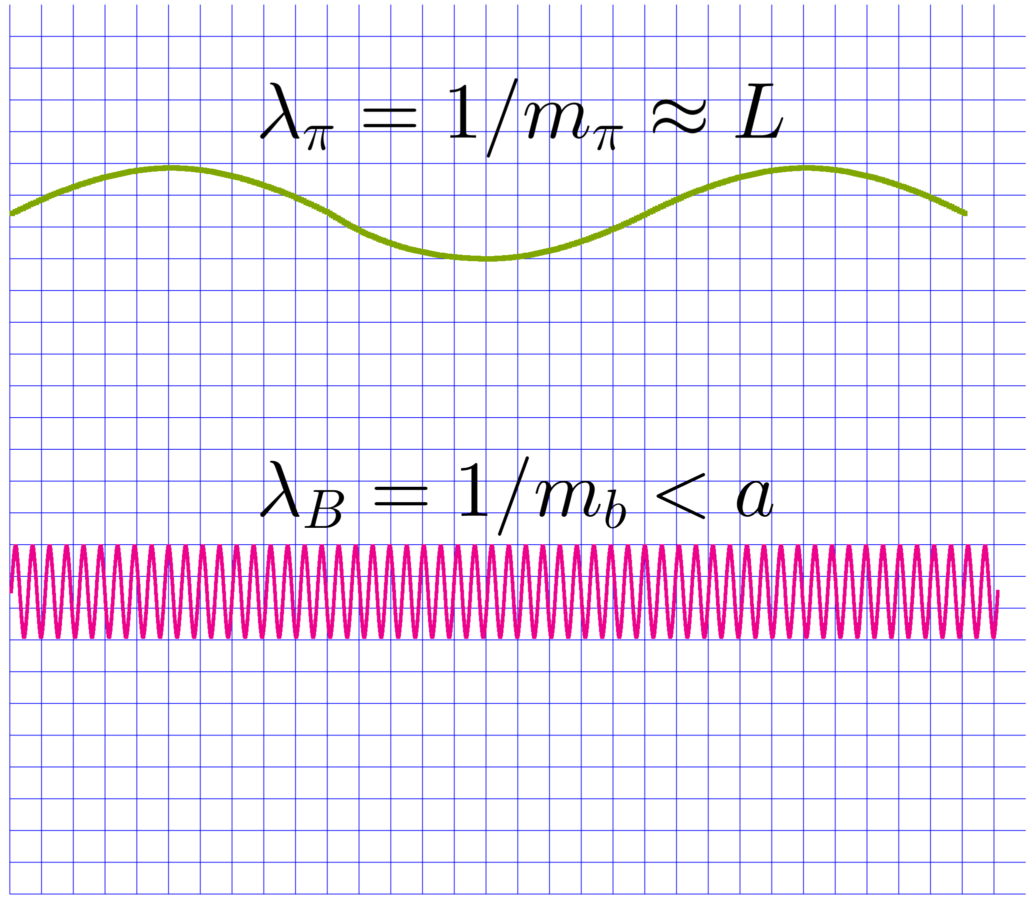
II. The challenges

Take a large lattice as it is possible in the quenched approximation

$L \approx 2.5\mathrm{fm}$

finite size effects due to light π's

$a \approx 0.07\mathrm{fm}$
discretization errors for B's



➥ **light quarks are too light** **b-quark is too heavy**

- light quarks are too light (for the computer ressources)
  - ➥ "chiral" extrapolations $\Longrightarrow$ brief discussion
  - ➥ hopefully new algorithms with better performance at small quark masses [M. Hasenbusch, 2001; M. Lüscher 2003]

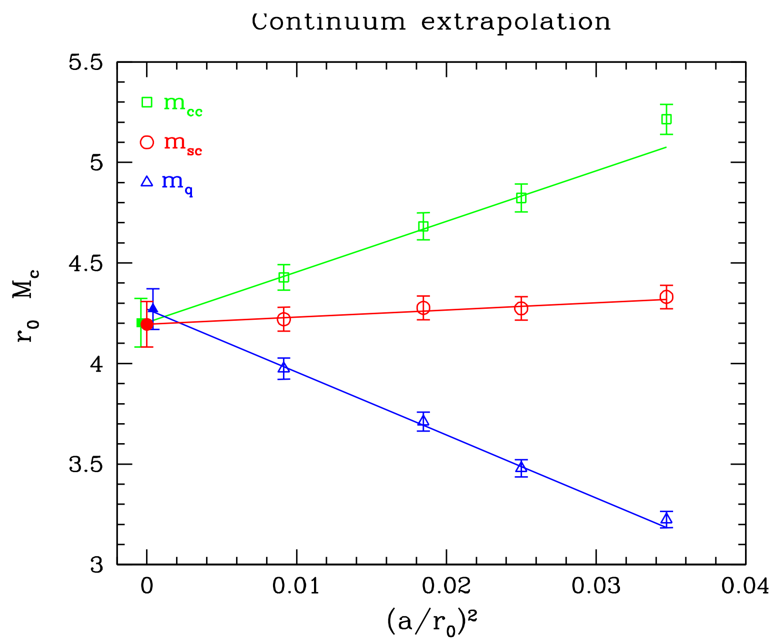
- b-quark is too heavy:

  charm: [Sint & Rolf]

