

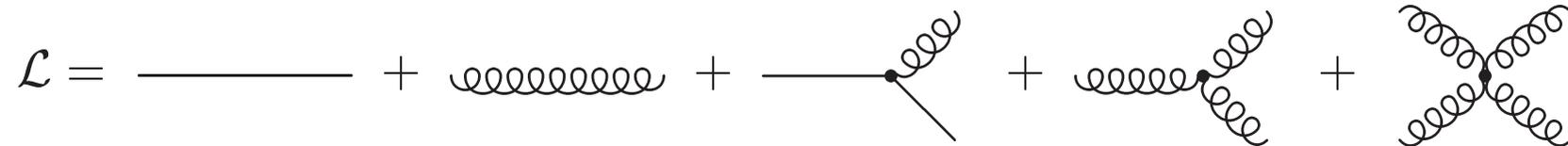
# QCD at $e^+e^-$ Experiments

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# Setting the Scene – The QCD Lagrangian

QCD Lagrangian – determined from  $SU(3)_c$  invariance

$$\mathcal{L} = \text{---} + \text{~~~~~} + \text{---} \cdot \text{~~~~~} + \text{~~~~~} \cdot \text{~~~~~} + \text{~~~~~} \cdot \text{~~~~~}$$


Free parameters: **coupling** and **quark masses**

Aim: measure parameters and test properties of the QCD Lagrangian

- verify basic diagrams and their relative coupling strength  
     $\iff$  QCD Colour Factors  $\iff$  verify  $SU(3)_c$  gauge group
- test quantum corrections (loops) causing asymptotic freedom & confinement  
     $\iff$  running of  $\alpha_s$  (& quark-masses),
- study rich phenomenology of strong interaction physics

# Modeling the Fragmentation Process

$e^+e^- \rightarrow q\bar{q}$ : simple initial QCD state

Perturbative QCD phase:

$$\mathcal{O}(\alpha_s^2)$$

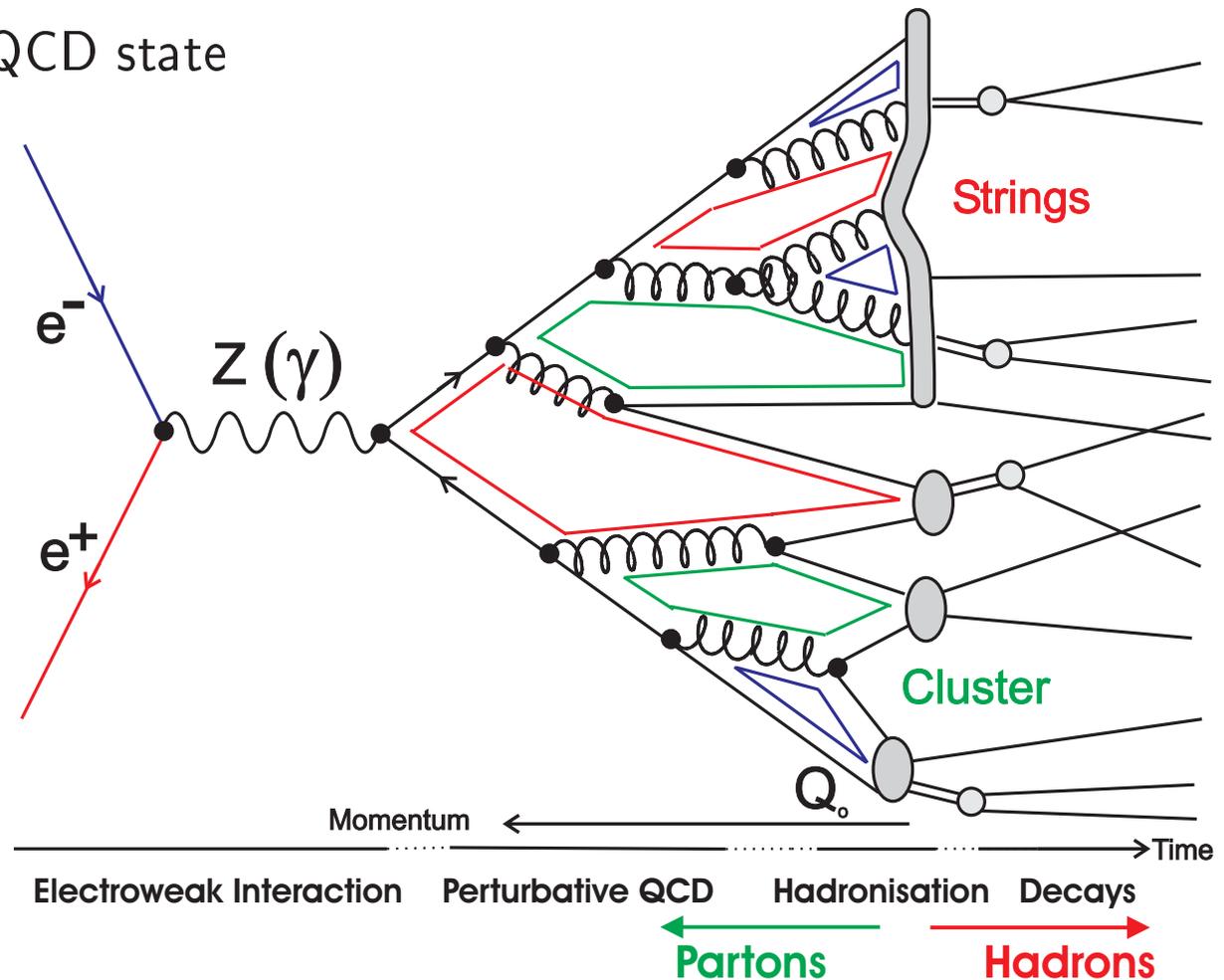
$$\mathcal{O}(\alpha_s) + LLA ***$$

$$\mathcal{O}(\alpha_s^4)_{Tree} + LLA \text{ NEW}$$

Hadronisation:

LUND string model \*\*\*

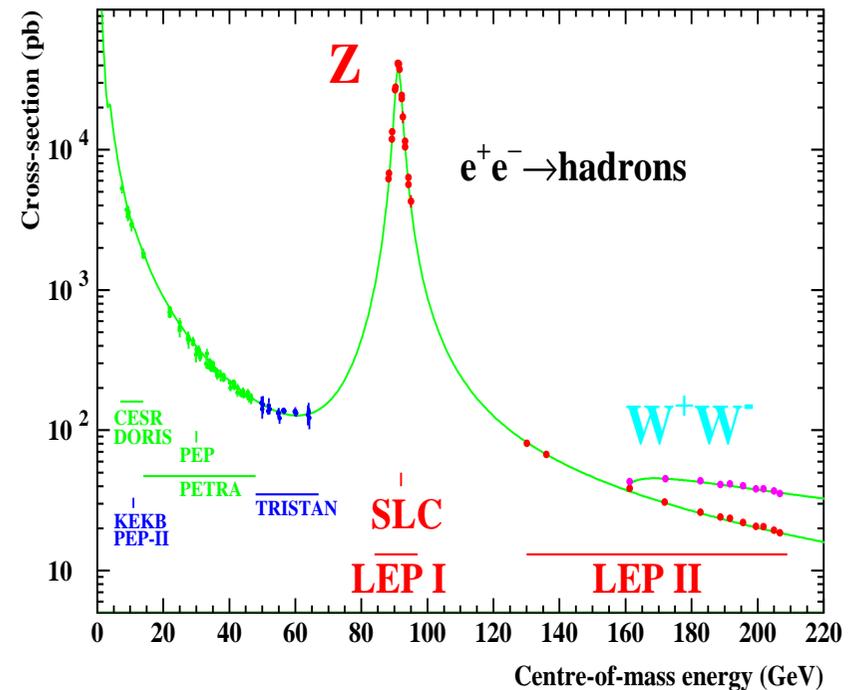
Cluster fragmentation



Models: PYTHIA, ARIADNE, HERWIG, APACIC++

# Setting the Scene – The $e^+e^-$ Experiments

- $e^+e^- \rightarrow q\bar{q}$  experiments provide simplest strongly interacting initial state
- many generations of accelerators: ADA ... LEP II
- recent results to cover ( $\sqrt{s} \gtrsim 10$  GeV): BABAR, BELLE, CLEO, (JADE) SLD, ALEPH, DELPHI, L3, OPAL



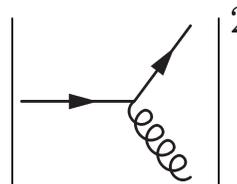
- B factories:  $10^{\mathcal{O}(7)}$  events/experiment
- Z data:  $\mathcal{O}(0.5 \rightarrow 4 \cdot 10^6)$  events/experiment  
“background free”; precise QCD studies even for “rare” events
- LEP II:  $\mathcal{O}(10^4)$  events/experiment  
background: bremsstrahlung  $\rightarrow$  “Z-return” events, WW and ZZ events

# Outline

- Momentum spectra and multiplicity (inclusive, b quarks, gluons)
- Heavy quark fragmentation (b, c)
- The mass of the  $b$  quark and its running
- 4 jet angular distributions and colour factors
- Critical review of  $\alpha_s$  from event shapes
- Measurement of the QCD  $\beta$  function
- Consequences for the gauge group of strong interactions
- Summary

