$(g-2)_\mu$

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- Introduction and Motivation
- Experiment; Design and Setup
- Data and Analysis
- a_{μ^+} to within 0.7 ppm uncertainty (2000 data)
- Theory; Status
- Conclusions and Outlook

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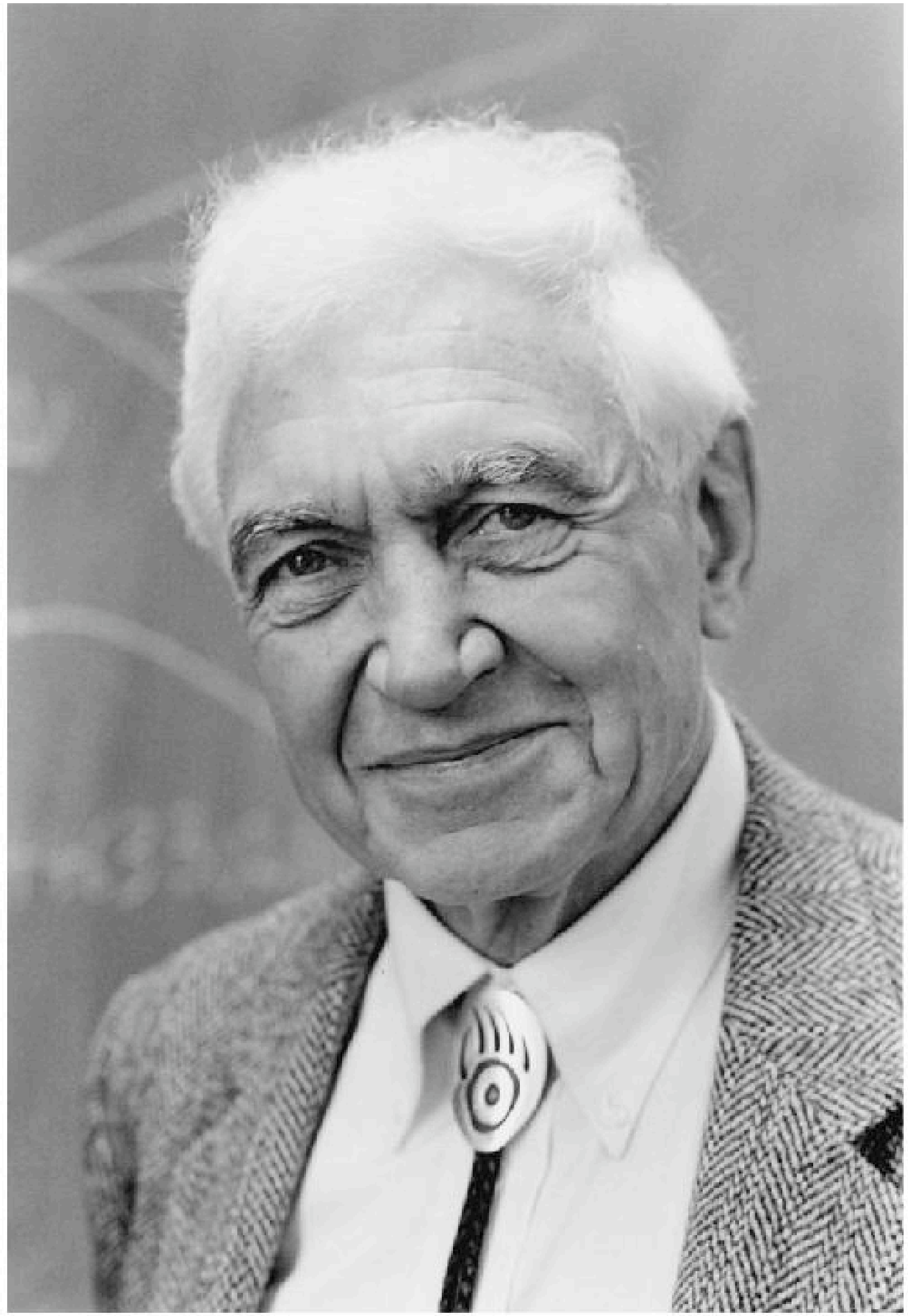
Vernon Willard Hughes

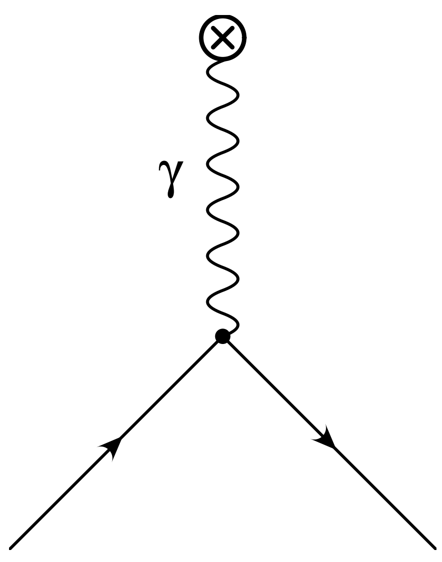


May 28, 1921 - March 25, 2003

The magnetic moment $\vec{\mu}$ of a particle is related to its intrinsic spin $\vec{S}$, according to:

$$\vec{\mu} = g \frac{e}{2mc} \vec{S}$$

Dirac theory predicts $g = 2$



for elementary spin $\frac{1}{2}$ particles.

Experimentally

$g_{\mathrm{p}} \simeq 5.6$

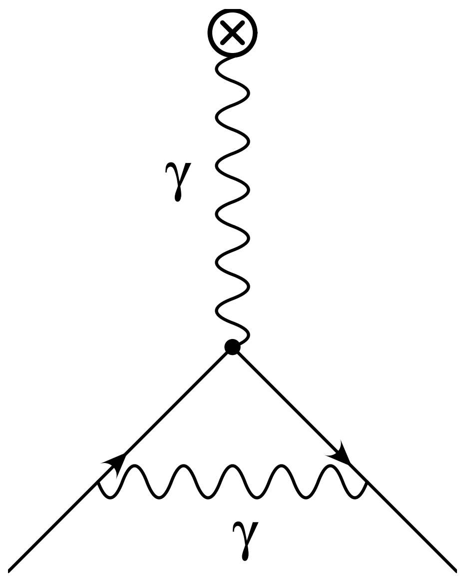
$g_{\mathrm{n}} \simeq -3.8$

and even

$g_{\mathrm{e}} \neq 2$

Instead,

$$a_{\mathrm{e}} = \frac{g_{\mathrm{e}} - 2}{2} \simeq \frac{\alpha}{2\pi}$$



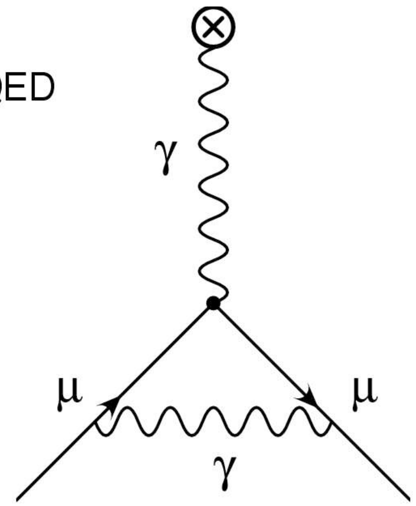
Presently,

$$\begin{aligned} a_{\mathrm{e}} &= 1\,159\,652\,188(4) \times 10^{-12} \ \ (3.4\,\mathrm{ppb}) \\ &\simeq a_{\mathrm{QED}}(e, \gamma) \ \text{at} \ \mathcal{O}(\alpha^4) \end{aligned}$$

To probe beyond QED, one needs *heavier* electrons; muons.

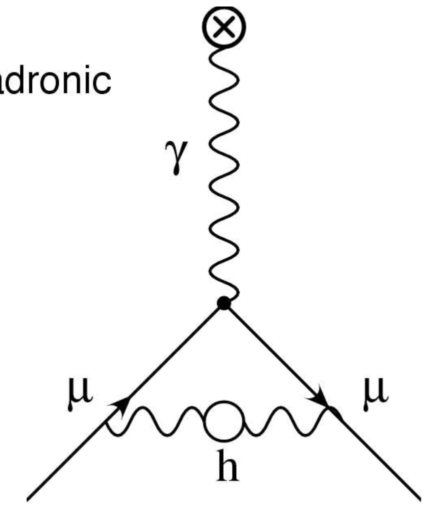
The sensitivity scales typically as $(m_\mu / m_{\mathrm{e}})^2 \sim 4 \cdot 10^4$

QED



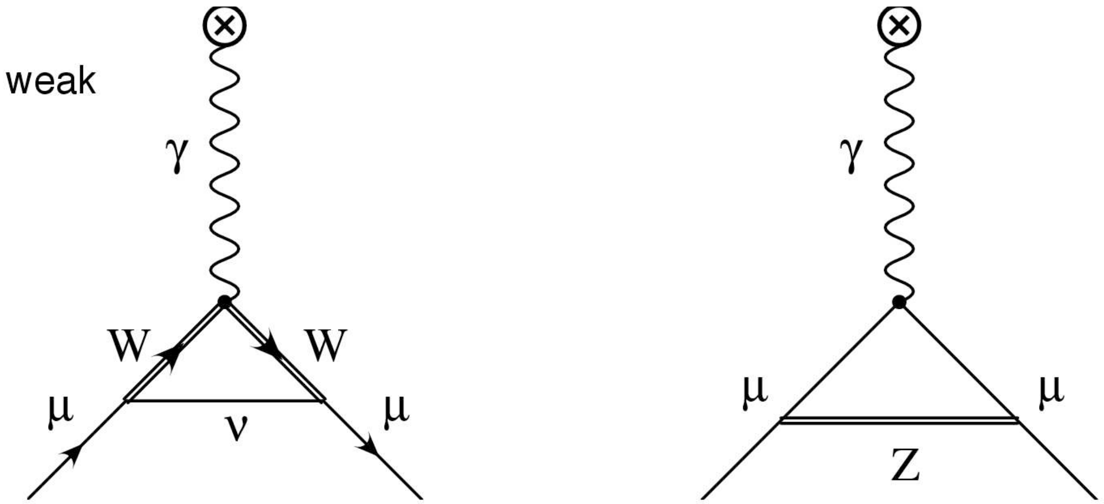
~α/(2π)