  3 definitions of charm quark mass differ by a-effects

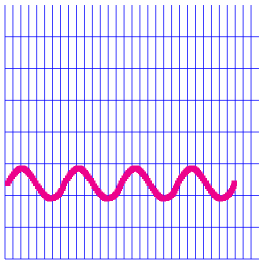
  charm just doable

  $aM_\mathrm{b} \approx 4aM_\mathrm{c}$!



Attempts to solve the problem of a heavy b-quark

❶ anisotropic lattices
beware: dropping space/time symmetry
$\rightarrow$ fine tuning necessary
subtleties under depate



❷ extrapolations
beware: order of limits:

$$\lim_{m_h \to m_\mathrm{b}} \lim_{am_h \to 0} F(m_h, am_h)$$

❸ effective theories
– NRQCD
– HQET

$$\mathcal{L}_\mathrm{HQET} = \overline{\psi}_\mathrm{h}\, D_0\, \psi_\mathrm{h} - \frac{1}{2m_\mathrm{b}} \overline{\psi}_\mathrm{h} \mathbf{D}^2 \psi_\mathrm{h} - \frac{c_\sigma}{2m_\mathrm{b}} \overline{\psi}_\mathrm{h} \mathbf{B} \cdot \sigma \psi_\mathrm{h} + \ldots$$

❹ combinations, in particular of ❷,❸

Dominant procedure in the last decade:

- try several approaches; if they agree, apply to phenomenology
- probably very much limited in precision (10%, 15%, ?)

can we do better?
➥ New developments: it seems so.

Chiral extrapolations

- One of the major problems in lattice QCD (also for light hadrons)

  computational effort $\propto m_\pi^{-(4+z)}$, $z \approx 5$!!
- New discussion for B-physics
  [Kronfeld & Ryan, 2002; Becirevic et al., 2002; Sanz-Cillero, Donoghue & Ross, 2003]
- old procedure:
  - fixed b-quark mass
  - data at various values of $m_{\text{quark}} \propto m_\pi^2$
  - linear (maybe quadratic) extrapolation to physical value of m_π^2
- But chiral perturbation theory gives (correct asymptotic expansion)

$$F_{\rm B} = F_0 \left[1 - \left(\frac{1 + 3g^2_{B^*B\pi}}{16\pi^2 F_\pi^2} \right) \frac{3}{8} m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} + C(\mu^2) + \mathrm{O}(m_\pi^4) \right]$$

(The physics: effective theory for π-loops) [Grinstein et al., 1992; Goity, 1992]
$g_{B^*B\pi} \approx 0.6$, [CLEO 2001; Abada et al, 2002]
$C(\mu^2)$ unknown.

- Use Chiral perturbation theory formula

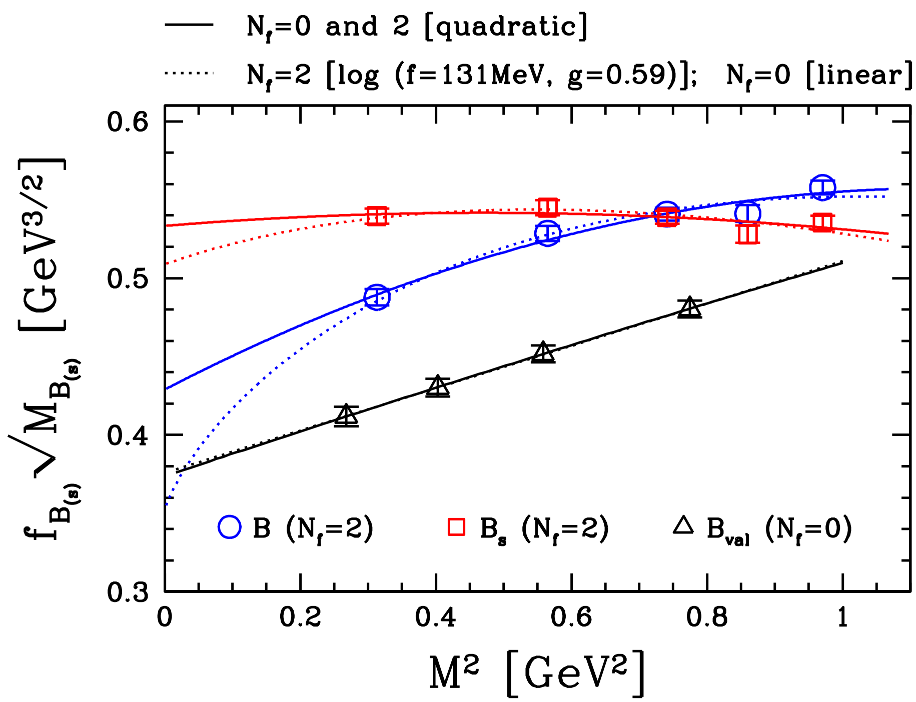
$$F_{\mathrm{B}} = F_0 \left[1 - \left(\frac{1 + 3g^2_{B^*B\pi}}{16\pi^2 F_\pi^2} \right) \frac{3}{8} m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} + C(\mu^2) + \mathrm{O}(m_\pi^4) \right]$$