# The Charged Multiplicity

Multiplicity increase in  $e^+e^- \rightarrow q\bar{q}$  due to **coherent gluon** bremsstrahlung off **quarks**



$$\left| \text{Diagram} \right|^2 \propto C_F \cdot \alpha_s$$

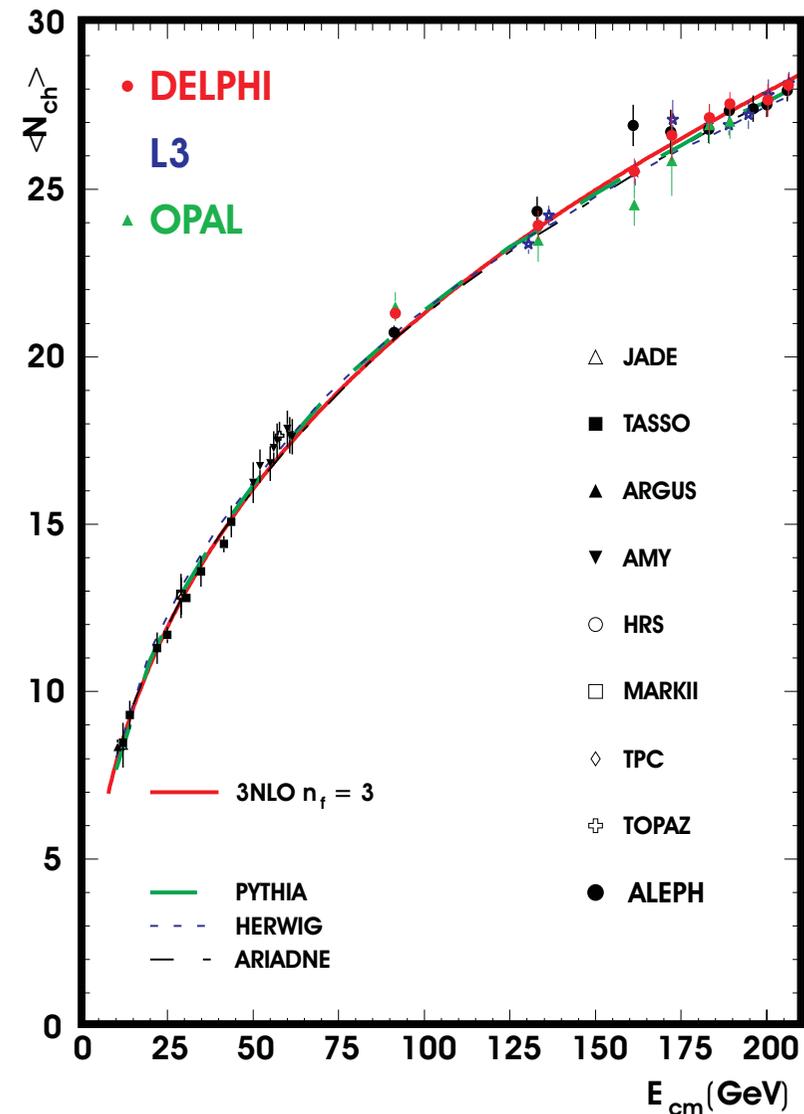
Consistent description of the energy dependence of the multiplicity by:

- fragmentation models
- MLLA (+ **LPHD**:  $\#_{\text{hadrons}} \propto \#_{\text{gluons}}$ )

$$\langle N_{ch} \rangle = K_0 \cdot \alpha_s(E_{cm})^{C_1} \cdot e^{\frac{C_2}{\sqrt{\alpha_s(E_{cm})}}}$$

- higher order (3NLO) predictions.

Dependence on flavour composition small.



# Momentum Spectra

Colour coherence limits gluon emission at small energies / large

$$\xi = -\ln x = -\ln \frac{2E_h}{E_{cm}}$$

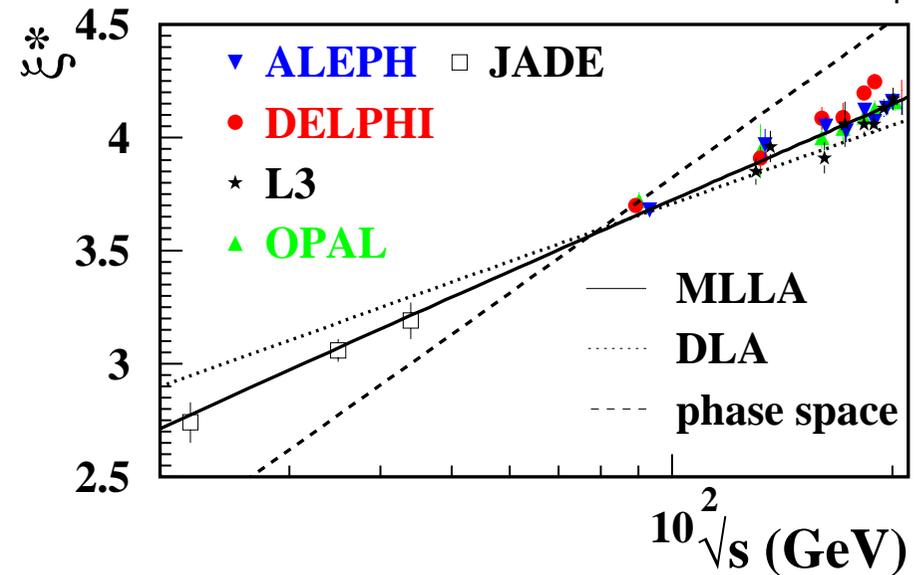
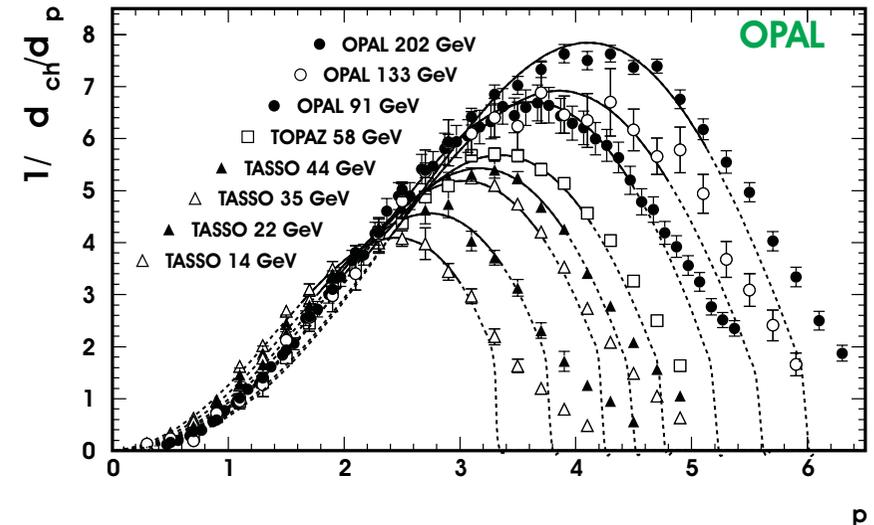
Predicted by MLLA “limiting spectrum”

Coherence → change of peak position

$$\xi^*(E) \sim \sqrt{\log E}$$

slower than expected from phase space

$$\xi^*(E) \sim \log E$$



# Multiplicity Difference of b and Light Quark Events

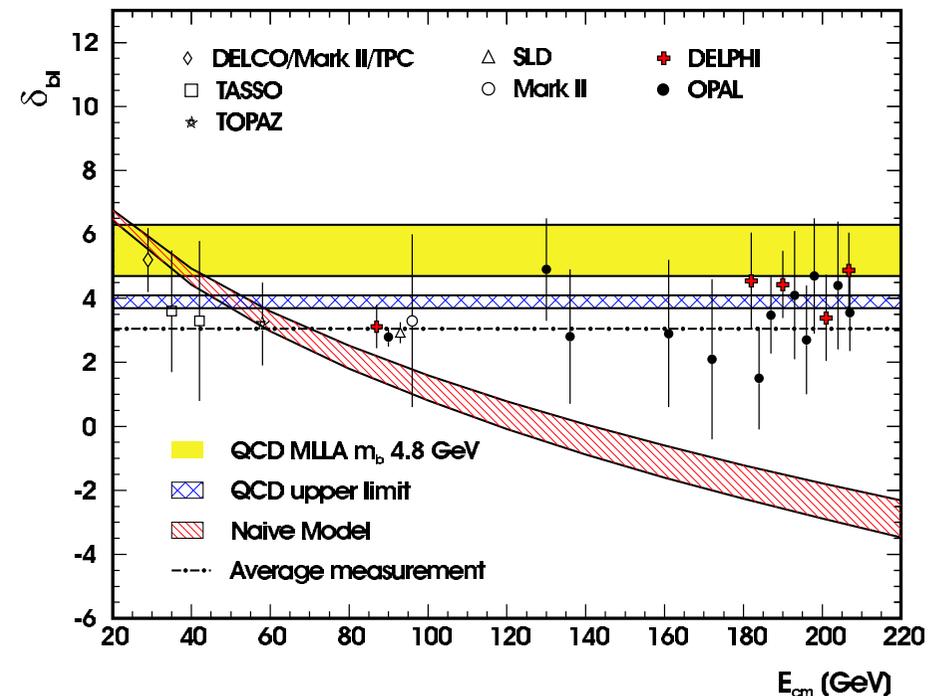
- Compare multiplicity of tagged heavy and light quark events
- Study energy dependence of multiplicity difference  $\delta_{bl}(s) = N_{ch}^b(s) - N_{ch}^{uds}(s)$

- Two predictions:

QCD (MLLA+LPHD):  $\delta_{bl} \sim \text{const.}$   
 due to coherence of gluon radiation

Naive incoherent model:  $\delta_{bl} \sim$   
 $2\langle N_B \rangle + N_{q\bar{q}}((1 - \langle x_B \rangle)^2 s) - N_{q\bar{q}}(s)$

→  $\delta_{bl}$  decreasing with energy !



Z:  $\langle \delta_{bl} \rangle = 2.98 \pm 0.21$        $\sqrt{s} > 130 \text{ GeV}: \langle \delta_{bl} \rangle = 3.99 \pm 0.56$

Measurements support QCD expectation → coherence of gluon radiation

# Multiplicity in Gluon Jets

Multiplicity increase in gluon jets due to coherent gluon bremsstrahlung off gluons.

$$\left| \text{diagram} \right|^2 \propto C_A \cdot \alpha_s$$

Expect increased multiplicity in gluon jets

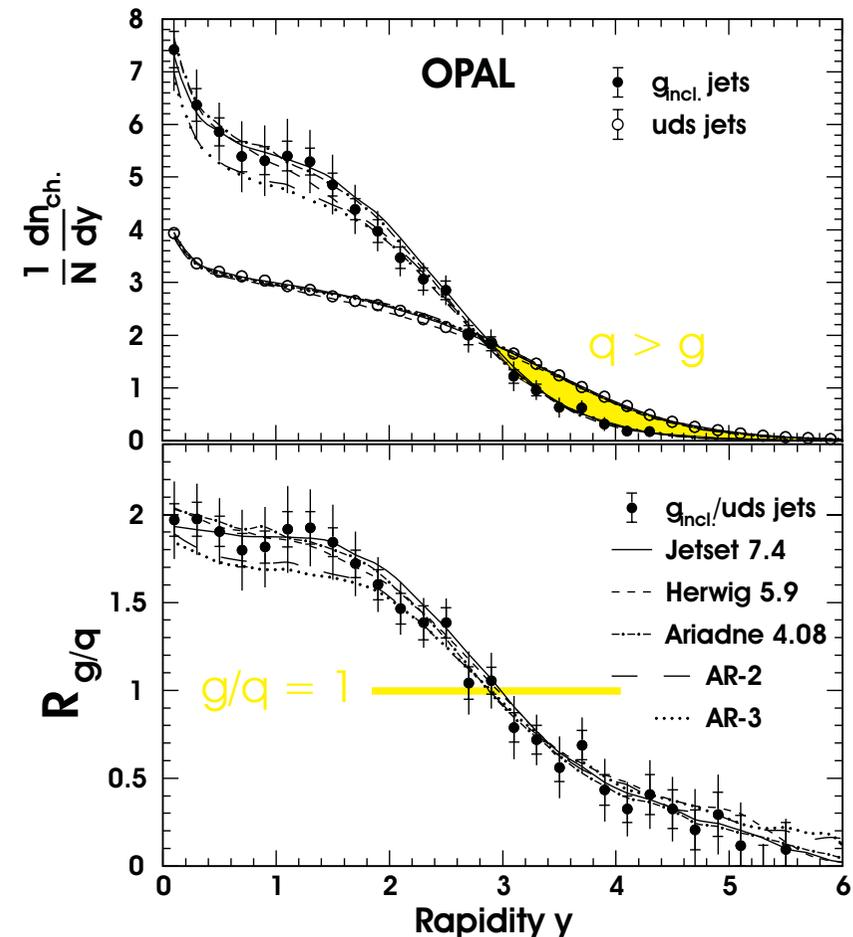
$$\frac{N_{ch}^{\text{gluon jet}}}{N_{ch}^{\text{quark jet}}} = \frac{C_A}{C_F} = \frac{3}{4/3} = 2.25$$