hadronic

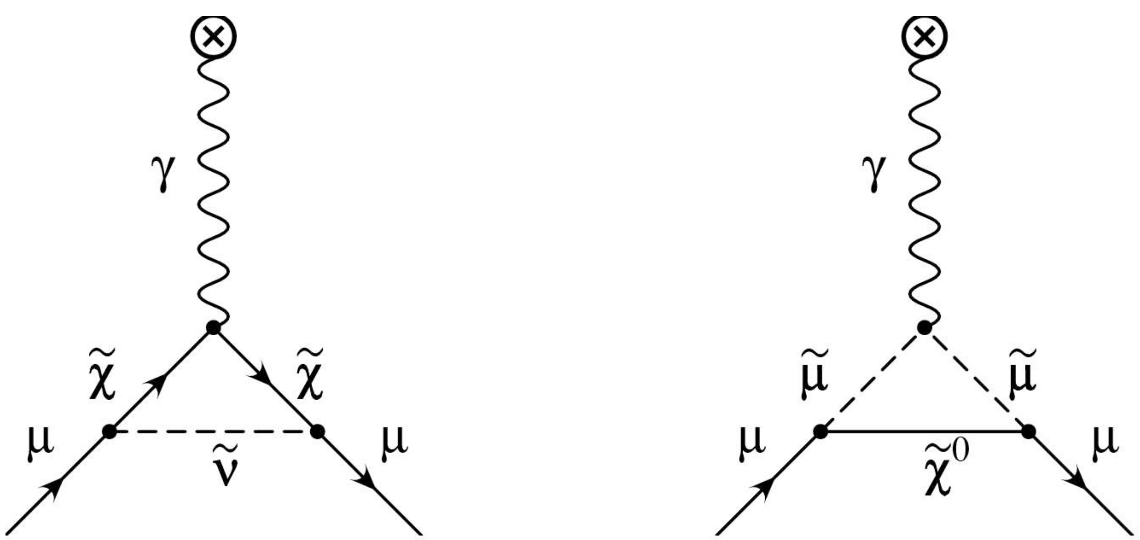


~60 x 10^{-6} of QED

weak

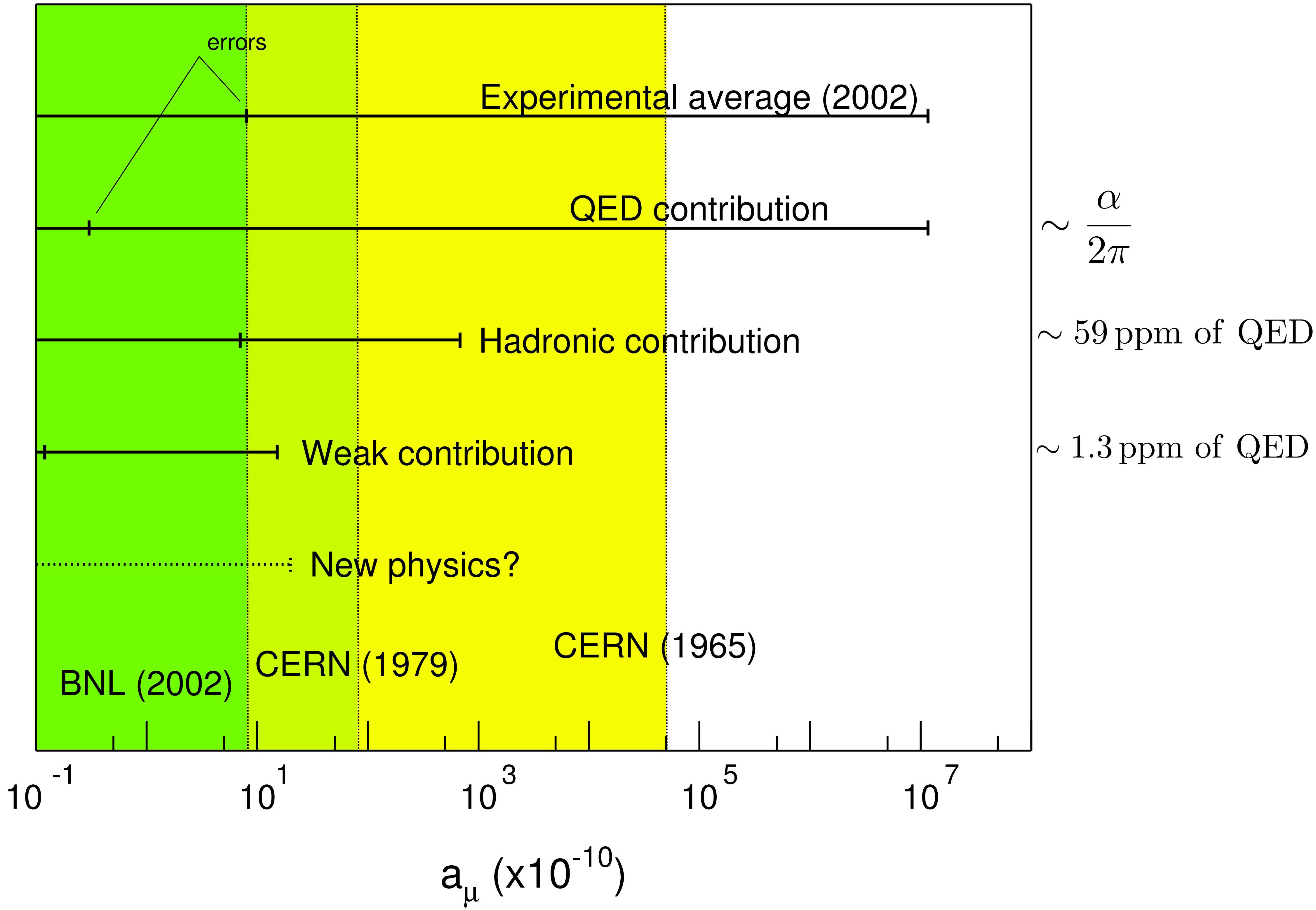


~1 x 10^{-6} of QED



possibly as large as ~ 1 x 10^{-6} of QED

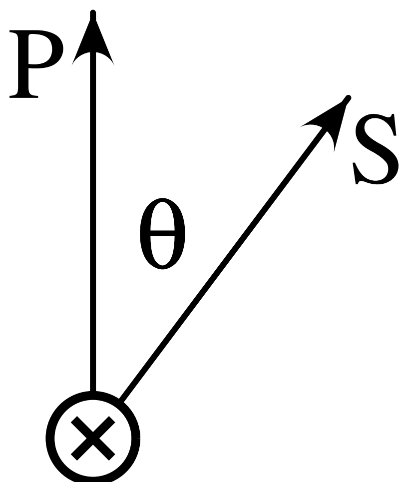
Or other speculative physics...



We have measured a_{μ^+} of the positive muon to within 0.7 ppm uncertainty (mostly statistical!); twice smaller than the Weak contribution and comparable in size to the uncertainty in theory evaluations.

Experiment - Technique

Store longitudinally polarized muons in a magnetic dipole field B,



$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}t} = \frac{e}{mc}\vec{S} \times \left(\left[a + \frac{1}{\gamma}\right]\vec{B} - a\left[\frac{\gamma}{\gamma+1}\vec{\beta}\cdot\vec{B}\right]\vec{\beta} - \left[a - \frac{1}{1+\gamma}\right]\vec{\beta} \times \vec{E}\right)$$

Measure the field and the difference of the spin precession and momentum rotation frequencies,

$$\omega_a = \omega_s - \omega_c = \frac{\mathrm{d}\theta}{\mathrm{d}t} = a_\mu \frac{e}{m_\mu c} B + \mathcal{O}(0.7\,\mathrm{ppm} \pm 5\%) \text{ for } \gamma = 29.3$$

+ e.d.m. (negligbly small in SM)

The field B is measured as the proton NMR frequency ω_{p}, so that:

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

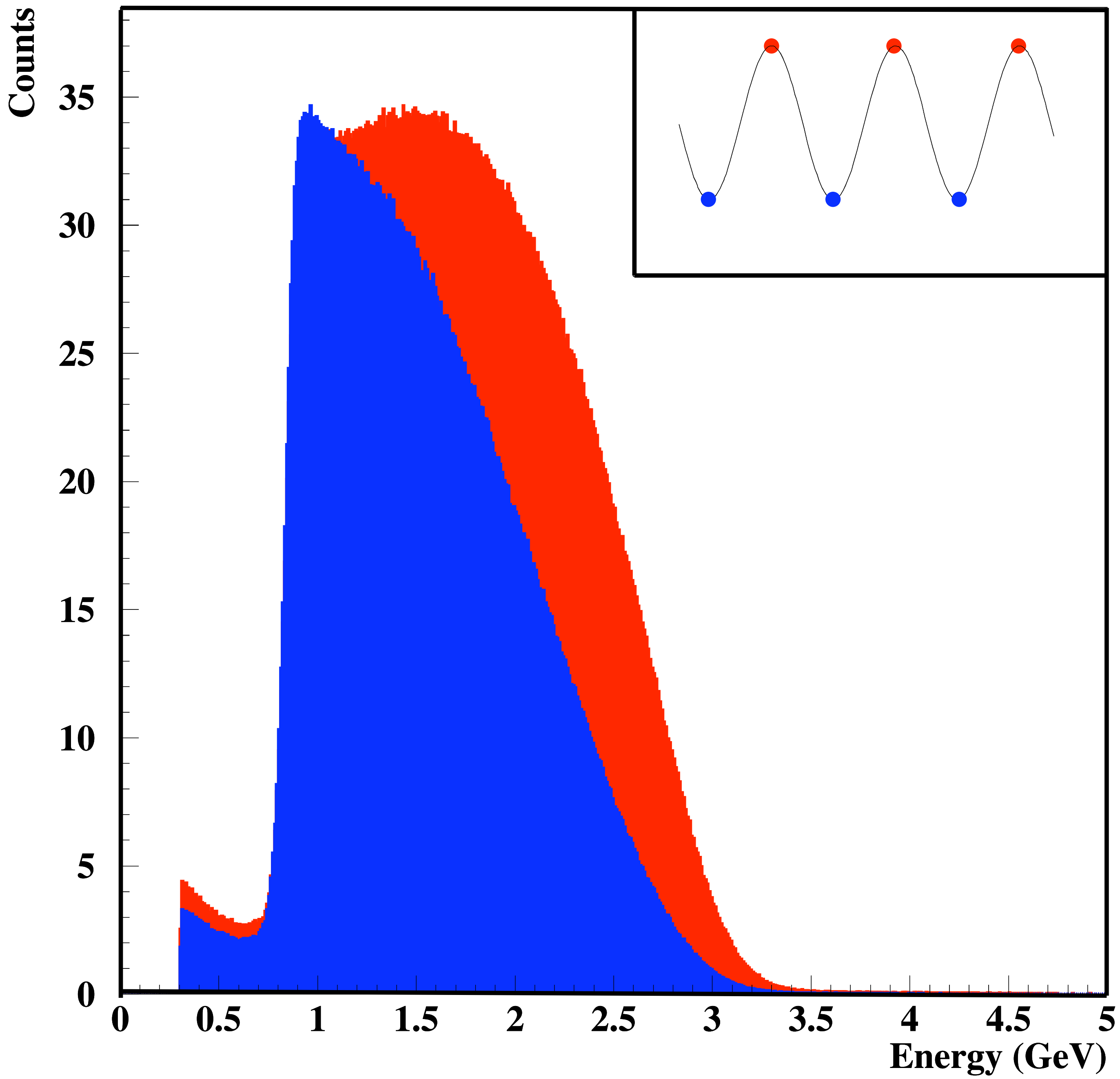
The frequency ω_a is observed via the *weak* decay,

$$\vec{\mu} \longrightarrow e \, \nu \, \nu$$

by counting decay electrons (positrons),

$$N_e(t) = N_0 \exp(-\frac{t}{\gamma\tau}) \, [1 + A \cos(\omega_a t + \phi)]$$

above an energy threshold *E*.



In the decay $\vec{\mu} \rightarrow e\,\nu\,\nu$, highly energetic electrons (positrons) are emitted preferentially along the muon spin direction.