$g_{B^*B\pi} \approx 0.6$ [CLEO 2001; Abada et al, 2002]

fit $C(\mu^2)$, $\mathrm{O}(m_\pi^4)$ dropped

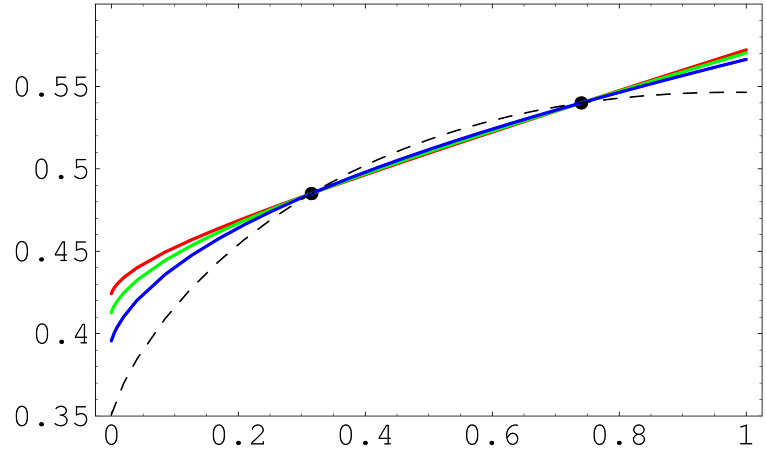
done by [JLQCD 2002]:

is chiral PT applicable
for these π-masses?



graph by [Lellouch, 2003]

- Donoghue et al.:
  - Chiral perturbation theory with a **finite** cutoff, $\Lambda = \mathrm{O}(\mathrm{GeV})$
  - argue:
    extended applicability domain because $m_\pi^2 \rightarrow \infty$ limit is properly treated(π decouples!)



  - model (cutoff) dependent taming of $\mathrm{O}(m_\pi^4)$ terms

- present uncertainty (also in ξ)
  10 % : [Kronfeld & Ryan]
  5 % : [Donoghue et al.]

- unfortunately: these are all (clever but) rough estimates

- true solution requires smaller quark masses
  new algorithms [M. Hasenbusch, 2001; M. Lüscher 2003; ???] may help

III. New developments

Non-perturbative HQET

$m_{\rm b} \ll \Lambda_{\rm QCD}$: accurate expansion in $\Lambda_{\rm QCD}/m_{\rm b}$

possible stumble stones:

❶ statistical precision

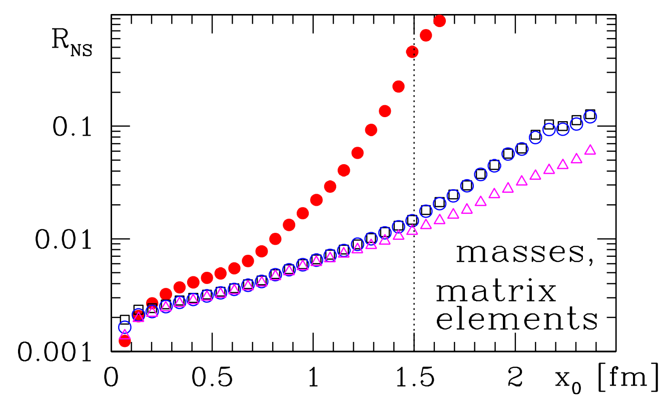
❷ number of parameters grows with the order in $\Lambda_{\rm QCD}/m_{\rm b}$

❸ **parameters** have to be fine-tuned **non-perturbatively** for $a \rightarrow 0$ to exist

considerable improvement on ❶
by change of the discretization of HQET
(discretization errors checked!)
[M. Della Morte et al. (ALPHA Collaboration), 2003]

best version makes use of "HYP-links"
[Hasenfratz & Knechtli, 2001]

error / signal



solution to ❷, ❸: matching of HQET & QCD in finite volume
[Heitger & S, 2001; ALPHA Collaboration , 2003]

Non-perturbative matching of HQET and QCD

- why non-perturbative matching ?