Find in  $q\bar{q}g$  events at PETRA ... LEP

$$\frac{N_{ch}^{\text{gluon jet}}}{N_{ch}^{\text{quark jet}}} \sim 1.1 \rightarrow 1.4$$

Small ratio is understood:

Coherence  $\rightarrow$  use “ $p_{\perp}$  like” scales,  
Biases due to 3 jet selection,  
Non perturbative / finite E effects



# Multiplicity of Three Jet Events

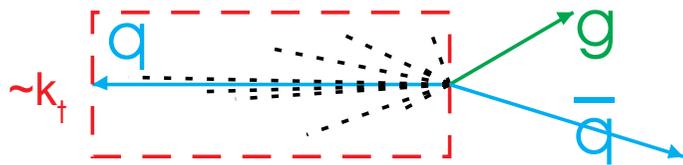
Measure **topology dependence** of the multiplicity of 3 jet  $q\bar{q}g$  events at the Z

Theory considers:

- coherence ( $p_{\perp}^2$  scales)

$$N_{q\bar{q}g}^{ch} = N_{q\bar{q}}^{ch}(s_{q\bar{q}}, y_{cut}) + \frac{1}{2} \cdot N_g^{ch}(p_{\perp g}^2)$$

- phase space ( $y_{cut}$ ) restriction of  $q\bar{q}$  system due to  $g$  jet

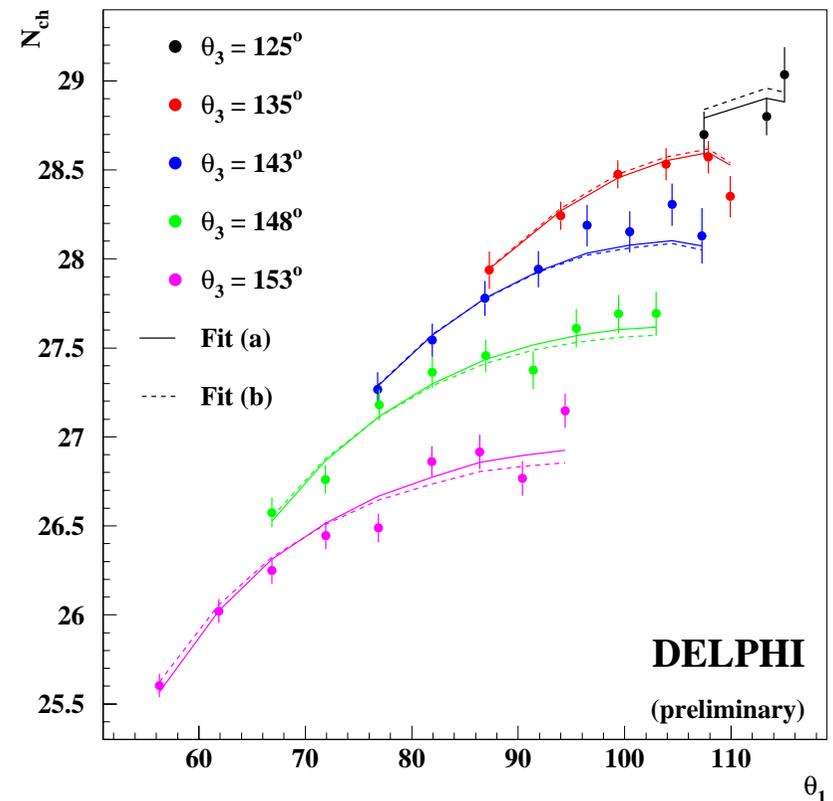
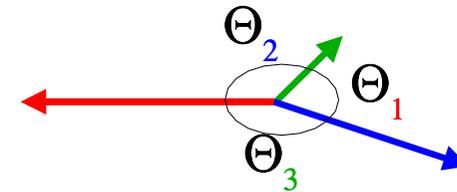


- Differential eqn. for multiplicity **slopes**

$$\frac{dN_{gg}(L')}{dL'} = \frac{C_A}{C_F} \left( 1 - \frac{\alpha_0 c_r}{L} \right) \frac{d}{dL} N_{q\bar{q}}(L)$$

$$L = \log s/\Lambda^2, \quad c_i = \text{const.}$$

→ **non-pert. integration const.** for  $N_g^{ch}$

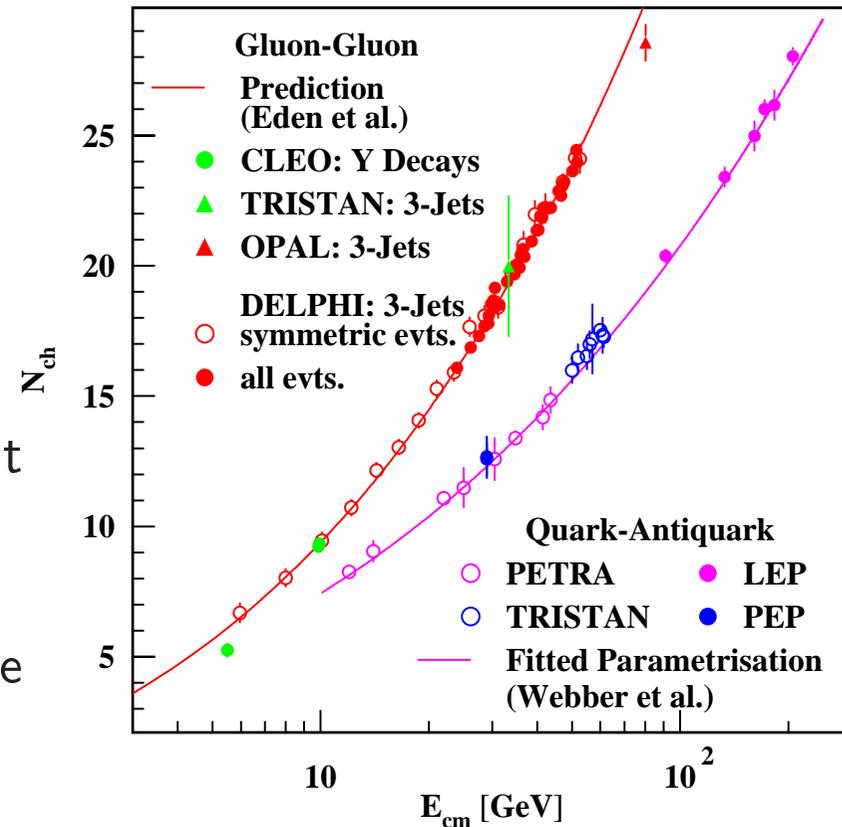


# Multiplicity of Colour Singlet $gg$ Systems

- Extract multiplicity of  $gg$  events:

$$N_g^{ch} = 2 \cdot (N_{q\bar{q}g}^{ch} - N_{q\bar{q}}^{ch})$$

- Good agreement of 3 jet analyses with **direct gluons** ( $\chi_b$  decays CLEO)
- Similar OPAL analysis uses different scheme
- Multiplicity for **gluons** increases  $\sim$  twice as fast as for **quarks**  $\implies C_A/C_F$  !



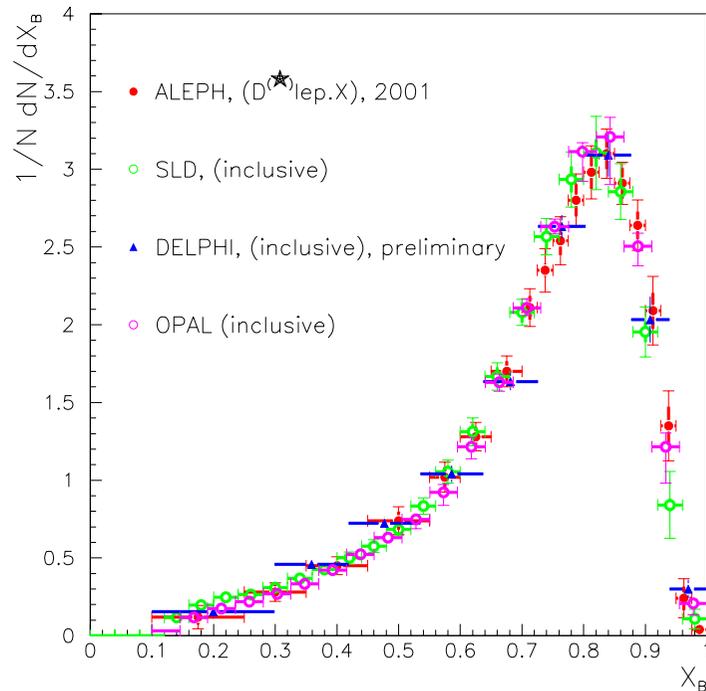
From symmetric events  $\Theta_1 = \Theta_2$  DELPHI obtains:

$$\frac{C_A}{C_F} = 2.221 \pm 0.032(stat.) \pm 0.047(exp.) \pm 0.058(hadr.c.) \pm 0.075(theo.)$$

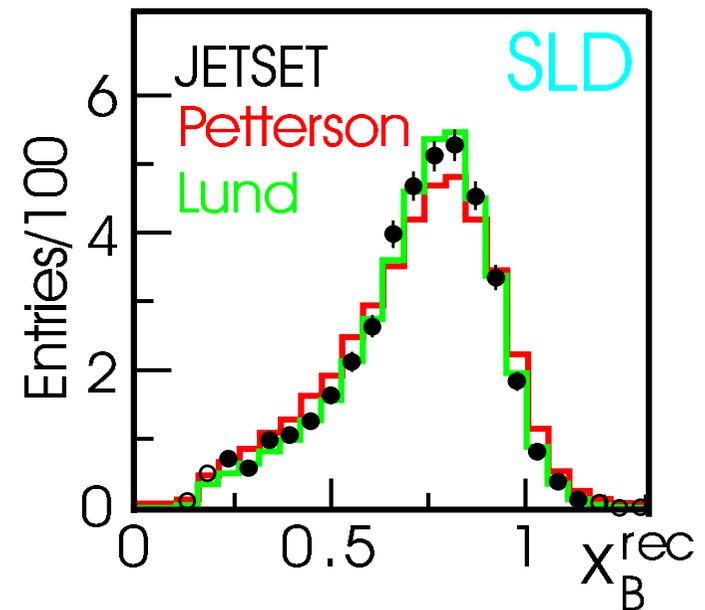
# Heavy Quark Fragmentation

- Fragmentation functions specify energy transfer: partons  $\rightarrow$  hadrons
- Simple case heavy quarks
- Recent b fragmentation measurements

$$D_b^B(x) = \frac{1}{\sigma_{tot}} \frac{d\sigma_B}{dx} \quad x = \frac{2E_B}{E_{CM}}$$



- partial/inclusive B reconstruction: high stat. precision  $\sim 0.3\%$   
good energy resolution  $\mathcal{O}(10)\%$
- New average (weakly decaying B's):  $\langle x_B \rangle = 0.715 \pm 0.003$
- Lund/Bowler ansatz for FF favoured **Peterson** ansatz disagrees.