The muon storage ring at Brookhaven National Laboratory:

$$p_\mu = 3.1\,\mathrm{GeV/c}, \quad B = 1.45\,\mathrm{T}, \quad r = 7.1\,\mathrm{m}$$

Time scales:

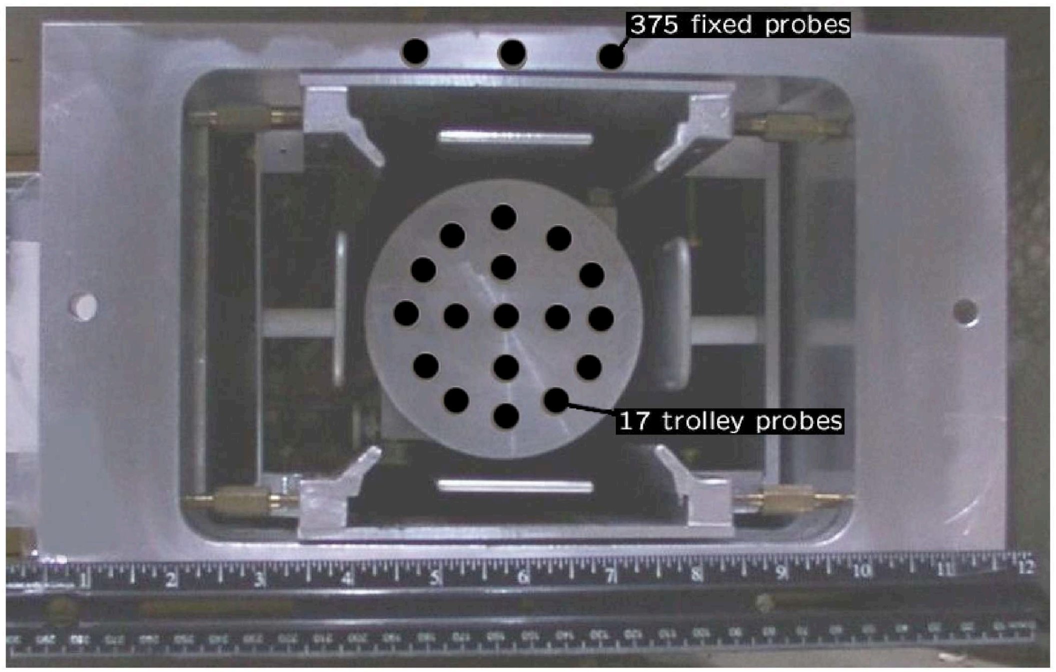
150 ns	cyclotron period τ_c
4.4 μs	τ_a
64 μs	dilated muon lifetime $\gamma\tau$

Experimental sequence:

$t = 0$	beam injection
35 − 500 ns	kick onto orbit
0 − 15 μs	beam scraping (collim
5 − 40 μs	calorimeters gate on
45 − 1000 μs	$g-2$ measurement
1000 − μs	data storage to tape
33 ms	cycle repeats

Inflector magnet	A. Yamamoto et al., NIM **A491** 23 (2002)
Ring magnet	G.T. Danby et al., NIM **A457** 151 (2001)
Kicker magnet	E. Efstathiadis et al., NIM **A496** 8 (2003)
Quadrupoles	Y.K. Semertzidis et al., NIM **A503** 458 (2003)
NMR system	R. Prigl et al., NIM **A394** 349 (1997)
Calorimeters	S. Sedykh et al., NIM **A455** 346 (2000)

Experiment - Field Measurement

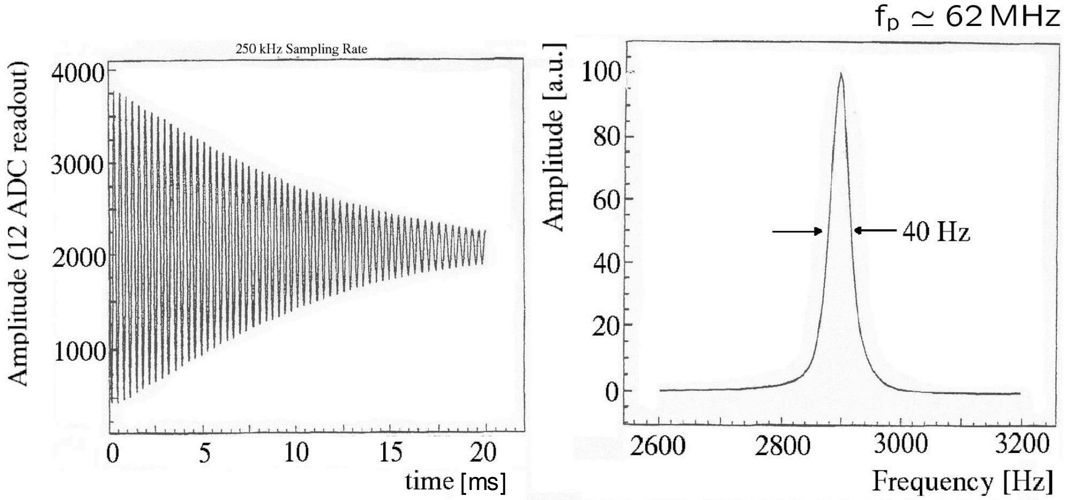


The field values along the muon trajectory are *measured* several times per week with 17 NMR probes mounted on a trolley.

The field is *tracked* continuously with ~160 out of 375 NMR probes in the top and bottom walls of the vacuum chamber.

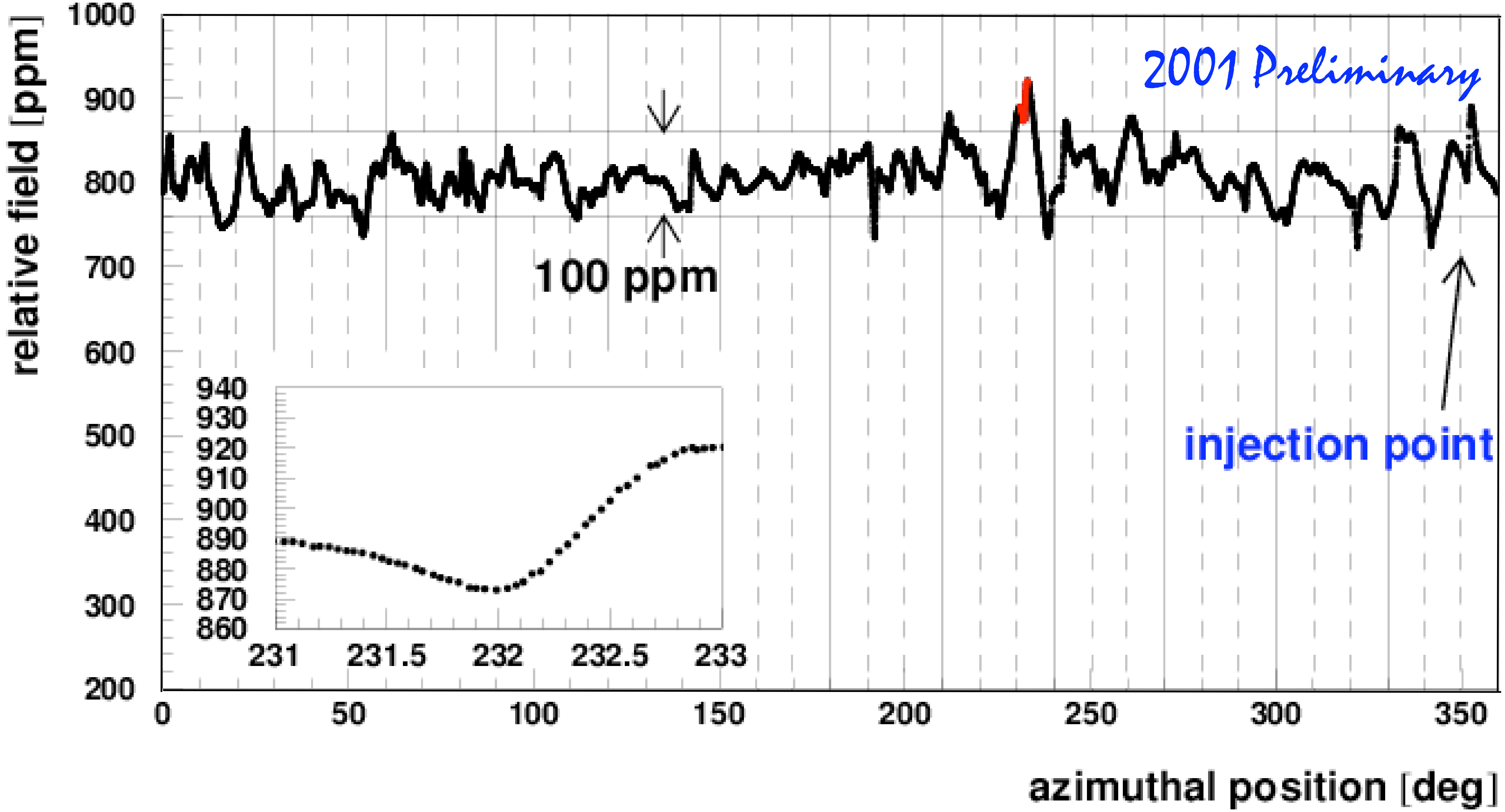
Free induction decay signals:

$f_p \simeq 62\,\mathrm{MHz}$

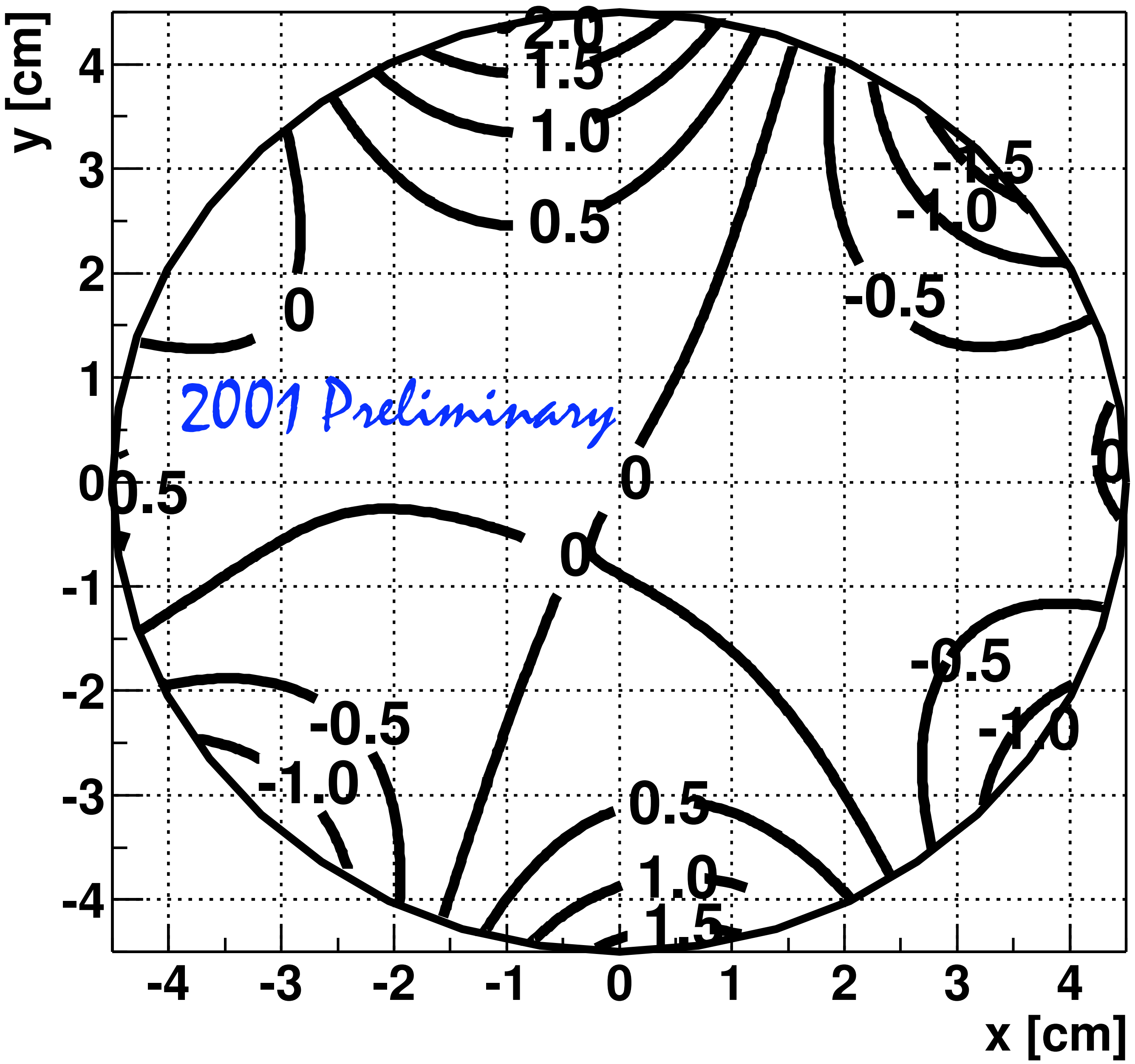


The system is *calibrated in situ* against a standard* before and after data taking with beam