HQET: effective theory, new operators at each order in $1/m_{\rm b}$
new free parameters c_k
parameters are computable from QCD:
transfer of predictivity QCD $\rightarrow$ HQET

this has to be done non-perturbatively, otherwise there are errors

$$\Delta c_k \sim \frac{g_0^{2(l+1)}}{a} \sim \frac{1}{a\,[\ln(a\Lambda)]^{l+1}} \xrightarrow{a\to 0} \infty$$

simple case
parameters computed to
l-loops

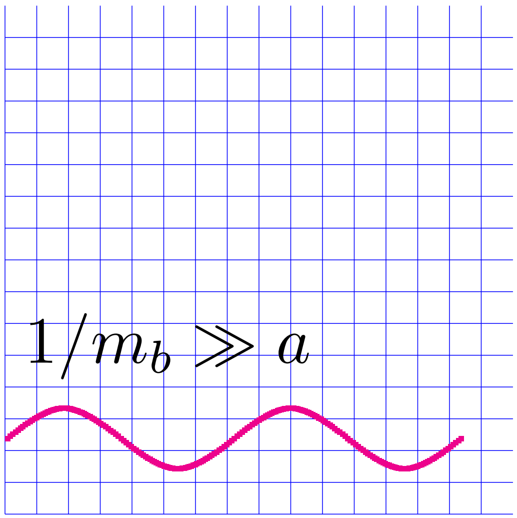
no continuum limit! (if c_k are computed at a finite order in g^2)

- non-perturbative matching: requires to be able to simulate the b-quark !

- The trick: start in small volume, $L \approx 0.2\,\mathrm{fm}$

- The trick: start in small volume, $L \approx 0.2\,\mathrm{fm}$: $L \ll 1/m_\pi$

QCD

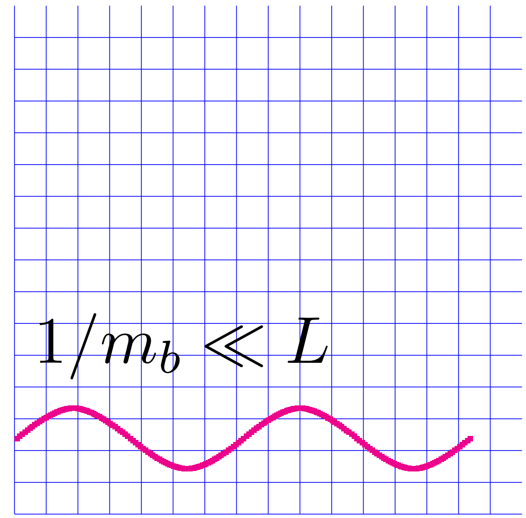


$$\Phi_k^{\mathrm{QCD}} = \Phi_k^{\mathrm{HQET}}$$

$$k = 1, 2, \ldots, N_{\mathrm{HQET}}$$

$N_{\mathrm{HQET}} =$ # of parameters

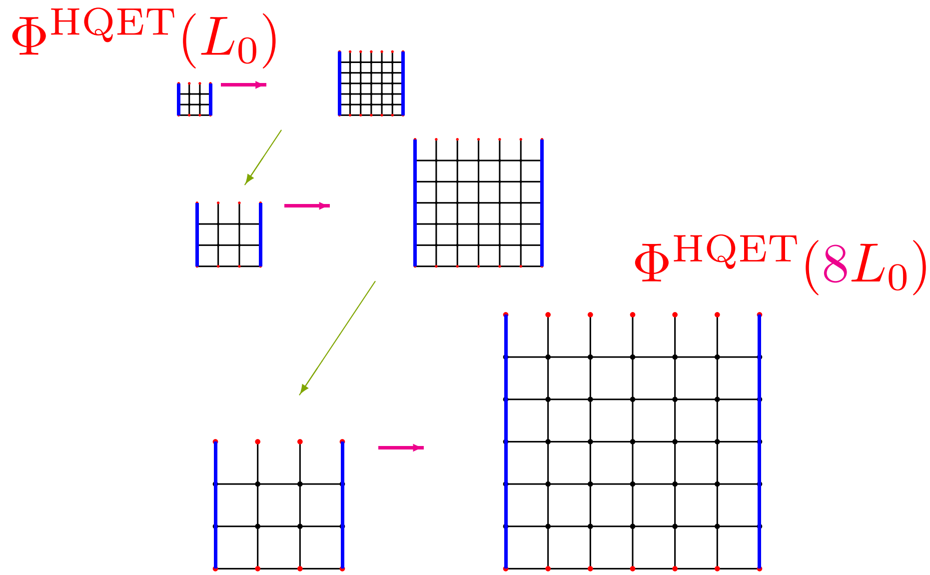
HQET



➥ HQET-parameters from QCD-observables in small volume

Physical observables (e.g. $B_{\mathrm{B_s}}, F_{\mathrm{B_s}}$) need a large volume, such that the B-meson fits comfortably: $L = L_0 \approx 2\,\mathrm{fm}$