# Interpretation of Heavy Quark Fragmentation

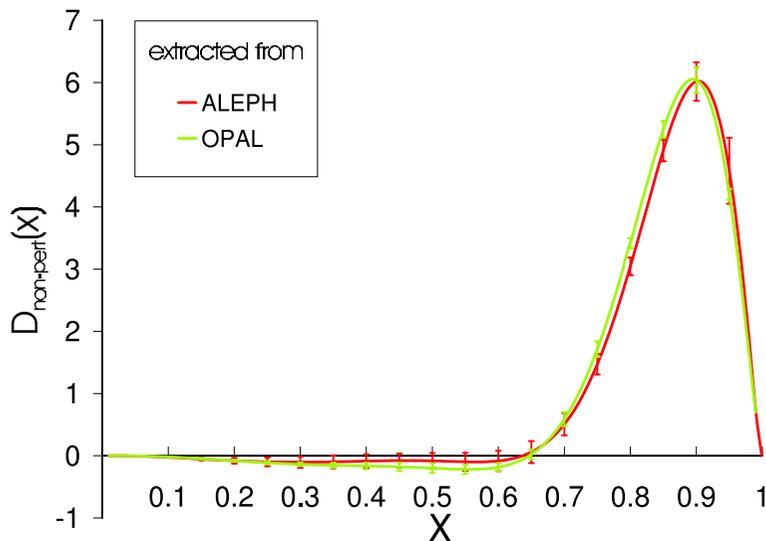
Observed FF is described by a convolution of a perturbative and a non-pert. part.

$$D_b^B(x) = \int_0^1 D_{pert}(z) \cdot D_{non-pert}\left(\frac{x}{z}\right) \frac{dz}{z}$$

a Mellin transform ( $\int dx x^{N-1} D(x)$ ) yields a simple product equation

$$\tilde{D}_b^B(N) = \tilde{D}_{pert}(N) \cdot \tilde{D}_{non-pert}(N) \rightarrow \text{solve for } \tilde{D}_{non-pert}(N)$$

Backtransformation  $\rightarrow$  model-independent  $D_{non-pert}(x)$



$D_{non-pert}(x)$  can be universally applied

Use similar assumptions for perturbative calculations !

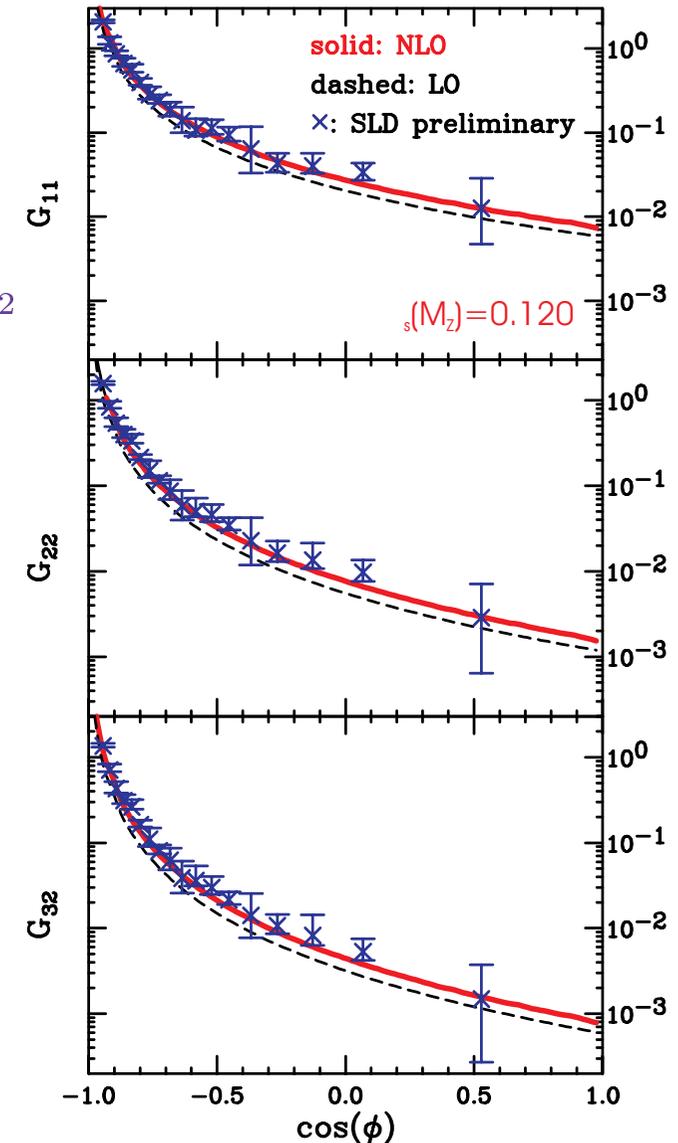
# Double Inclusive b Fragmentation

- Study both B hadrons, measure double moments (SLD, Brandenburg et al.)

$$\tilde{D}_{ij}(\phi) = \frac{1}{\sigma_B} \int x_{B_1}^{i-1} x_{B_2}^{j-1} \frac{d^3\sigma}{dx_{B_1} dx_{B_2} d\cos\phi} dx_{B_1} dx_{B_2}$$

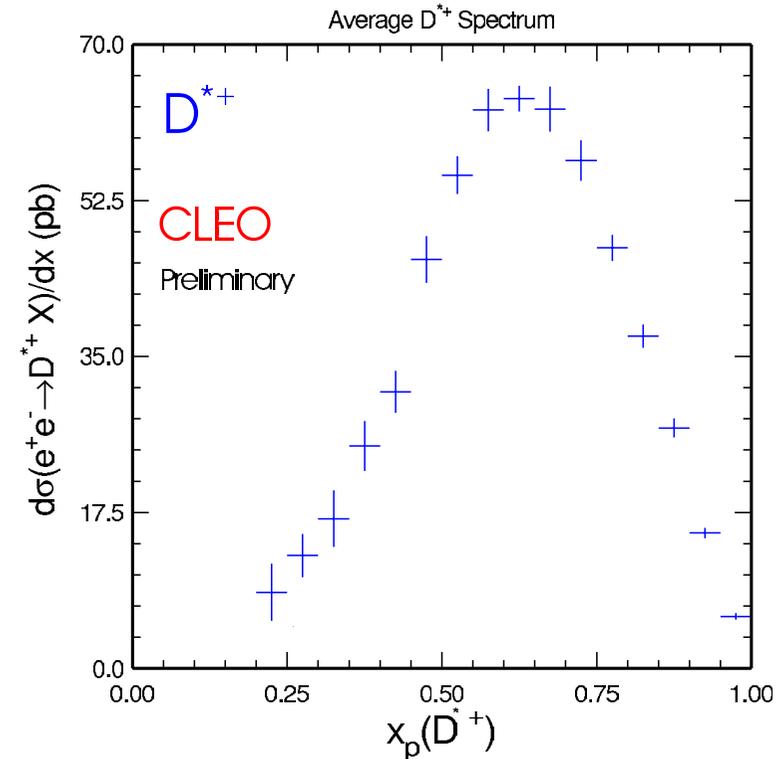
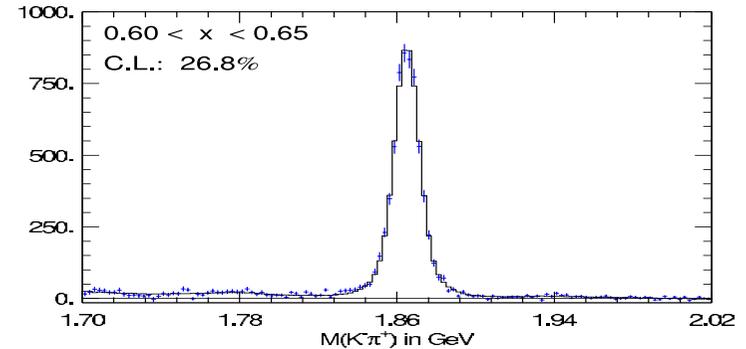
$$G_{ij} = \frac{\tilde{D}_{ij}}{\tilde{D}_i \tilde{D}_j} \quad \phi \text{ is } \angle \text{ between b's}$$

- b mass regulates collinear singularity, save pert. prediction for  $G_{ij}$
- Non-perturbative and large  $\log E_{CM}/m_b$  terms cancel to all orders
- Measurement implies test of **factorisation theorem** (SLD)
- Allows **new** measurement of  $\alpha_s$  (Brandenburg et al.)



# Charm Fragmentation

- New precise charm frag. function  
CLEO (preliminary)  
 $\sim 3fb^{-1}$  below  $b\bar{b}$  threshold,  
 $\sim 6fb^{-1}$  of  $\Upsilon(4S)$  data ( $x_p^D > 0.5$ )
- Decays:  $D^+ \rightarrow K\pi\pi$ ,  
 $D^0 \rightarrow K\pi, K3\pi, D^{*+} \rightarrow D^0\pi$
- Excellent resolution  $\Delta x \sim 0.006$ ,  
high efficiency  $\epsilon \sim 0.77$
- Pure charm fragmentation –  
not influenced by B decays
- C fragmentation softer than b
- To be analysed like b data
- Slightly different  $x$  spectra for  $D^0, D^+, D^{*+}$   
partly  $\rightarrow$  mass difference  
More important: resonance decays



# Measurement of the $b$ Quark Mass $m_b(M_Z)$

- Quarks **bound** inside hadrons  $\rightarrow$  masses known only to **limited precision**
- Assess masses via dynamical relations
- $m_b$ : reduction of gluon bremsstrahlung  $\propto \frac{m_b^2}{p_{\perp g}^2} = \frac{m_b^2}{M_Z^2} \cdot \frac{1}{y_{cut}} \sim 0.3\% \cdot \mathcal{O}(10)$
- Besides **pert. pole mass**  $M_b$  define **renormalised mass**  $m_b(Q)$  in  $\overline{\text{MS}}$  scheme. This **running** mass absorbs loop corrections like  $\alpha_s(Q)$ .