* X. Fei, V.W. Hughes, R. Prigl NIM A394 349 (1997)



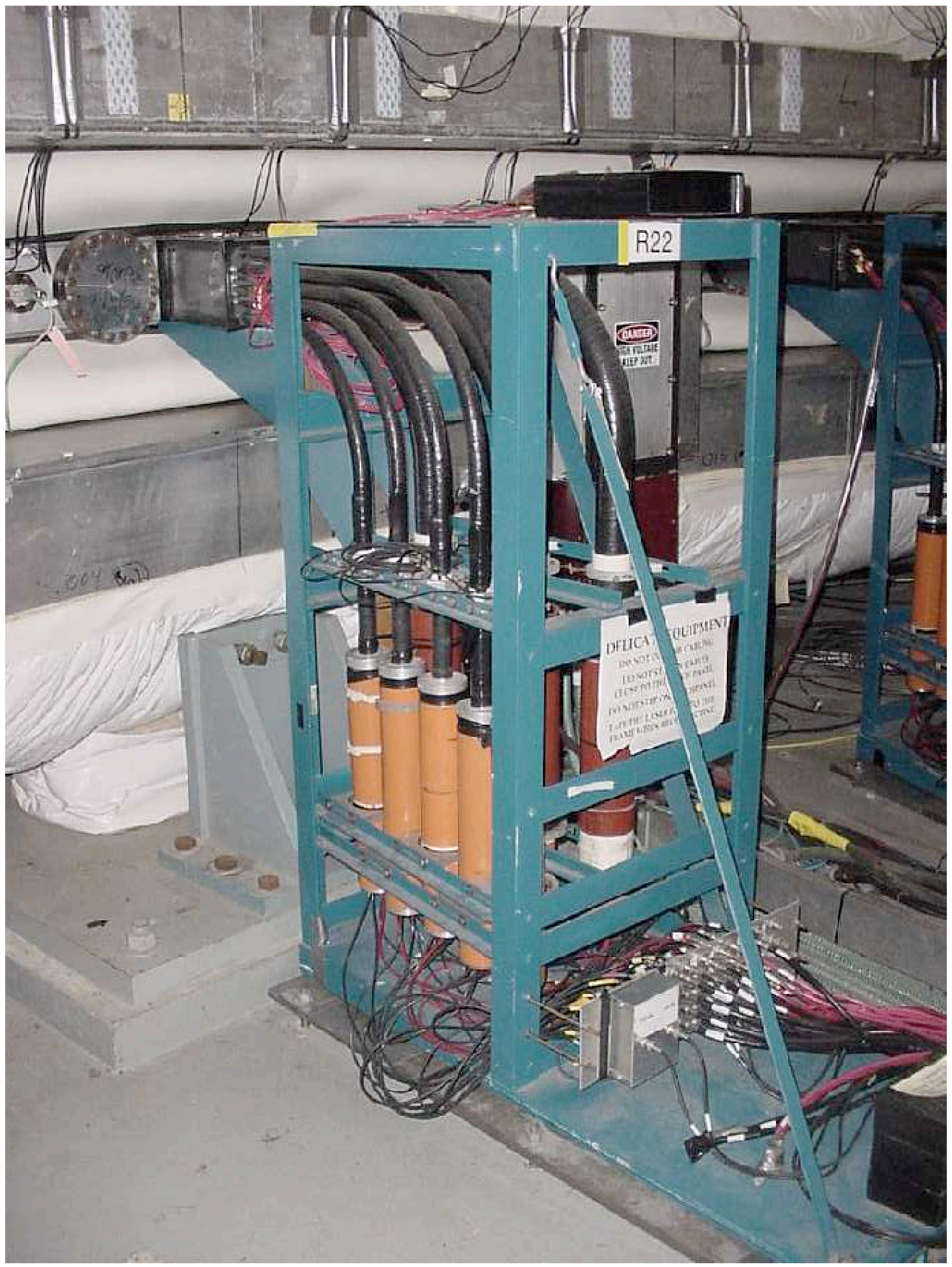
Recall that the cyclotron period (~150ns) is ~430 times shorter than the dilated muon lifetime (~64us).



Multipoles (ppm)

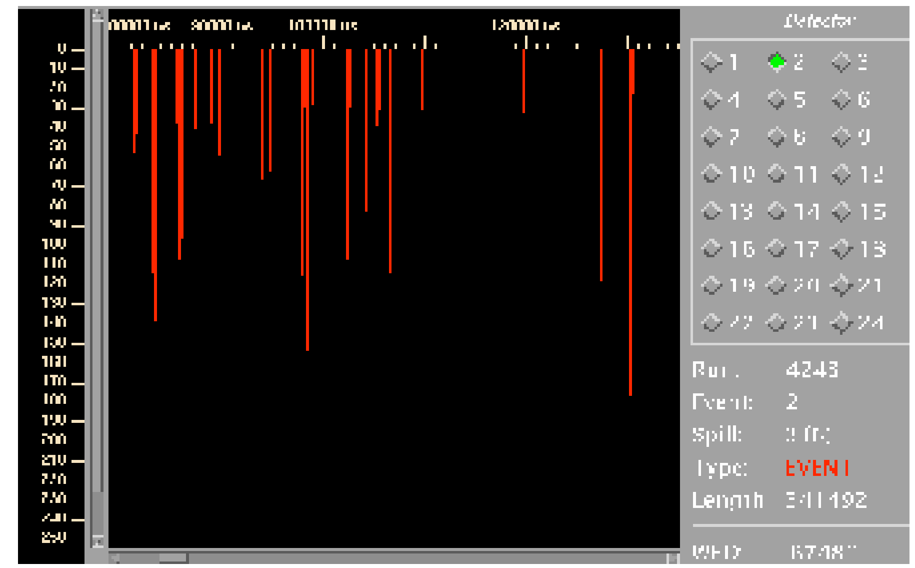
	normal	skew
Quad	-0.29	0.15
Sext	-0.71	-0.48
Octu	0.06	0.01
Decu	1.08	0.39

Experiment - Calorimeters

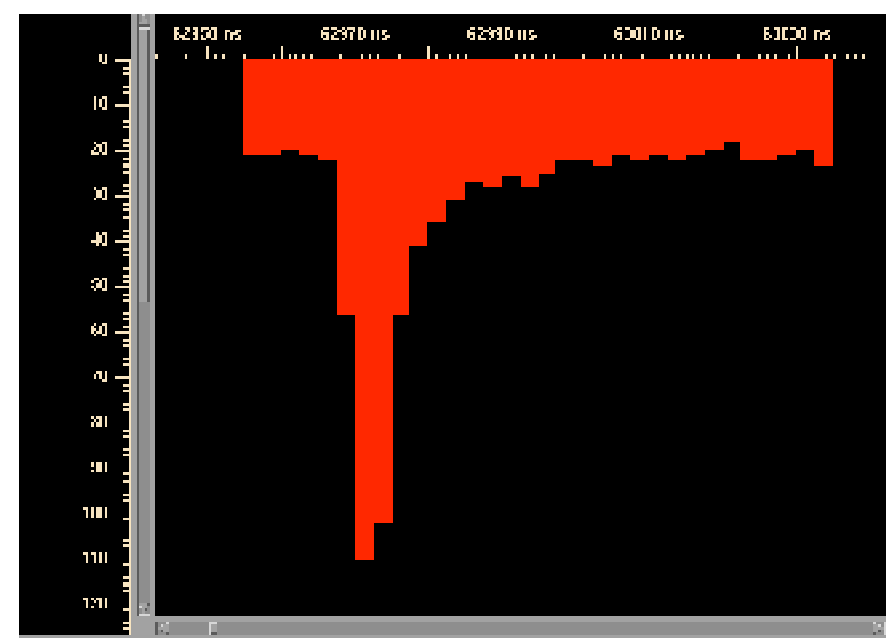


One of 24 Pb/scintillating-fiber calorimeters on the inner side of the storage ring.