Connection achieved by recursive method:
[Lüscher, Weisz & Wolff, 91; ALPHA Collaboration 1993-2003]



– first fully non-perturbative formulation of HQET
– continuum limit can be taken in all steps

At lowest order in $1/m_{\rm b}$ (static approximation) simple equations result:

example: computation of the b-quark mass

$$\begin{aligned}\Gamma(L) &= \text{finite volume B-meson "mass"}\\ &= \text{energy of a state with quantum \# of a B in an } L^4 \text{ world}\\ L_2 &= 4L_1 = 2L_0\end{aligned}$$

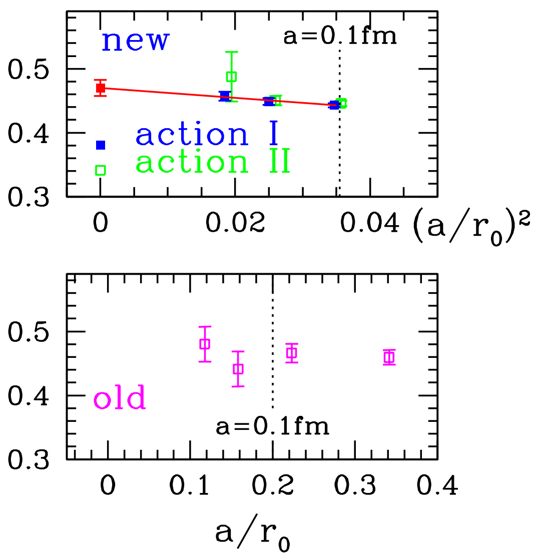
$$m_{\rm B} = \underbrace{E_{\rm stat} - \Gamma_{\rm stat}(L_2)}_{a\to 0 \textbf{ in HQET}} + \underbrace{\Gamma_{\rm stat}(L_2) - \Gamma_{\rm stat}(L_0)}_{a\to 0 \textbf{ in HQET}} + \underbrace{\Gamma(L_0, M_{\rm b})}_{a\to 0 \textbf{ for } M_{\rm b}L_0 \gg 1\textbf{:}\ L_0 \approx 0.2{\rm fm}}$$

➥ Solve the above equation for $M_{\rm b}$ (the RGI b-quark mass)

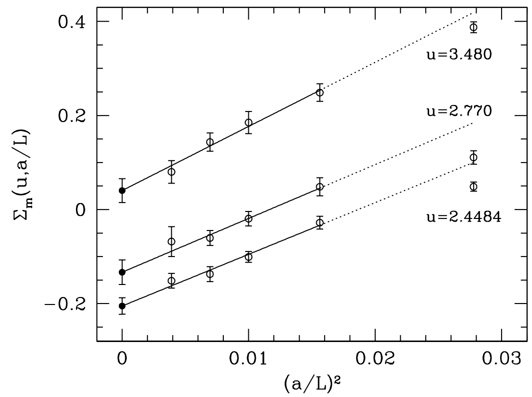
$$m_{\rm B} = \underbrace{E_{\rm stat} - \Gamma_{\rm stat}(L_n)}_{a\to 0 \textbf{ in HQET}} + \underbrace{\Gamma_{\rm stat}(L_n) - \Gamma_{\rm stat}(L_0)}_{a\to 0 \textbf{ in HQET}} + \underbrace{\Gamma(L_0, M_{\rm b})}_{a\to 0 \textbf{ for } M_{\rm b}L_0 \gg 1:\ L_0 \approx 0.2\text{fm}}$$

continuum extrapolations (results still in quenched approximation):

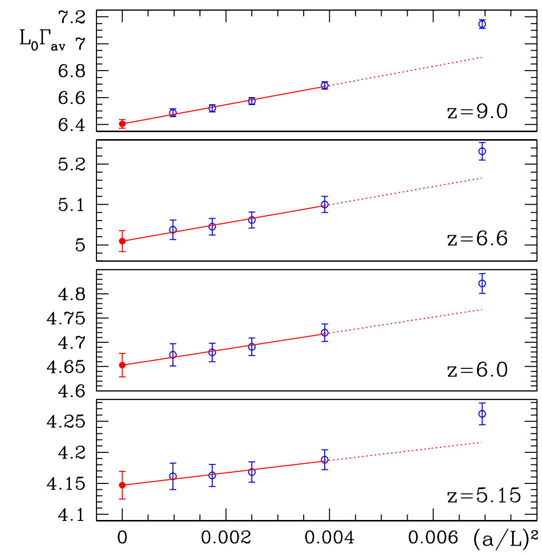
$L_0\,(E_{\rm stat} - \Gamma_{\rm stat}(L_2))$