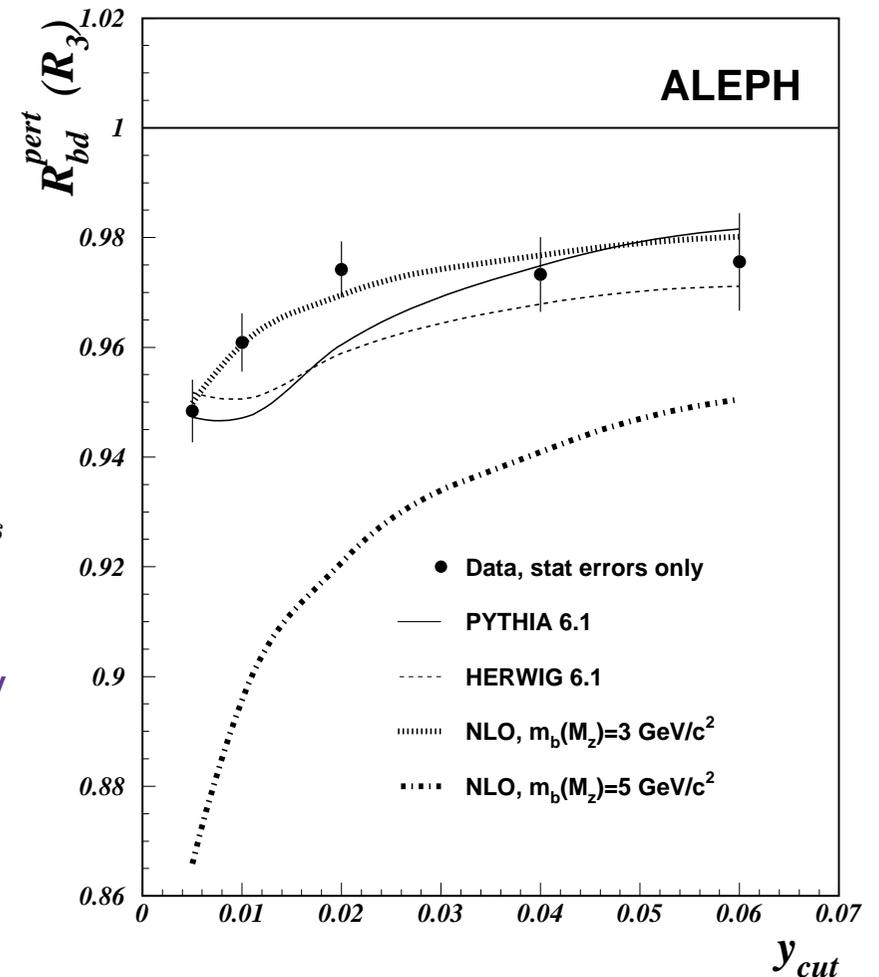
## Measurement:

- Compare **b/inclusive** ratio of jet rates/event shapes obs.
- b-identification using impact param. tag. purity 65-90%
- Corrections: acceptance, tag bias, **hadronisation** (partly neglected)
- **ALEPH**: several shapes use  $R_3, \langle y \rangle$
- **Brandenburg et al (SLD)**: several jet algos (assign syst. for discr.)
- **Opal**:  $T, M_h, B_w, y_3, C$  exploiting correlation btw. observables
- **DELPHI 2003**:  $R_3$  Durham, Cambridge

## Theory: four independent calculations

# Measurement of the $b$ Quark Mass $m_b(M_Z)$

- Observe expected  $y_{cut}$  dependence.
- Recent improvements:  
 understanding of hadronisation correction  
 new important error:  $b$  mass in model  
 overall  $\rightarrow$  reduced uncertainty
- Theory error substantial:  
 Mass ambiguity, renormalisation scale,  $\Delta\alpha_s$
- Combined ADO/SLD result  
 $m_b(M_Z) = 2.95 \pm 0.15 \pm 0.24_{hadc./theo}$  GeV
- Alternatively: flavour independence of  $\alpha_s$   
 $\frac{\alpha_s^b}{\alpha_s^l} = 0.996 \pm 0.009$



# The Running of $m_b(Q)$

- Compare LEP/SLD average  $\Rightarrow$  to b-mass from bound states:

$$m_b(m_b) = 4.24 \pm 0.11 \text{ GeV}$$

$\Rightarrow$  running of the b-mass

- Soon improved measurements of  $m_b$  from leptonic B decays

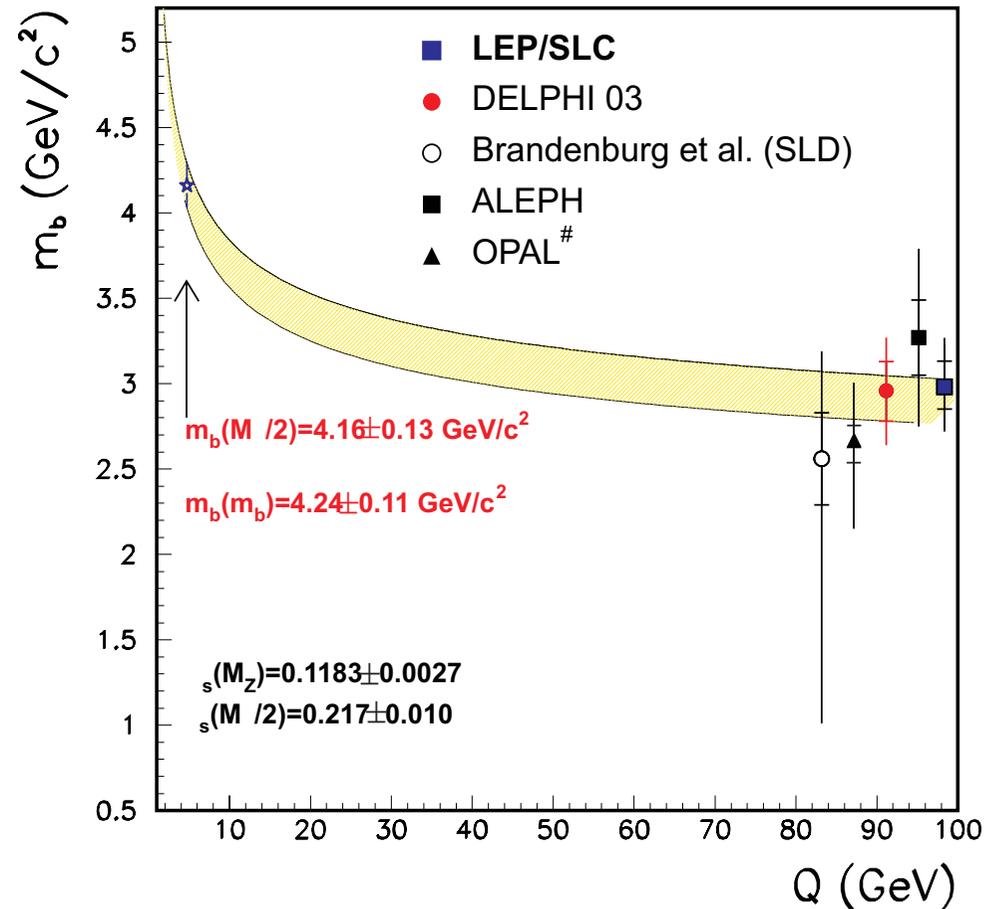
- Tests RGE for the mass

$$\frac{\partial m}{\partial \ln Q^2} = -\gamma_m(\alpha_s) \cdot m(Q^2)$$

mass anom. dimension



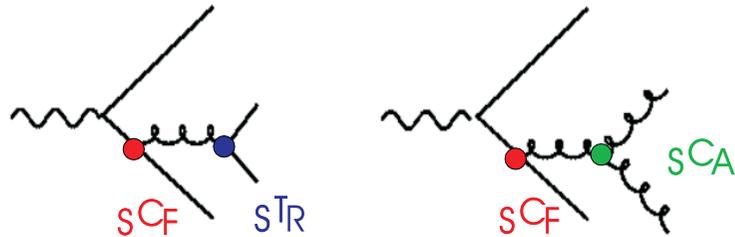
contains loop corrections to b quark propagator



$m_b(M_Z)$  measurement implies a successful test of QCD loop structure!

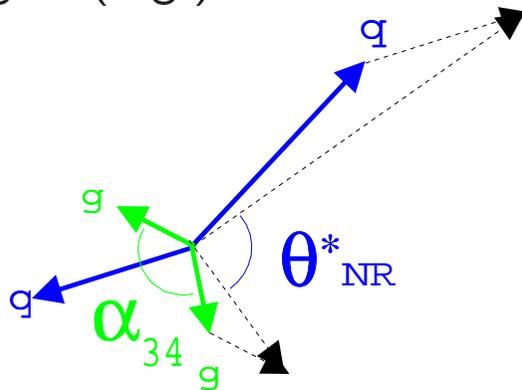
# Colour Factors from Four Jet Events

4 jet events give access to full QCD gauge structure



Connect colour to helicity structure.