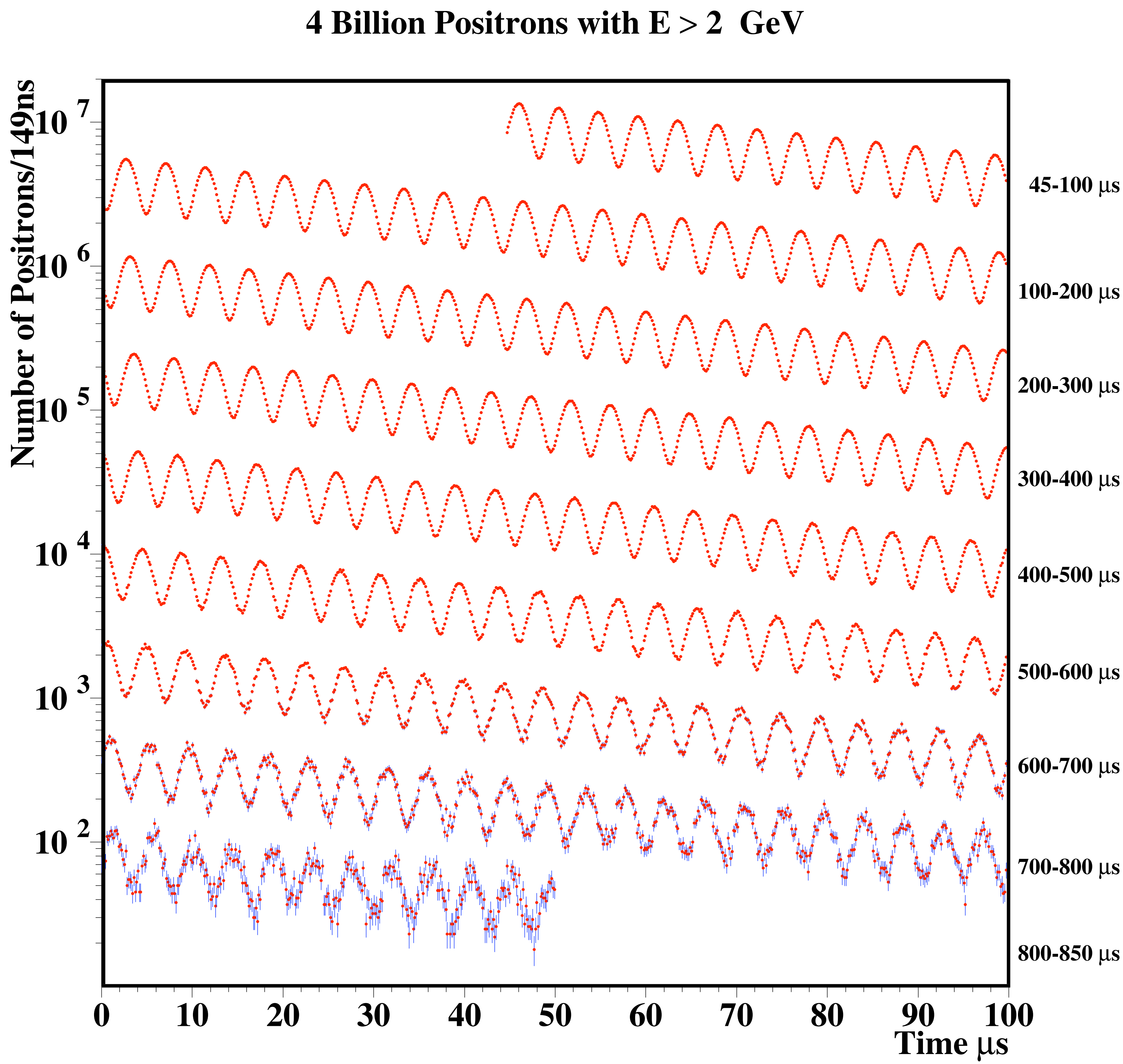
55 μs



80 ns



2000 Data Sample and Analysis



The leading behavior,

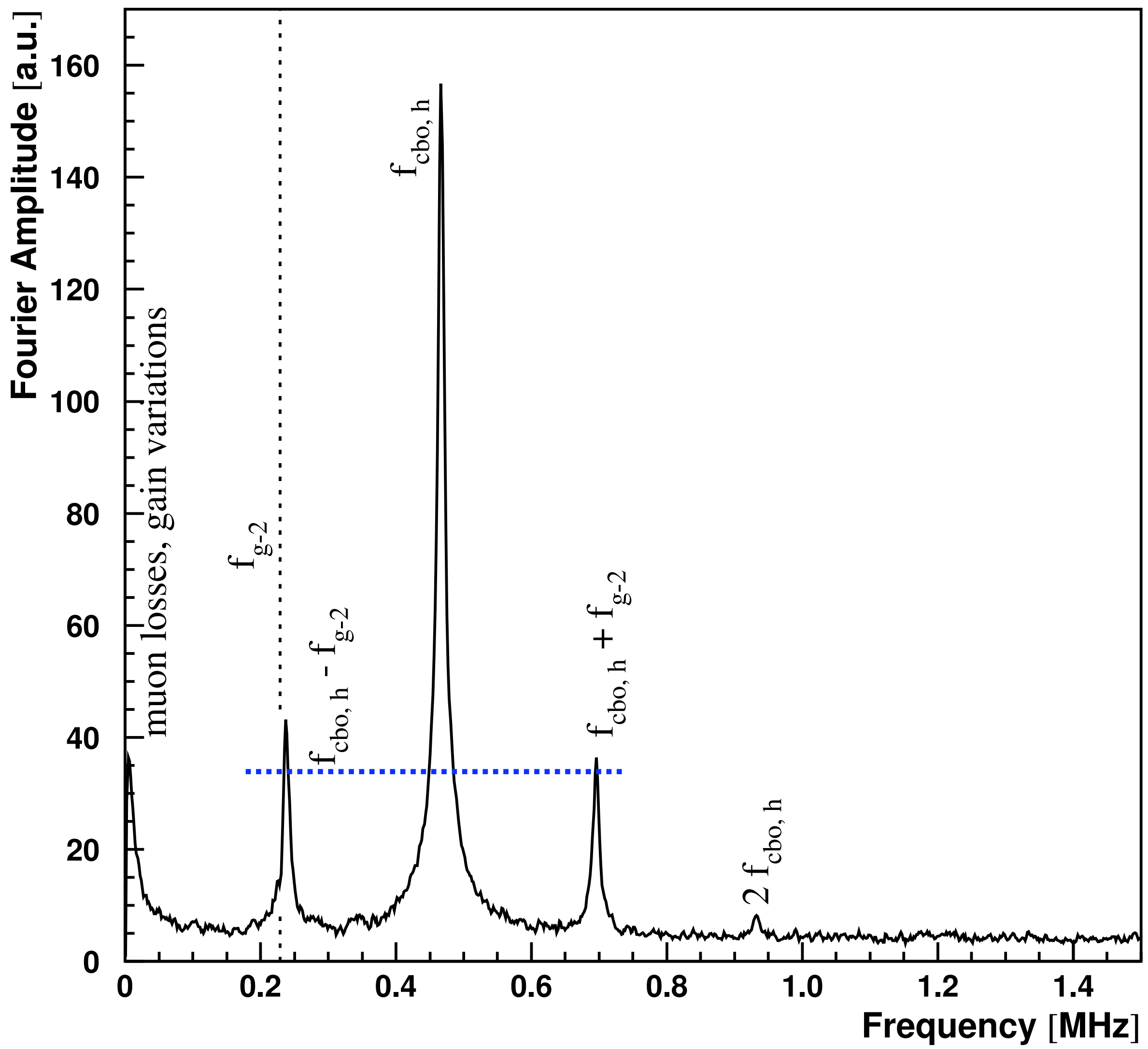
$$N_e(t) = N_0 \exp(-\frac{t}{\gamma\tau})\,[1 + A\cos(\omega_a t + \phi)]$$

is a *partial* description of the data.

Specifically, the very high statistics sample requires careful consideration of:

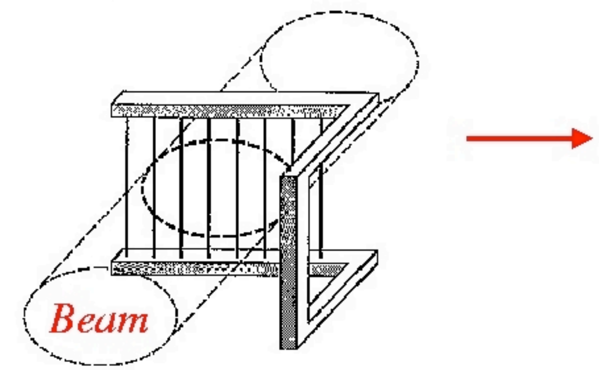
- Coherent Betatron Oscillations
- Muon losses (muon loss detectors)
- Detector gain and time stability (UV laser)

A Fourier transform of the positron time spectrum,

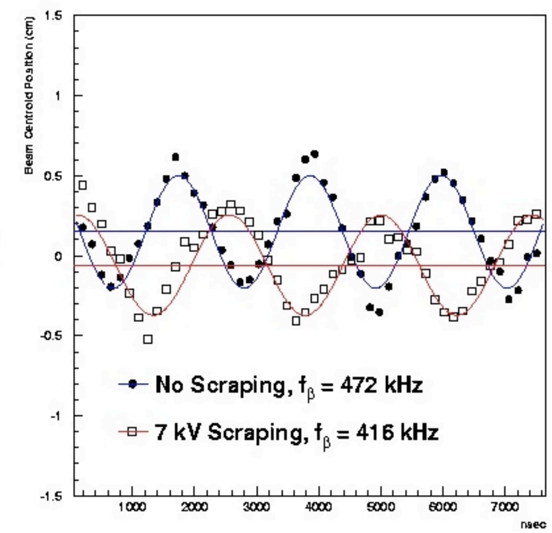


Direct measurement:

Scintillating fiber plunging beam monitor:
- 7 x 0.5 mm thick fibers
- 13 mm spacing



Direct observation of CBO



$$\omega_{CBO} = \omega_c (1 - \sqrt{1-n})$$

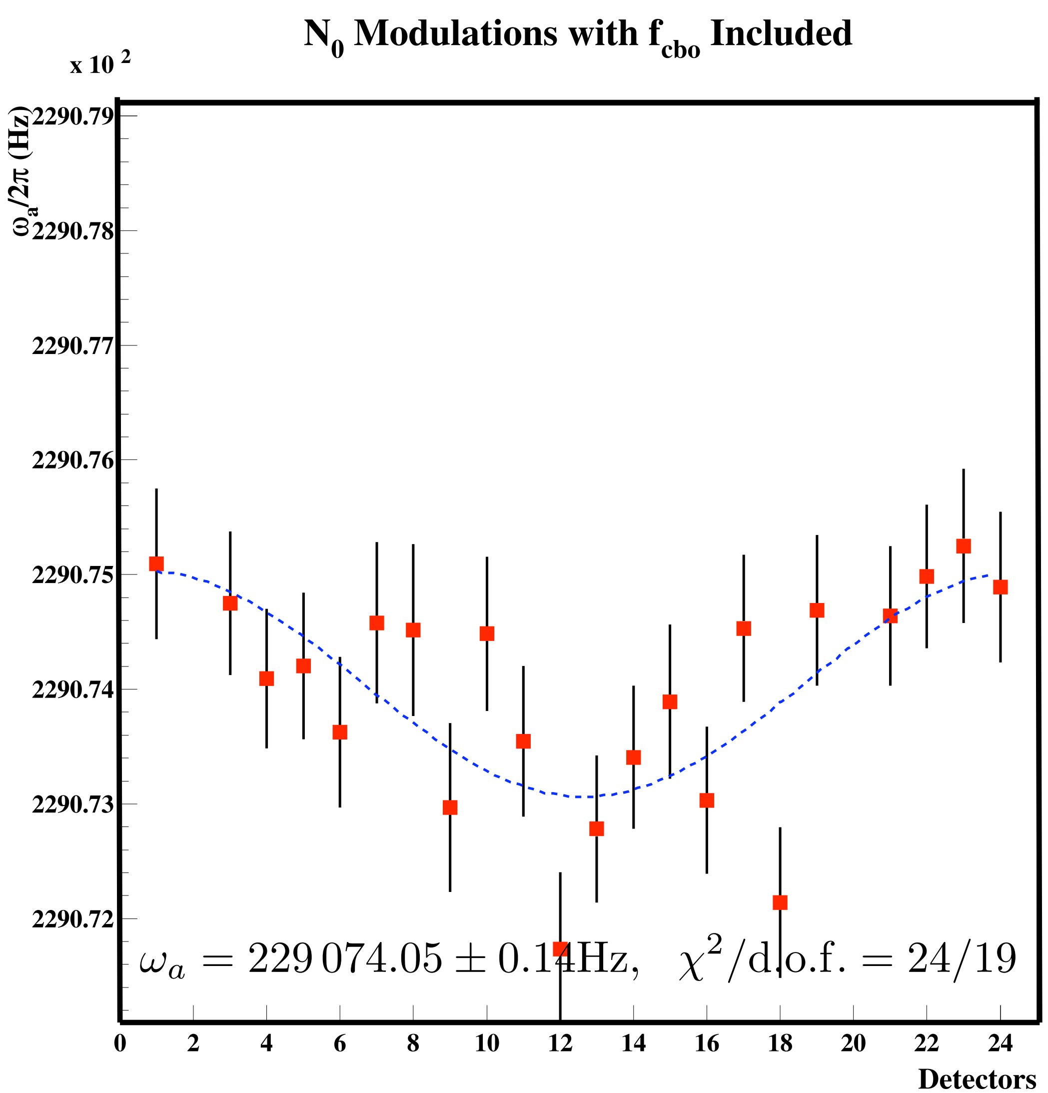
indicates significant modulation of the number *and energy* distributions
~ $1 + \mathcal{A} \sin(\omega_{\rm cbo} t + \phi_{\rm cbo})$