$L\,(\Gamma(2L) - \Gamma(L))$



$L_0\,\Gamma(L_0, M)\,,\quad z = L_0 M$



Solve for $M_{\rm b}$
[ALPHA Collaboration, 2003, preliminary]

$$M_{\rm b} = 4.13(2)(4)\,\text{GeV} + \text{O}\left(\frac{\Lambda^2_{\rm QCD}}{M_{\rm b}}\right)$$

new quality: NP renormalization, continuum limit (but still quenched!)

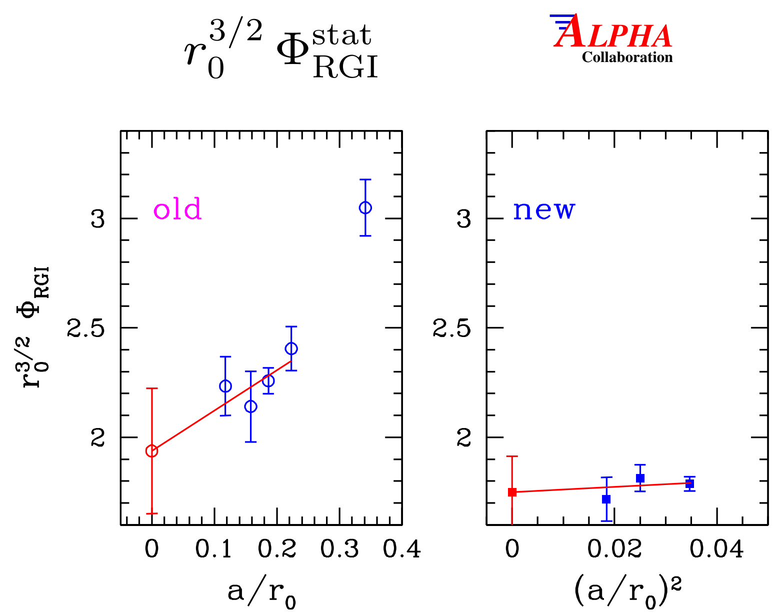
Continuum limit for $F_{\mathrm{B_s}}$

$$F_{\mathrm{PS}}\sqrt{m_{\mathrm{PS}}} = C_{\mathrm{PS}}(m_{\mathrm{PS}})\Phi^{\mathrm{stat}}_{\mathrm{RGI}} + \mathrm{O}(1/m_{\mathrm{PS}})$$

$C_{\mathrm{PS}}(m_{\mathrm{PS}})$ perturbatively computable weak (logarithmic) m_{PS}-dependence
[Shifman & Voloshin, 87; Politzer & Wise, 88; ... Chetyrkin & Grozin, 2003 (3-loop)]

- Computation of $F_{\mathrm{B_s}}$ in lowest order in HQET quenched!
  - NP renormalization [Heitger,Kurth & S; 2003]
  - action with reduced statistical errors
  - linear a-effects removed
  - preliminary continuum extrapolation

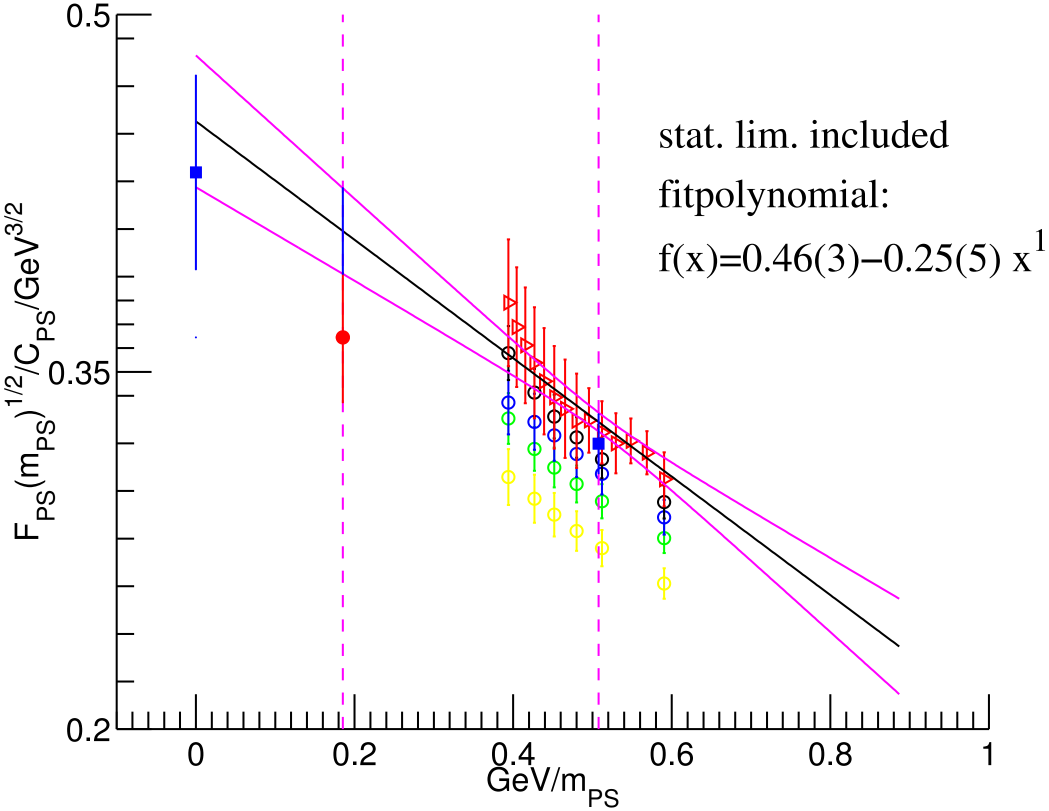
$r_0^{3/2}\,\Phi^{\mathrm{stat}}_{\mathrm{RGI}}$