Sensitive angles (e.g.):

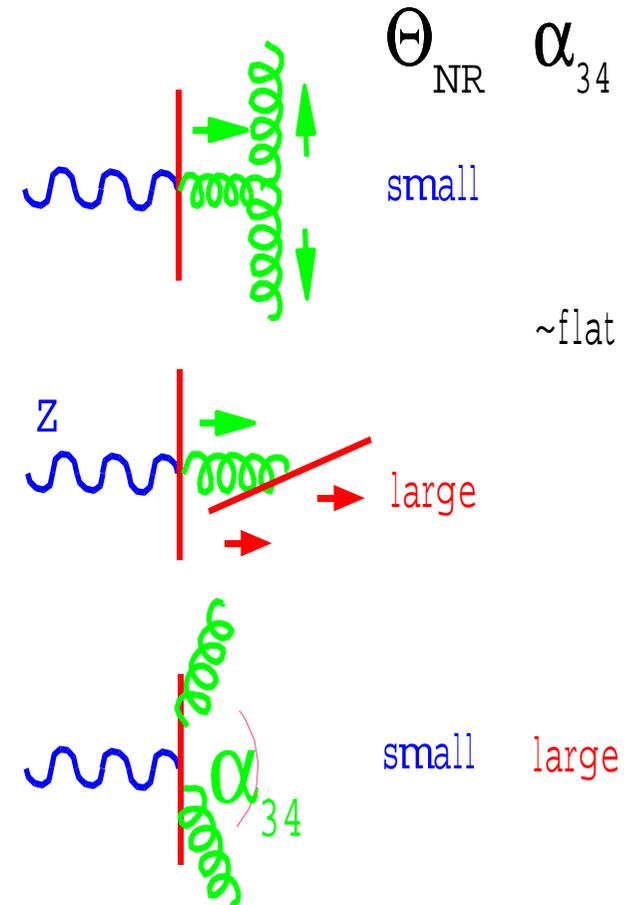


$\Theta_{NR}^* \sim \angle(\text{betw. primary and secondary partons})$

$\alpha_{34} \sim \angle(\text{betw. secondary partons})$

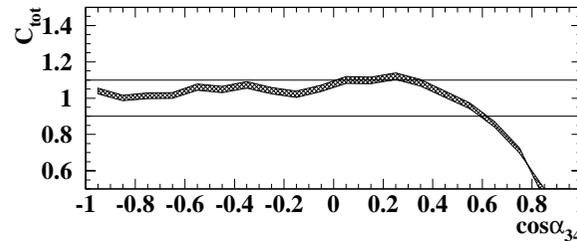
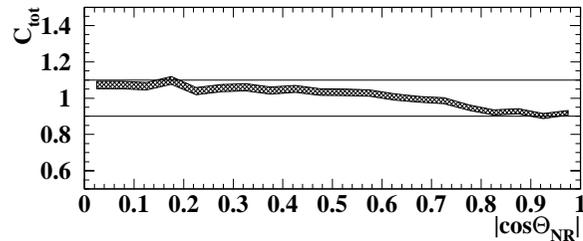
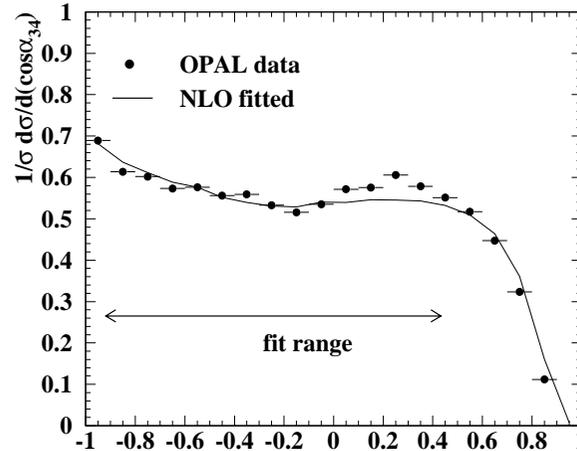
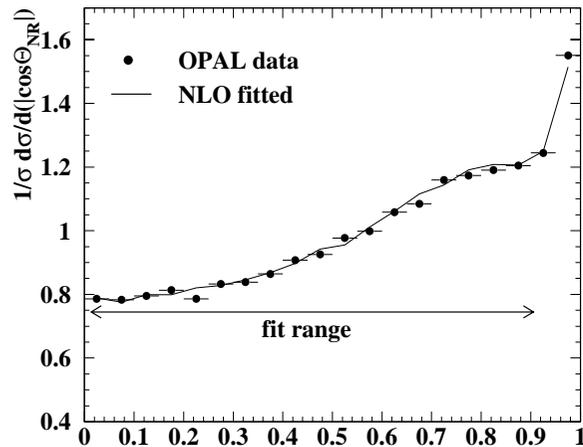
Initial gluon polarised, like any vector particle. Spin 0 or 2 case different.

Initial analyses used LO theory; recent years NLO calculation.



# Colour Factors from Four Jet Events

OPAL & ALEPH use (matched) NLO 4 jet calculation, inc. also 3 jet rate.



Reconstruct jets,  
unfold detector and  
**hadronisation**  
(implies QCD models)

Primary fit yields:

$$\frac{C_A}{C_F}, \frac{n_f \cdot T_F}{C_F} \text{ and } C_F \alpha_s.$$

Solve for individual  
colour factors in NLO.

$$\text{A \& O:} \quad C_A = 2.97 \pm 0.50 \quad C_F = 1.34 \pm 0.23$$

$$\text{QCD:} \quad C_A = 3 \quad C_F = 1.33$$

But : Correlations between  $C_A$  and  $C_F$  are **large!**

# Measuring $\alpha_s$

## Event Shape Observables

Event shapes observables measure the amount of gluon bremsstrahlung

Sensitive to  $\alpha_s$  (and QCD colour factors)

Inensitive to electroweak physics.

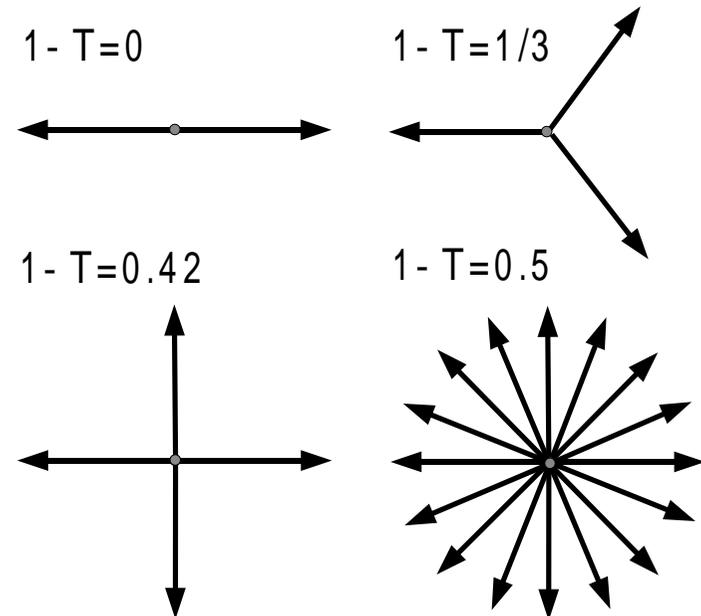
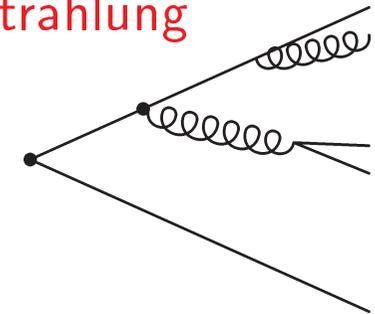
- Infrared safe
- Collinear safe
- insensitive to non-perturbative effects.

Example Thrust:

$$y = 1 - T := 1 - \max_{|\vec{n}|=1} \frac{\sum_{i=1}^n |\vec{p}_i \cdot \vec{n}|}{\sum_{i=1}^n |\vec{p}_i|}$$

Furtermore:

Jet-Masses, Jet-Broadenings,  $y_{cut}$ , . . .



# The Procedure – Guide through the $\alpha_s$ Maze

## Experiment

- Measure event shape distribution
- Correct detector acceptance/resolution, ISR, backgrounds . . .

## Theory

- Calculate perturbative predictions
  - NLO  $\alpha_s$  M.E.
  - NLLA
  - matched NLO  $\alpha_s$ /NLLA
- Correct for non-pert. hadronisation by
  - hadronisation models
  - power corrections

## Compare common level

- Fit  $\alpha_s$
- Determine experimental uncertainties
- Determine theoretical uncertainties

## Next to Leading (Fixed) Order pQCD

- $\mathcal{O}(\alpha_s^2)$  prediction:

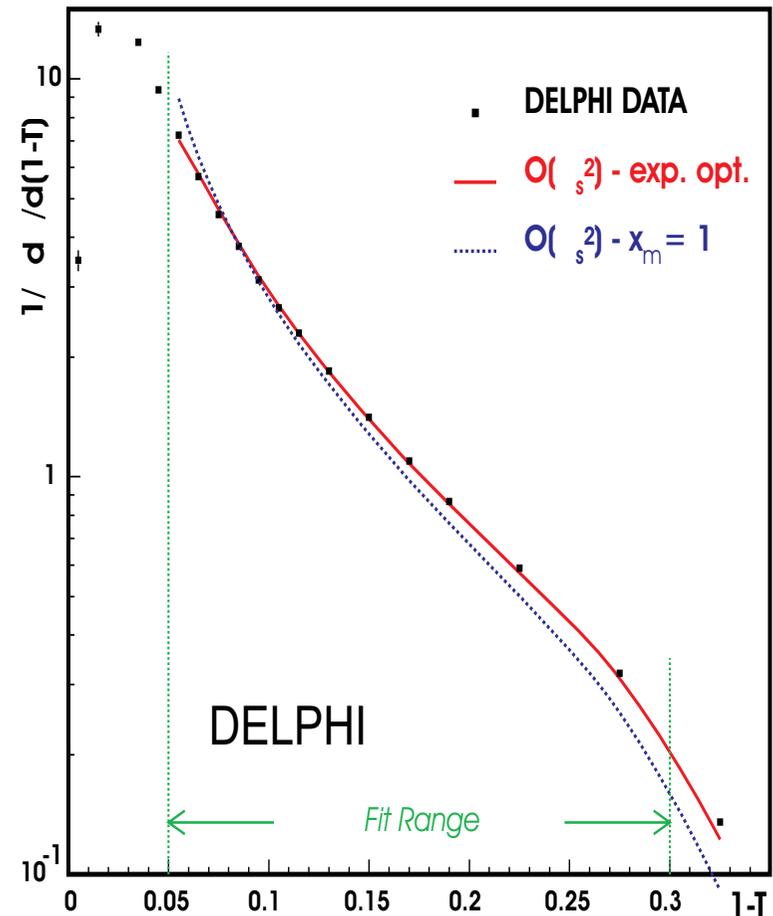
$$\mathcal{D}(y)_{\text{pert}} = \frac{1}{\sigma} \frac{d\sigma}{dy} =$$

$$A_y \cdot \bar{\alpha}_s(\mu) + (A_y \cdot \beta_0 \log x_\mu + B_y) \bar{\alpha}_s(\mu)^2$$

- Uncalculated higher order terms  $\rightarrow$  (unphysical)  $x_\mu = \mu^2/s$  renormalisation scale (factor) dependence
- Valid for 3 jet region – large  $y$  HO terms important for  $y \rightarrow 0$
- Apply MC hadronisation corrections

$$C = \frac{\mathcal{D}_{\text{parton}}(y)}{\mathcal{D}_{\text{hadron}}(y)}$$

- NLO predictions in general show different slope of  $\mathcal{D}_{\text{hadron}}(y)$  than data!
- Except  $x_\mu$  is optimised!