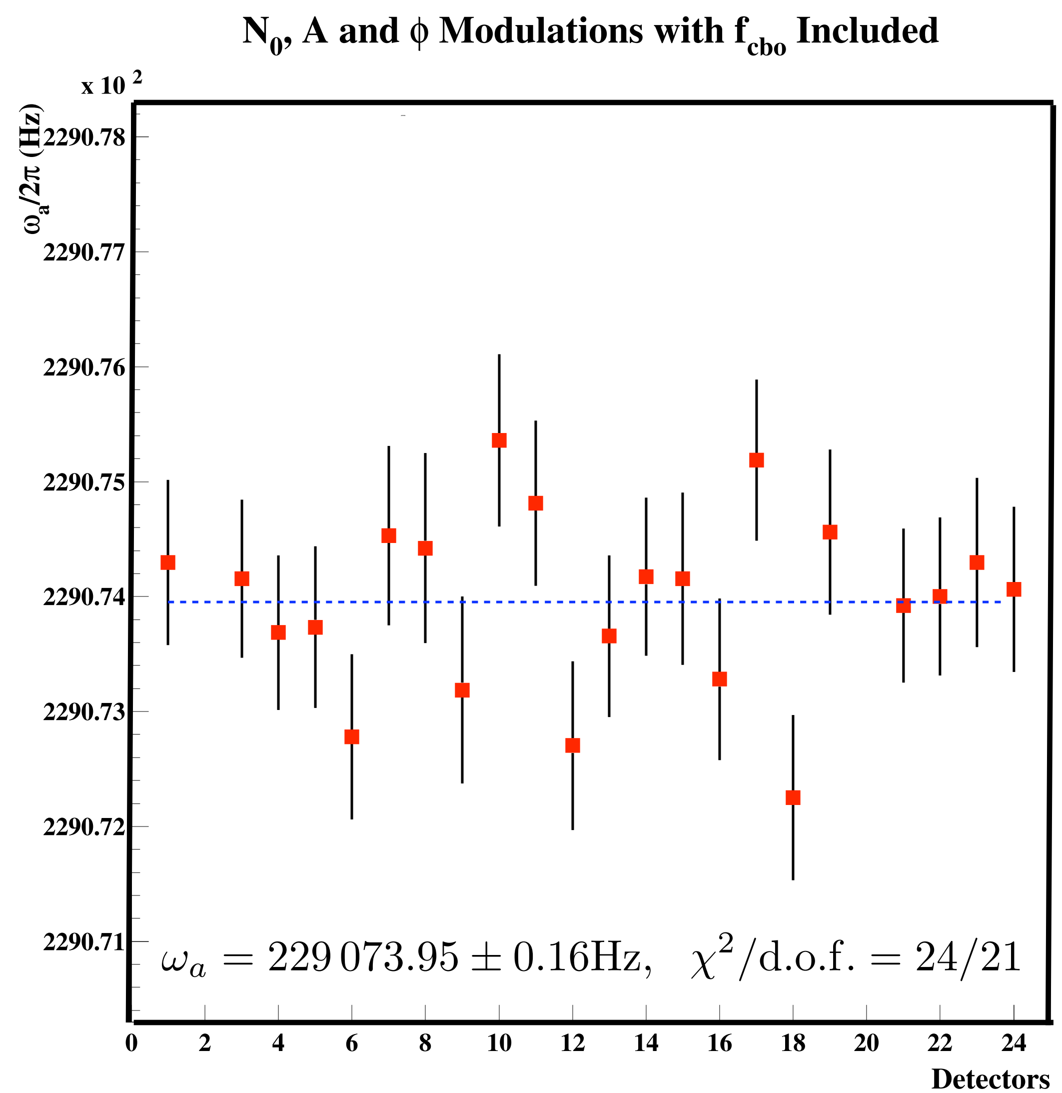
Hence, $N_0(E), A(E),$ and $\phi(E)$ in

$$N_{\rm observed}(t) = N_0(E) \exp(-\frac{t}{\gamma\tau}) \left[1 + A(E) \sin(\omega_a t + \phi(E))\right]$$

vary with t. *This is a considerable complication of the analysis*

The proximity of 2 x ω_a and ω_{cbo} causes a relatively large artificial shift (bias) in the fitted frequency ω_a from individual calorimeters if the modulation of $A(E)$ and/or $\phi(E)$ are not accounted for in the fitting function.





This bias cancels by an order of magnitude in the summed detector spectra (established in independent ways).

Four analyses to determine ω_a have been performed,

- Averaged result from fits to the data from individual calorimeters in energy intervals. The fit function incorporates number, asymmetry, and phase modulation. The fit start-times are chosen so that there is no apparent need to incorporate e.g. detector gain effects.

- A fit to the summed calorimeter time spectrum for energies larger than 2 GeV and a start-time of 50μs. The fitting function incorporates the leading modulations (number and asymmetry) and uses an empirical description of detector gain effects.

- As above, using different methods to determine systematic effects.

- A fit to the ratio,

$$r(t) = \frac{n_1(t-\tau_a/2) + n_2(t+\tau/2) - n_3(t) - n_4(t)}{n_1(t-\tau_a/2) + n_2(t+\tau/2) + n_3(t) + n_4(t)}$$

  in which $n_1 — n_4$ are formed by randomly splitting the summed calorimeter time spectrum (for energies larger than 2 GeV and a start-time of 50μs).

After the 4 analyses of ω_a had been completed,

$$\omega_a/(2\pi) = 229\,074.11\,(14)(7)\,\mathrm{Hz}\ \ (0.7\,\mathrm{ppm}),$$

After the 2 analyses of ω_p had been completed,

$$\omega_p/(2\pi) = 61\,791\,595\,(15)\,\mathrm{Hz}\ \ (0.2\,\mathrm{ppm}),$$

separately and independently, the anomalous magnetic moment was evaluated,

$$a_\mu = \frac{R}{\lambda - R} = 11\,659\,204\,(7)(5) \times 10^{-10} \quad (0.7\,\mathrm{ppm})$$

where $R = \dfrac{\omega_a}{\omega_p}$ and $\lambda = \dfrac{\mu_\mu}{\mu_p} = 3.18\,334\,539\,(10)$[1]

[1] W. Liu et al., PDG

TABLE I: Uncertainties for the ω_a analysis.

Source of errors	Size [ppm]	
	2000	1999
Coherent betatron oscillations	0.21	0.05
Pileup	0.13	0.13
Gain changes	0.13	0.02
Lost muons	0.10	0.10
Binning and fitting procedure	0.06	0.07
AGS background		0.10
Others†	0.06	
Total systematic error on ω_a	0.31	0.3
Statistical error on ω_a	0.62	1.3

† Timing shifts, E field and vertical oscillations, beam debunching/randomization.

TABLE II: Uncertainties for the ω_p analysis

Source of errors	Size [ppm]	
	2000	1999
Absolute calibration of standard probe	0.05	0.05
Calibration of trolley probe	0.15	0.20
Trolley measurements of B_0	0.10	0.10
Interpolation with fixed probes	0.10	0.15
Inflector fringe field	–	0.20
Uncertainty from muon distribution	0.03	0.12
Others†	0.10	0.15
Total systematic error on ω_p	0.24	0.4

† higher multipoles, trolley temperature and voltage response, eddy currents from the kickers, and time-varying stray fields.

Theory - Status

The Standard Model prediction is evaluated as

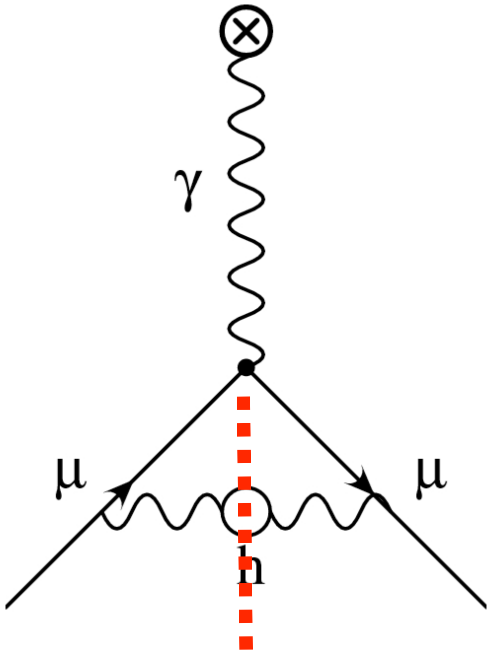
$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{had}) + a_\mu(\mathrm{weak})$$

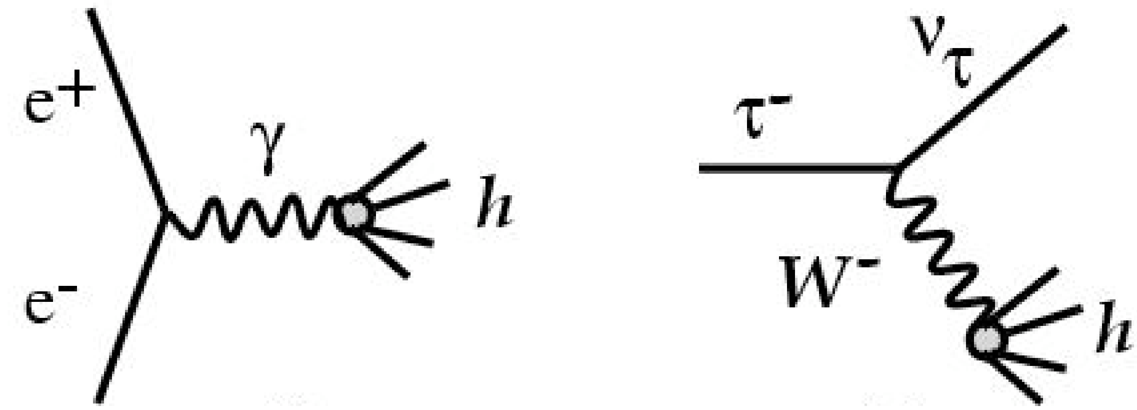
in which the QED and weak contributions have been calculated to high accuracy,

$$a_\mu(\mathrm{QED}) = 11\,658\,470.6(3) \times 10^{-10}\ (0.03\,\mathrm{ppm})$$
$$a_\mu(\mathrm{weak}) = 15.1(4) \times 10^{-10}\ (0.03\,\mathrm{ppm})$$

The hadronic contribution is *by far* less certain (~0.6 ppm).