- $F_{\mathrm{PS}}(m_{\mathrm{PS}})$ for $m_{\mathrm{PS}} \approx (0.8-1.8)\times m_{\mathrm{D_s}}$
  - details as above
- combine by interpolation (linear in $1/m_{\mathrm{PS}}$)

- combine by interpolation

$$F_{\rm PS}\sqrt{m_{\rm PS}} = C_{\rm PS}(m_{\rm PS})\Phi^{\rm stat}_{\rm RGI} + \frac{c_-}{m_{\rm PS}}$$



$$F_{\rm B_s} = 217(10)\,{\rm MeV}$$

preliminary, quenched!

ALPHA Collaboration

- improvements in progress
- compares well with the result of second new method (•), to be discussed...

Extrapolation of finite volume effects in the quark mass

[Guagnelli, Palombi, Petronzio & Tantalo, 2002-2003]

- Same starting point as NP HQET
  b-quark can be simulated in small volume ($L_1 = 0.4\,\mathrm{fm}$)

- Observation: $\sigma_\Phi(m_h, L) = \dfrac{\Phi(m_h, 2L)}{\Phi(m_h, L)}$ has weak dependence on m_h

  (e.g. $\Phi = F_\mathrm{B}$)

  ➥ $\lim_{m_h \to m_\mathrm{b}} \lim_{am_h \to 0}$ easier to take for such ratios (finite size effects)

  in practise: $\sigma_\Phi(m_h, L) = \sigma_0(L) + \frac{\sigma_1(L)}{m_h}$

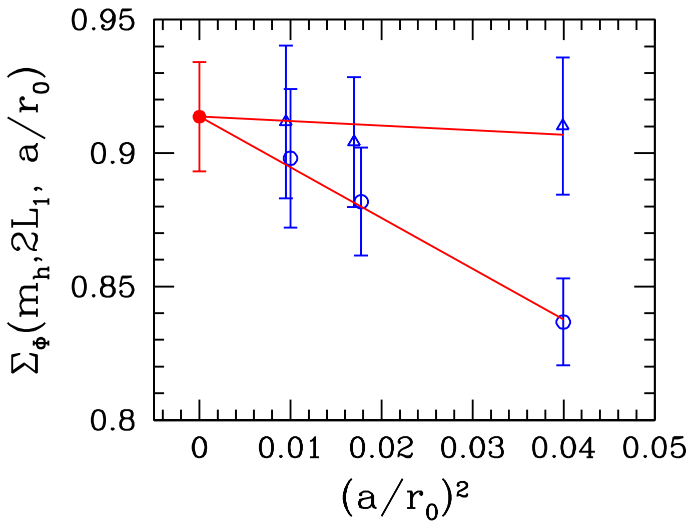
- in detail (e.g. $\Phi = F_{\rm B}$)