All arguments also apply for the  $\mathcal{O}(\alpha_s^3)$  calculation (4 jets)

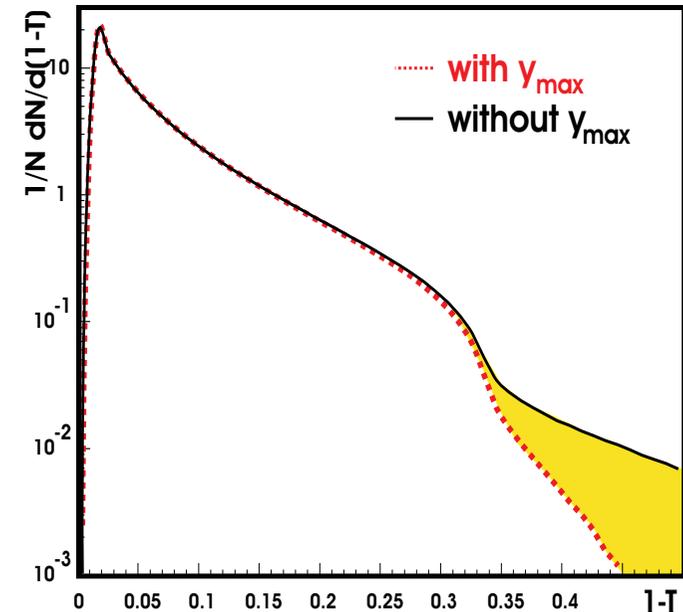
## Next to Leading Log Approximation (NLLA)

$$R_y(y) = \int_0^y dy \mathcal{D}_y(y) = e^{(Lg_1(\alpha_s L) + g_2(\alpha_s L))}$$

$$L = \log(1/y)$$

- Resums leading & next to leading log. terms
- Use for small  $y$ , collinear rad., “2-jet region”
- Problem:  $R_y(y_{\max}) \neq 1$ ,  $R'_y(y_{\max}) \neq 0$
- Solution:

$$L \rightarrow L' = \frac{1}{p} \ln \left[ \frac{1}{(x_L \cdot y)^p} - \frac{1}{(x_L \cdot y_{\max})^p} + 1 \right]$$



## Matching Fixed Order and NLLA

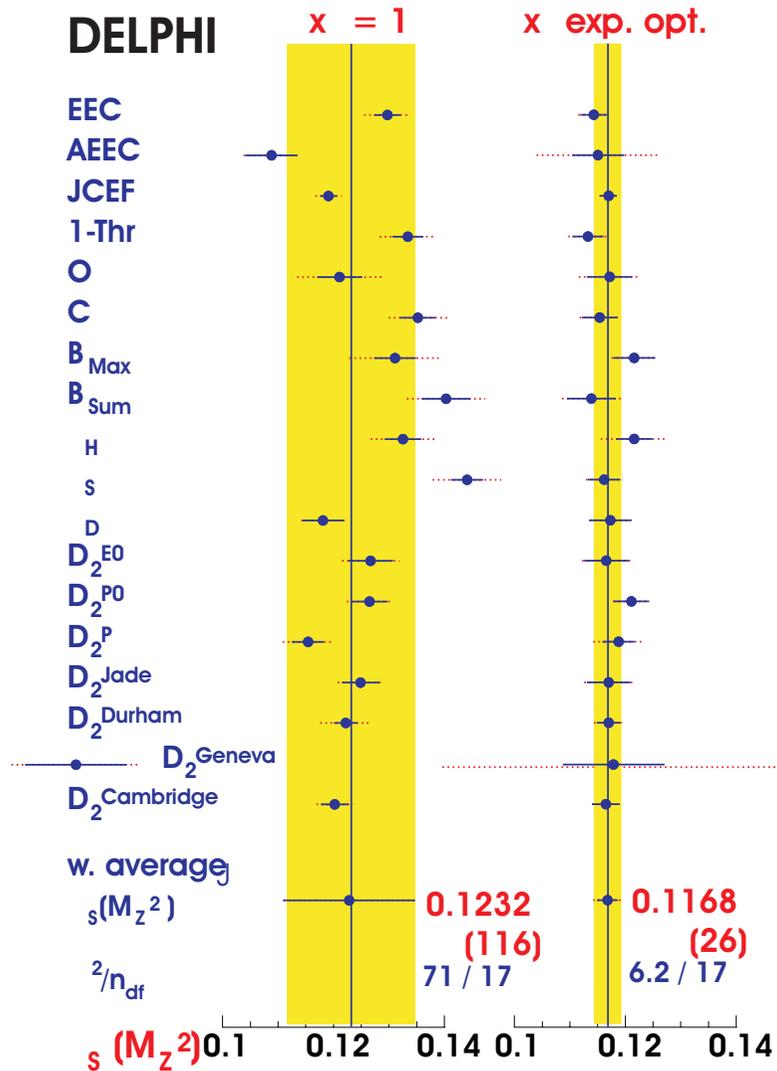
Aim: combine NLLA and  $\mathcal{O}(\alpha_s^2)$  ( $\overline{\text{MS}}$  scheme,  $x_\mu = 1$ ) calculations.

Avoid double counting:

$$R_{\mathcal{O}(\alpha_s^2)+NLLA} = R_{NLLA} + A \cdot \alpha_s + B \cdot \alpha_s^2 - \text{double counting}$$

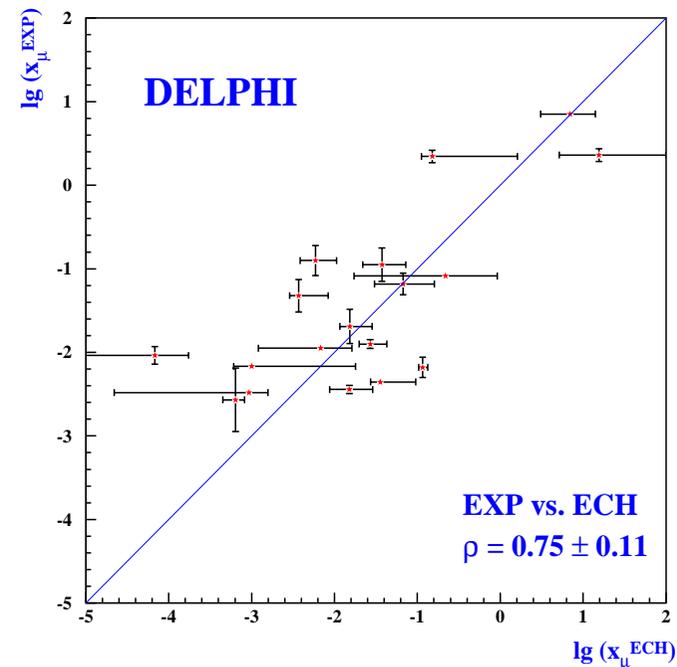
Problem: ambiguity  $\rightarrow$  perform matching in  $R(y)$  or  $\log R(y)$ .

# $\mathcal{O}(\alpha_s^2)$ Results

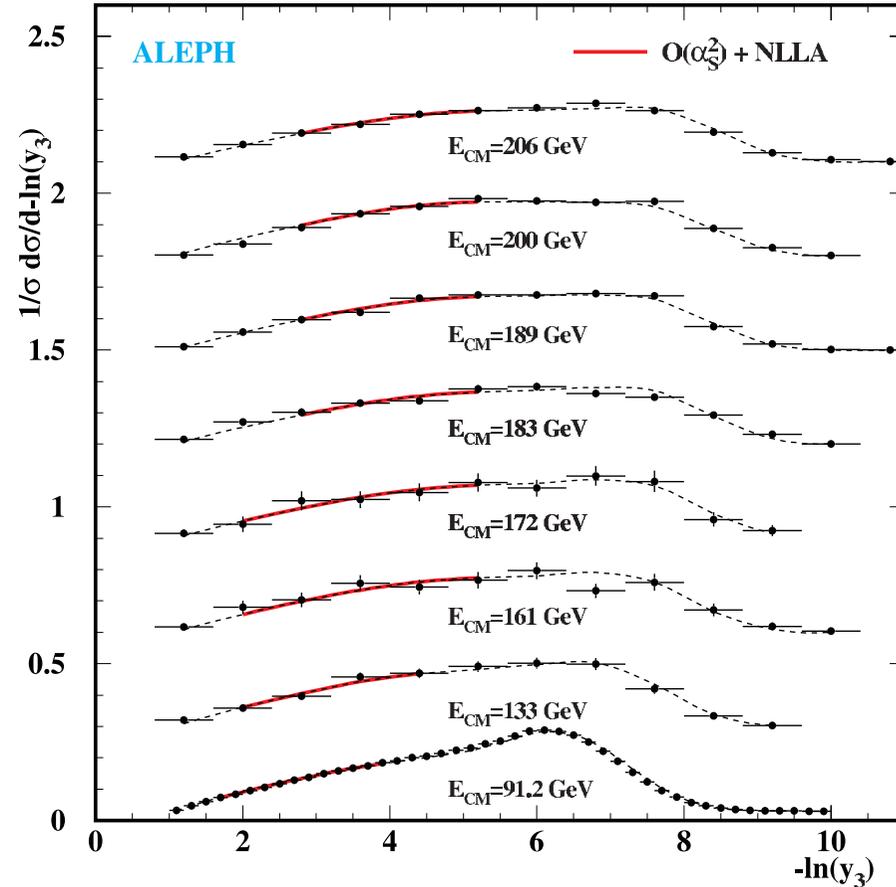
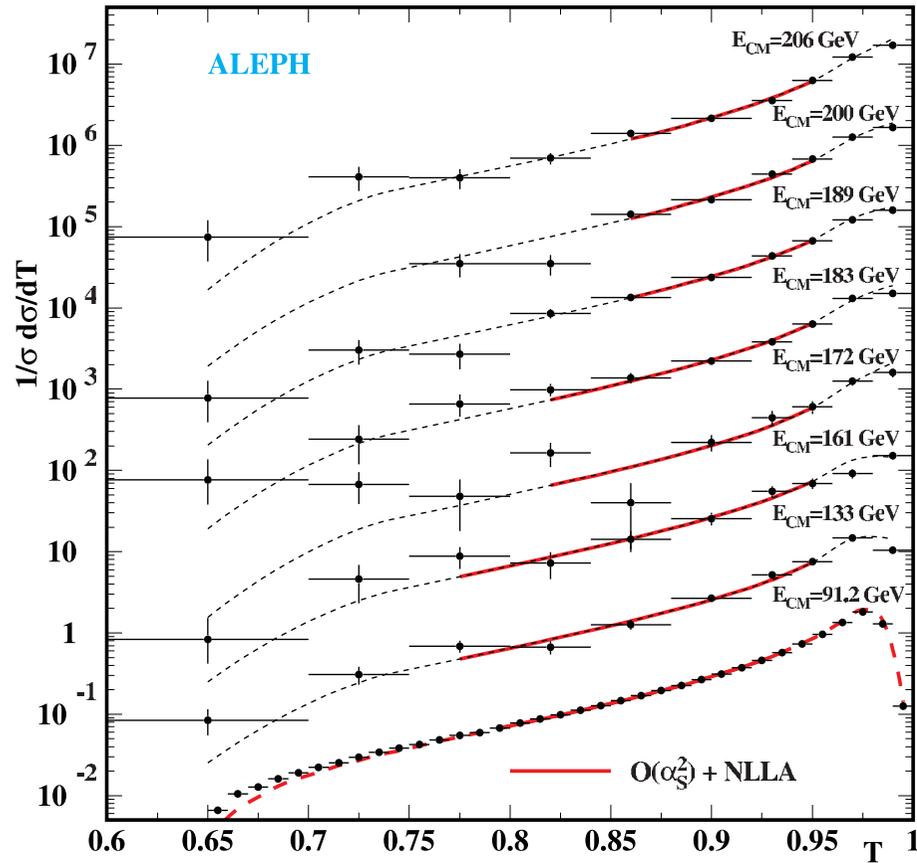