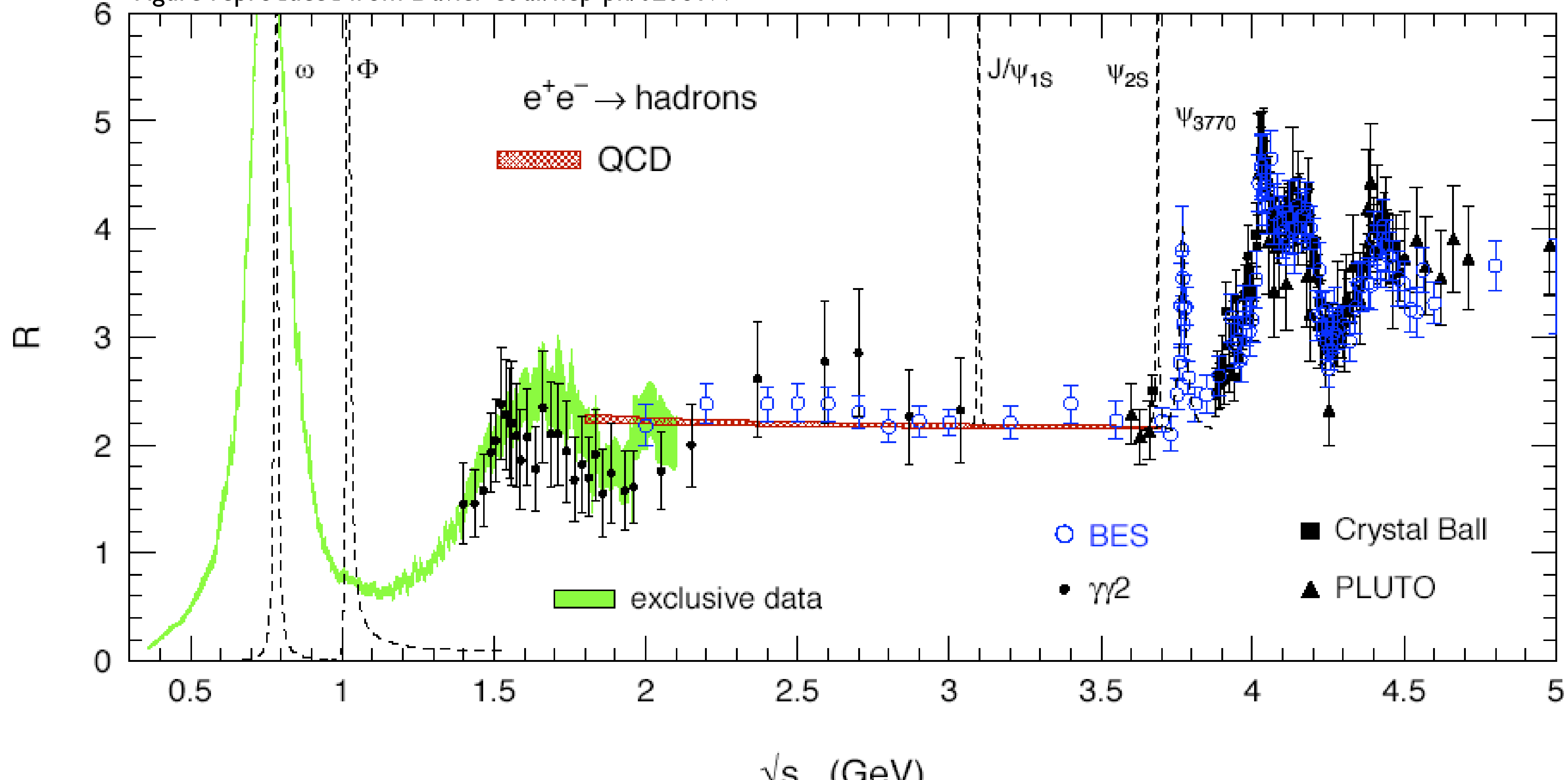
In lowest order it is evaluated with input, mostly from e^+e^- collision and τ^- decay *data* at low c.m. energies.





$$a_\mu(\text{had; l.o.}) \propto \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$$

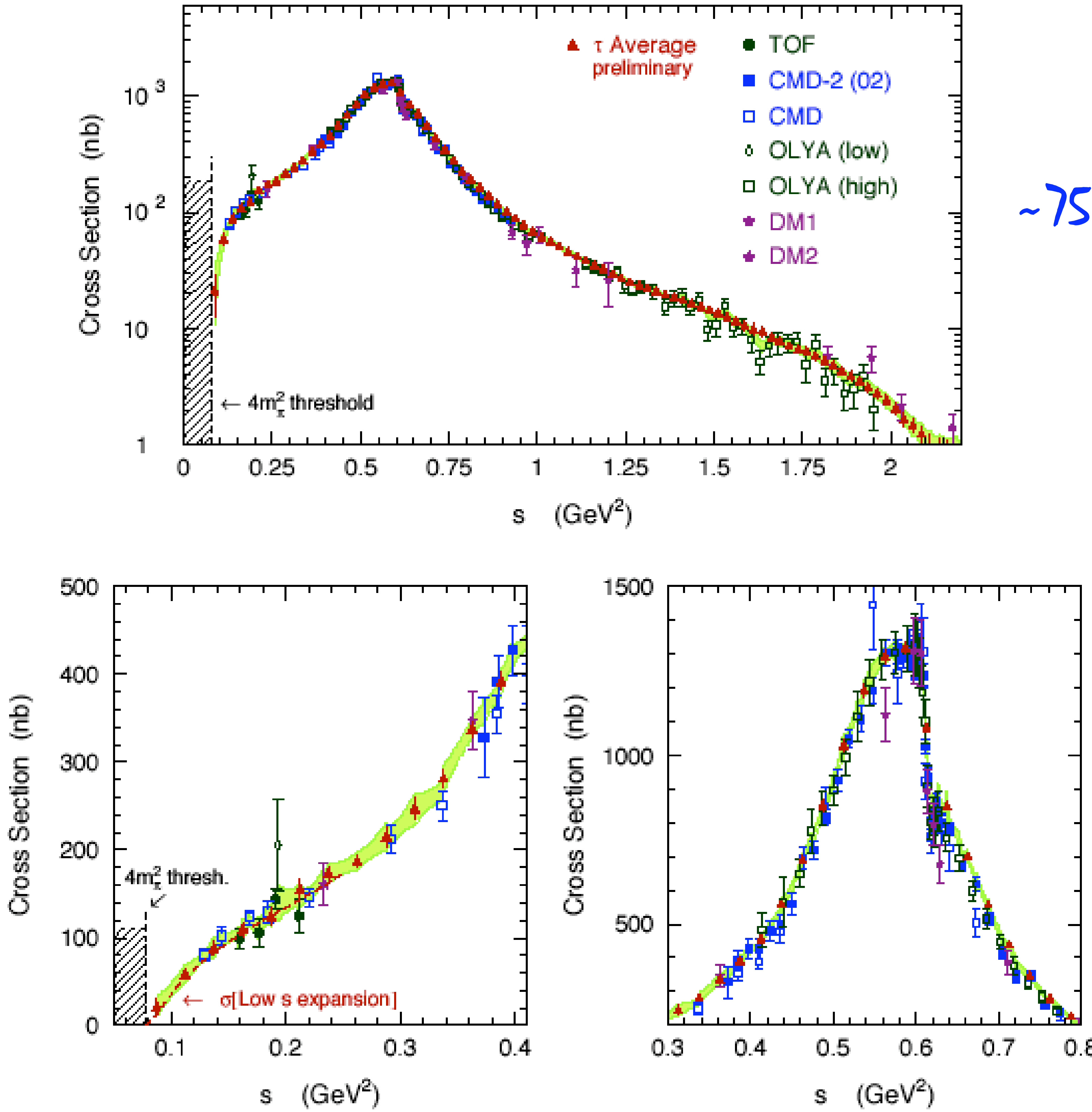
Figure reproduced from Davier et al. hep-ph/0208177



It is largely an experimental question, weighted towards lower energies...

The recent analysis by Davier et al includes the most precise data to date:

Figures reproduced from Davier et al. hep-ph/0208177

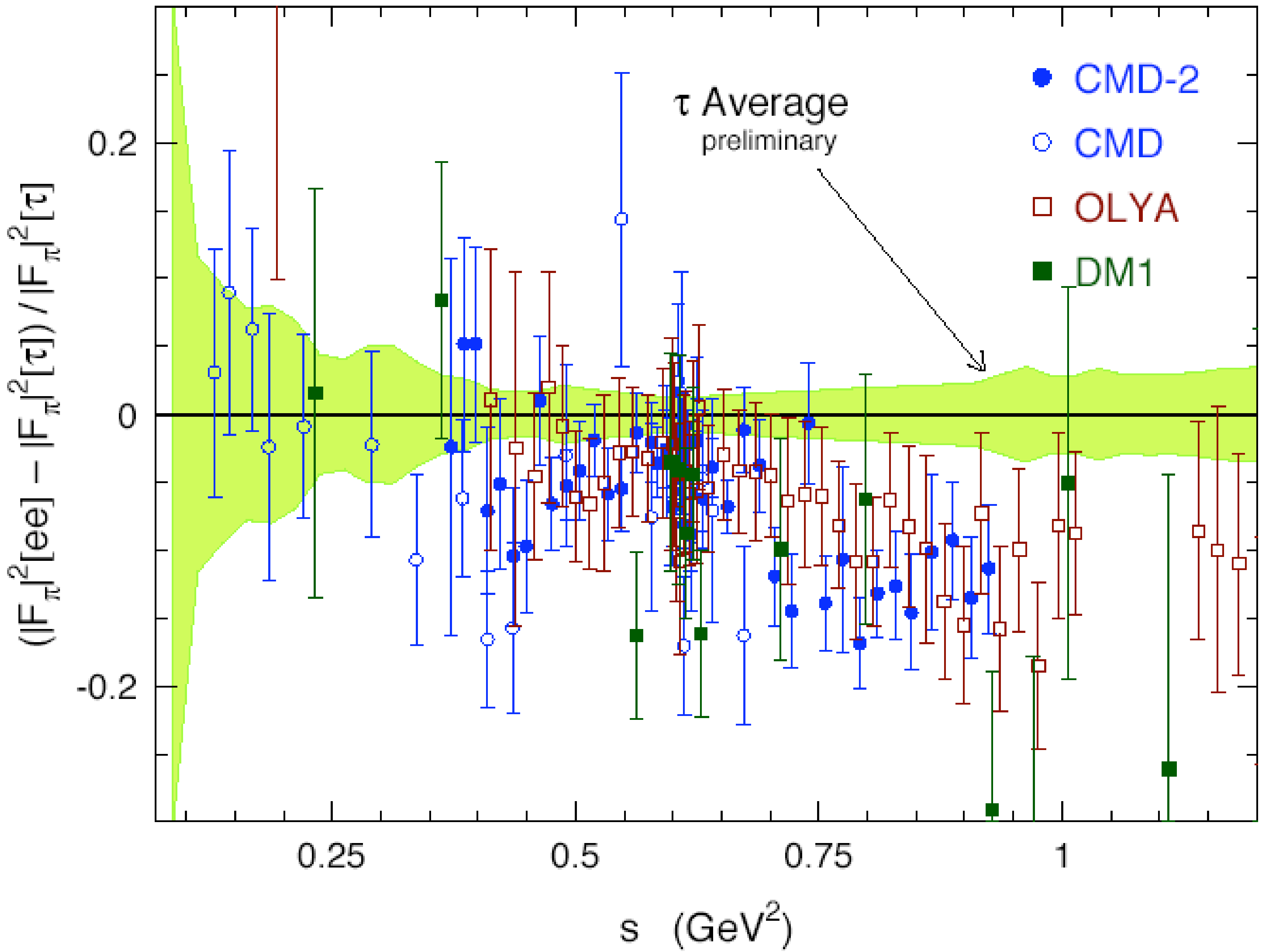


~75% of a_μ(had)

At first sight, the agreement is strikingly good. We need it to 1% however...