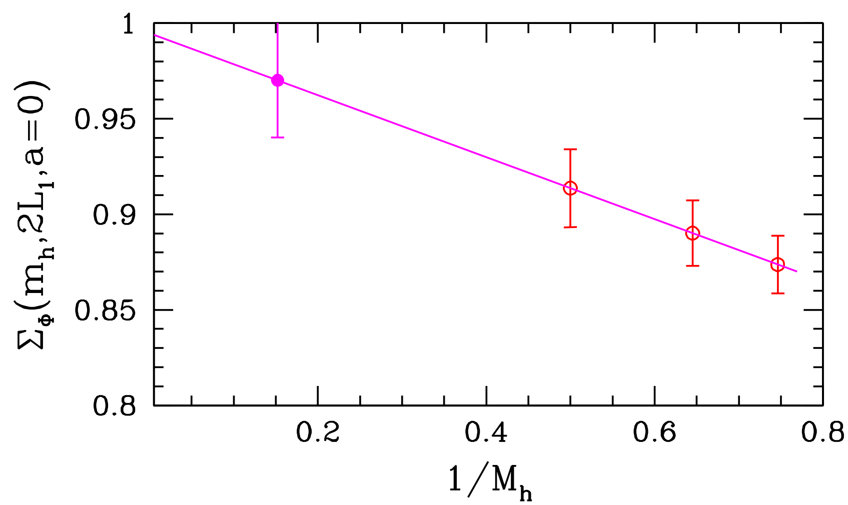
$$\Phi(m_h, 4L_1) = \underbrace{\frac{\Phi(m_h, 4L_1)}{\Phi(m_h, 2L_1)}}_{= \sigma_\Phi(m_h, 2L_1)} \times \underbrace{\frac{\Phi(m_h, 2L_1)}{\Phi(m_h, L_1)}}_{= \sigma_\Phi(m_h, L_1)} \times \Phi(m_h, L_1)$$

$m_h \geq m_{\rm b}/4$	$m_h \geq m_{\rm b}/2$	$m_h \geq m_{\rm b}$
$a \leq 0.1\,{\rm fm}$	$a \leq 0.05\,{\rm fm}$	$a \leq 0.025\,{\rm fm}$

- essential step: $\sigma_\Phi(m_h, 2L_1)$

continuum extrapolation — mass extrapolation





- Also a determination of b-quark mass has been performed by this method.
- $32^3 \times 64$ lattices needed; difficult for dynamical fermions

IV. Perspectives

- $m_l \to m_{\rm phys}$ extrapolations with dynamical quarks (,d,s)
  Exciting paper [“High-Precision Lattice QCD Confronts Experiment”, Davies et al., hep-lat/0304004]
  claim: “small quark masses in $N_{\rm f} = 3$ theory under control” no more quenching

  some questions about the first principles remain (locality, unitarity, continuum limit)

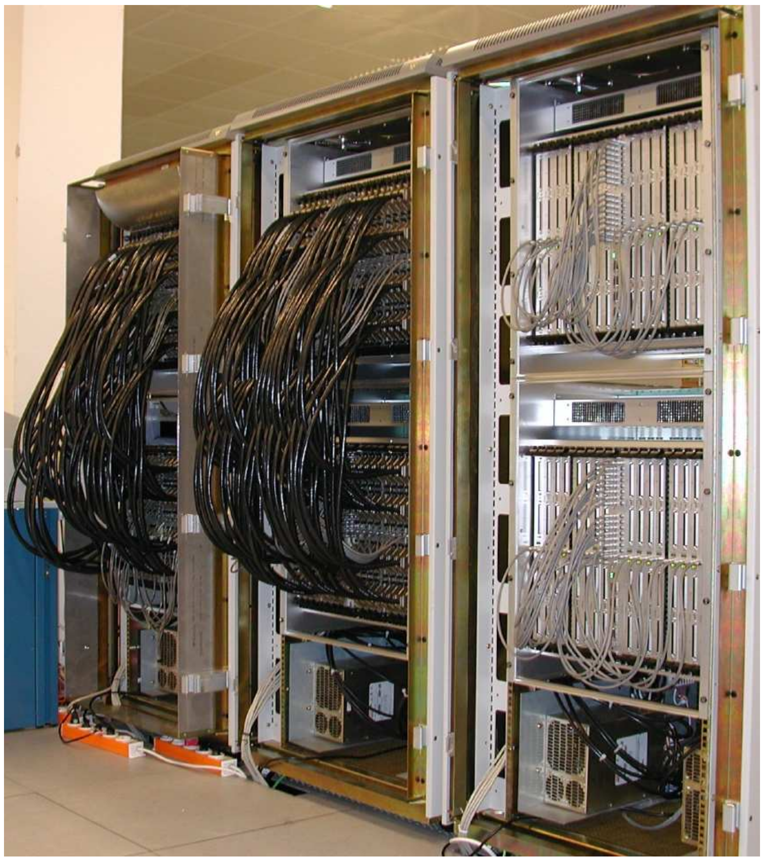
- new methods are available which are promising for dynamical quarks do not need huge lattices

  – Non-perturbative HQET
  – Extrapolation of observables in L and $m_{\rm b}$

- Chances that methods will be developed which lead to precision matrix elements ➥ match experimental precision within this decade ?!
  $1/m_{\rm b}$ corrections ...

- in general: **Progress** through a combination of **new methods** and **new computers**

have a look at our massively parallel **computer**: APEmille



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