Relative comparison of the $\pi^+\pi^-$ spectral functions from e^+e^- data and from isospin-breaking corrected τ data.

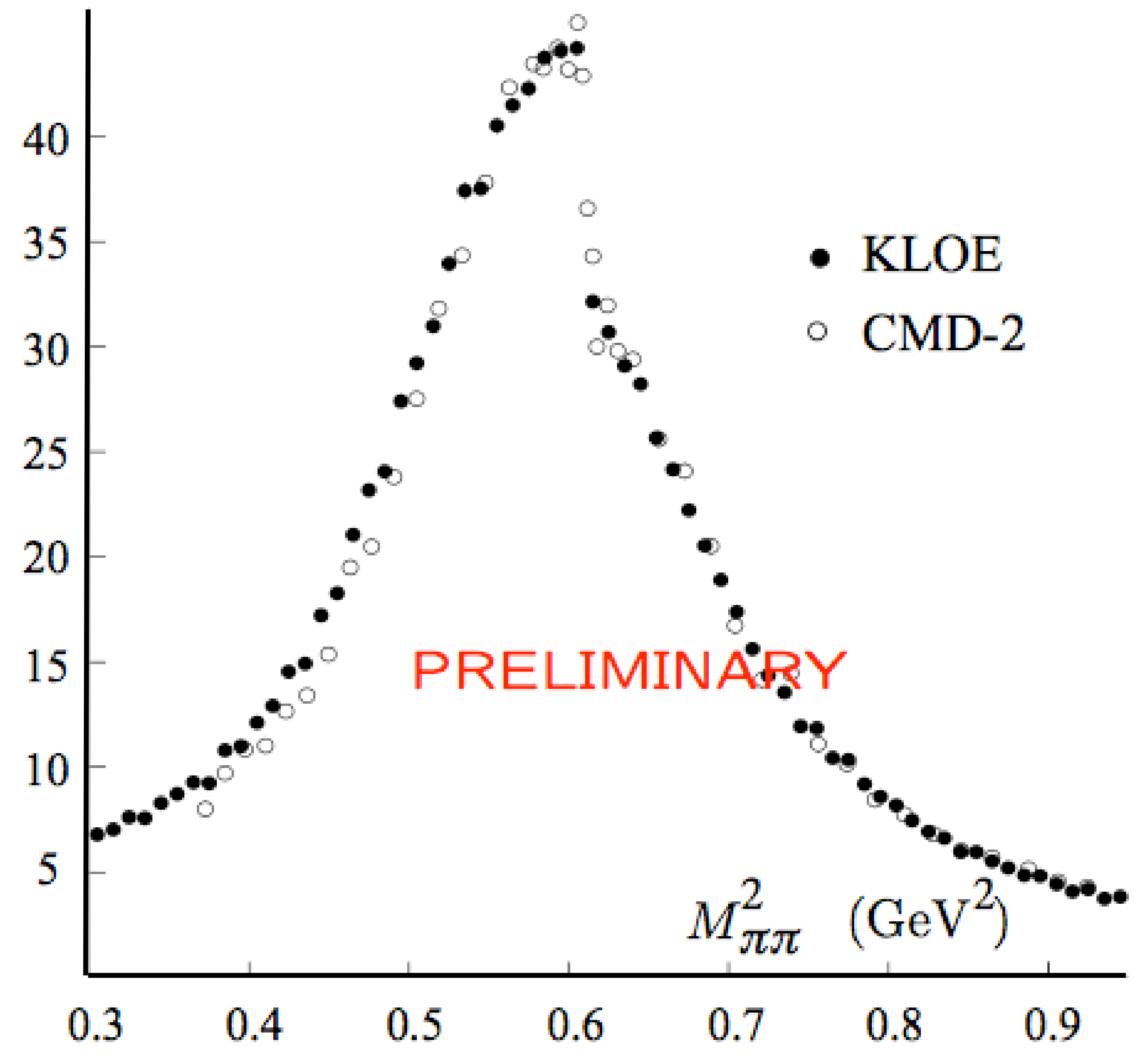
Figure reproduced from Davier et al. hep-ph/0208177



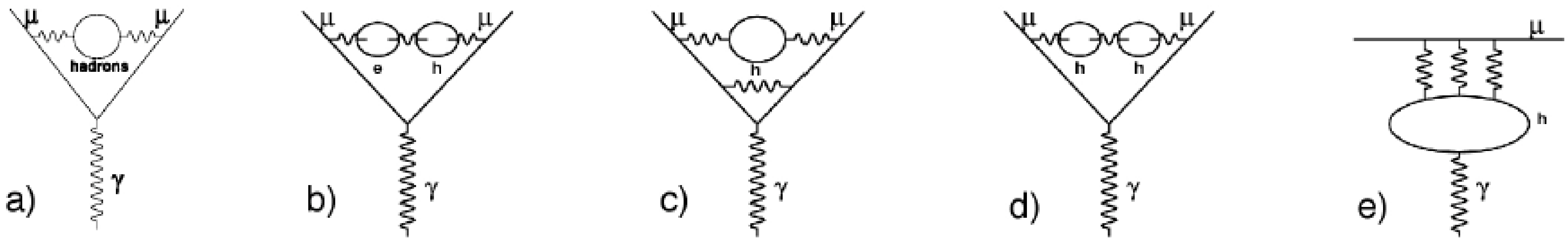
Davier et al: e^+e^- and τ data cannot presently be combined.

Radiative return data from e^+e^- facilities may help resolve the discrepancy.
E.g. KLOE at Frascati (J. Lee-Franzini at Lepton Moments, June 2003):

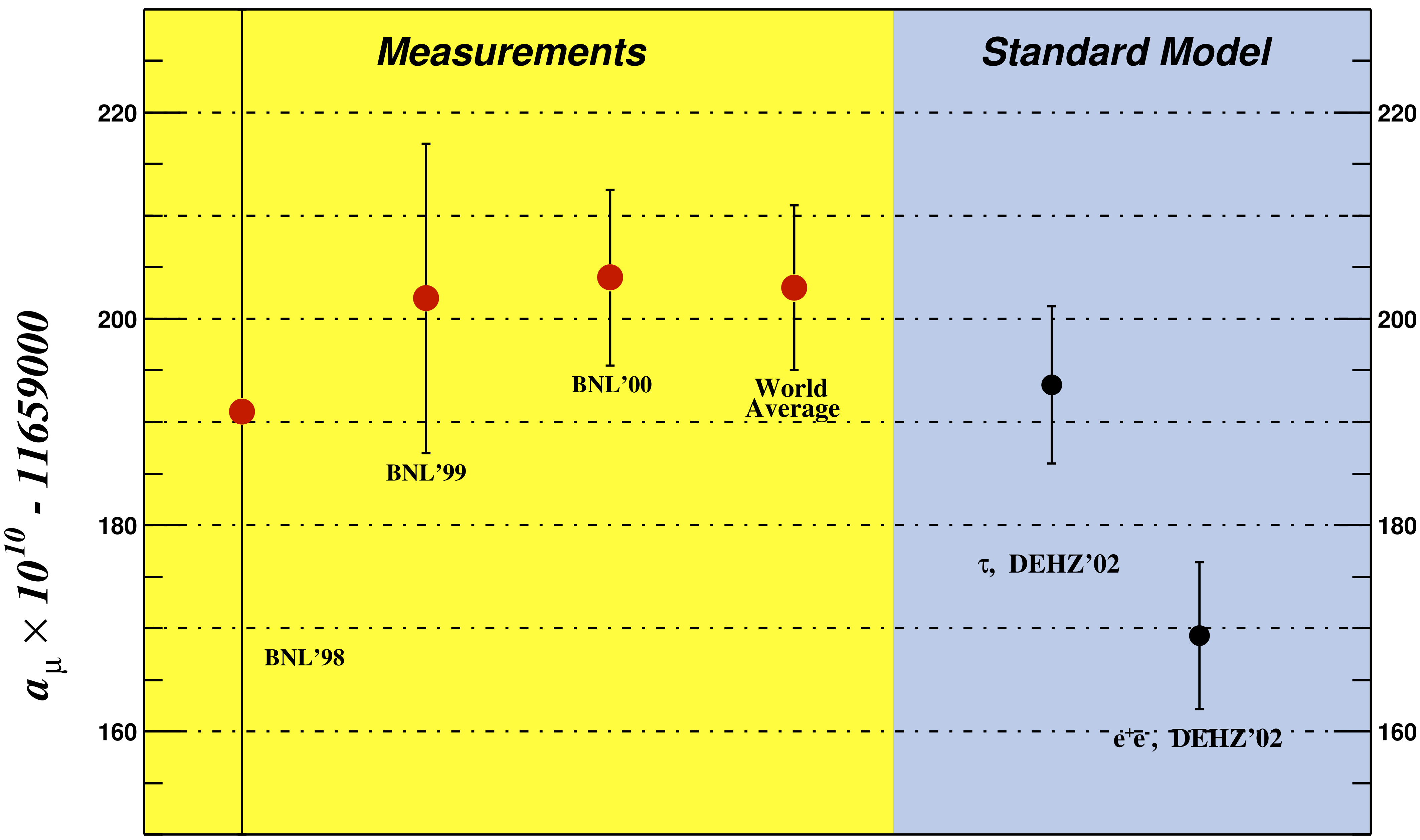
The KLOE $e^+e^- \to \pi^+\pi^-$ cross section extracted from the measured $\pi^+\pi^-\gamma$ cross section above, compared with CMD-2 results.



Also, the higher order hadronic contributions,



continue to be scrutinized. Numerically most significant a *mistake* of sign was discovered in the contribution from hadronic light-by-light scattering, which resulted in a ~1.3 ppm shift (stated uncertainty ~0.4 ppm). M. Knecht et al., Phys. Rev. **D65**, 073034, 2002.



References:

BNL'98 PRL 86 2227
BNL'99 PR 62D 091101
BNL'00 PRL 89 101804

τ, DEHZ'02 a_μ(had;1) from hep-ph/0208177
e^+e^-, DEHZ'02 a_μ(had;1) from hep-ph/0208177

Concluding remarks

Our present measurement and data:

- confirm previously measured values,
- form the most precise determination of a_{μ^+} of the positive muon to date (0.7 ppm)
- have an uncertainty that is mostly statistical and has a size about half the size of the weak contribution to $a_\mu(\mathrm{SM})$

The Standard Model evaluation:

- has improved by about an order of magnitude in accuracy since the last CERN experiment (1979),

- has shifted significantly in the past year because of a corrected sign in the evaluation of hadronic light-by-light scattering (Knecht et al.),

- agrees/differs from the experimental value by 0 to 3 times the combined experimental and *stated* theoretical uncertainty, depending if e^+e^- or τ spectral functions are used to evaluate the contribution from hadronic vacuum polarization (Davier et al.),

- may benefit from *final* τ data, re-examination of the e^+e^- normalization, radiative return measurements (e.g. KLOE, BaBar), and possibly lattice QCD calculation (T. Blum).

Future:

- Analysis of our last data set - on the *negative* muon (our first) collected in 2001 - is well underway; we expect completion with an event sample of 3 billion analyzed electrons and further reduced systematics. For now, the collaboration continues the study of systematics in the muon precession frequency.

- Ongoing analysis of out-of-plane precession *may* result in a somewhat sharper direct limit on the muon EDM; a follow up experiment is being proposed.