For 18 observables,

- $\overline{\text{MS}}$  scheme:  $\sim 10\%$  scatter  
→ require additional theory error
- Using **experimentally optimised** scales:  
Consistent results –  $\sim 2\%$  scatter
- Observe **correlation** with **theoretically** motivated scales (ECH, PMS)



# Matched LLA/ $\mathcal{O}(\alpha_s^2)$ Fits



Successfully describes small  $y$  “2jet” region

Fit range: **solid lines**

Residual slope (?) between data and theory for Thrust

# $\alpha_s$ Combination

LEP QCD Working Group:

**Aim:** average ADLO  $\alpha_s$  & running (high energy data)

**Input:**  $\alpha_s$  values (using  $\log R$  matching) + exp. uncertainties

**Work:** Compare theory implementation, hadronisation corrections  
Define theory uncertainty  
Treat correlations

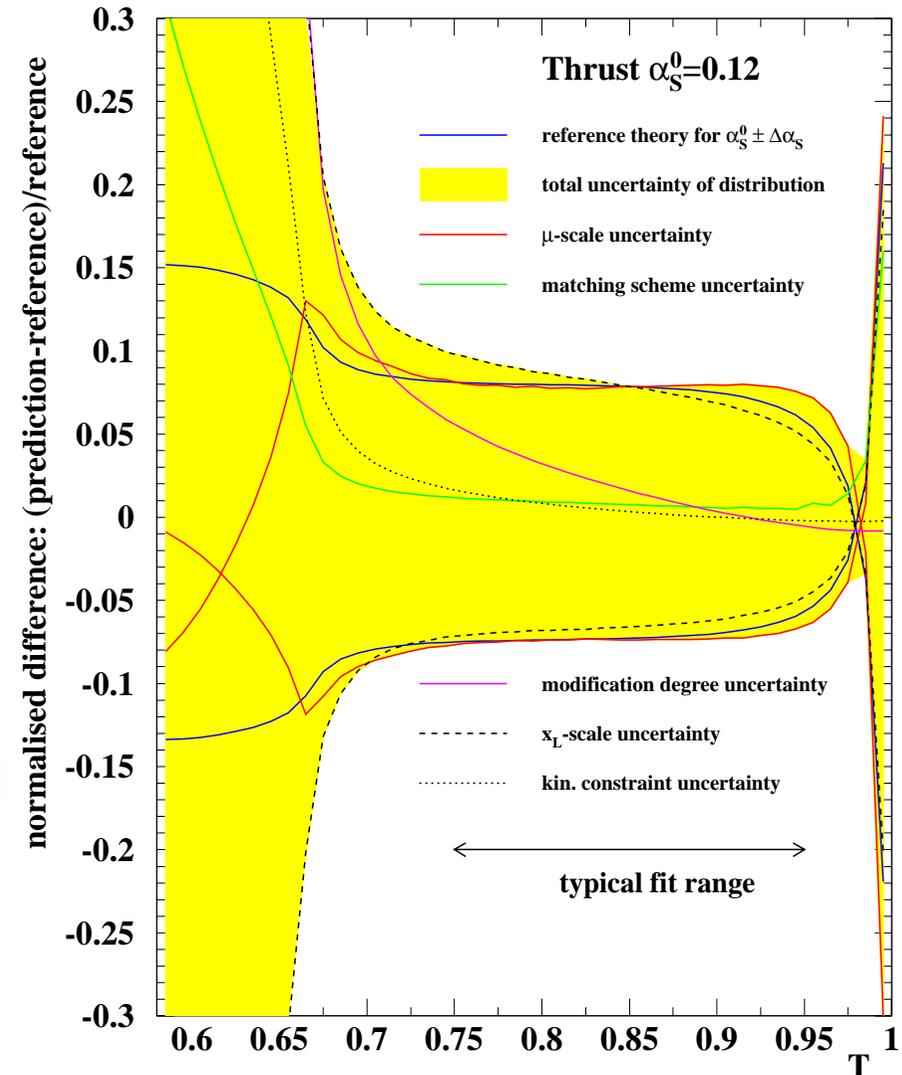
**Output:**  $\langle \alpha_s(E) \rangle \pm \Delta_{exp.} \pm \Delta_{theo.}$

Problems observed:

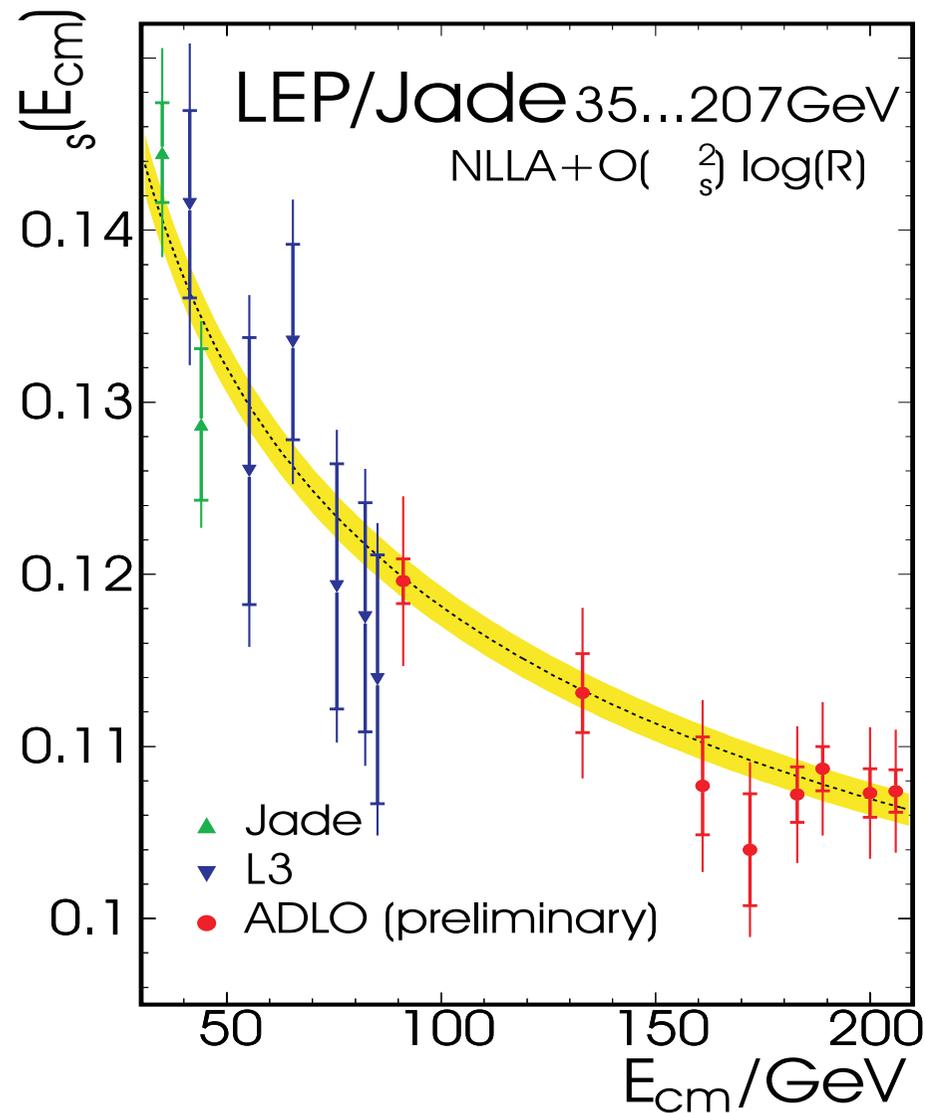
- Z:  $\alpha_s$  often depends on fit range ( $\mathcal{O}(\alpha_s^2 | x_\mu = 1)$  remnant ?)
- Badly described observables obtain smallest error
- $\alpha_s$  downwards biased as  $\Delta_{theo} \alpha_s \propto \alpha_s^3$
- negative weights  $\leftrightarrow$  correlations inaccurate

# The Uncertainty Band

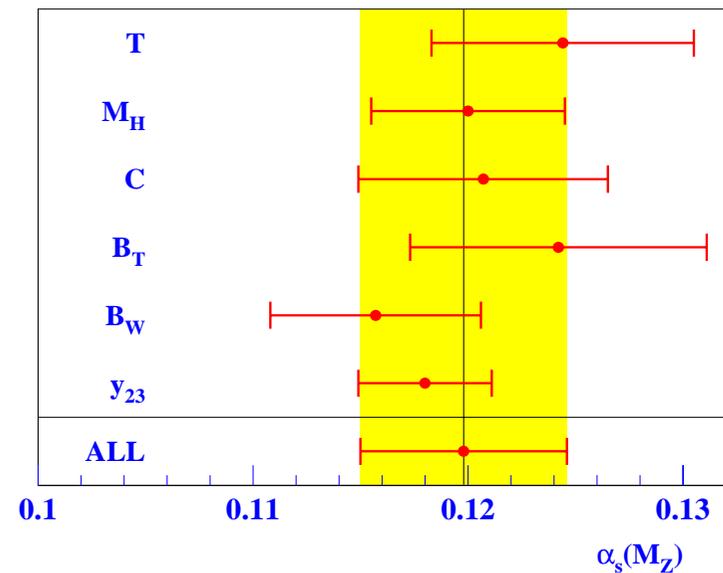
- Take reference theory with fixed  $\alpha_s(M_Z)$
- Vary reference theory:  
Renormalisation scale  $1/2 < x_\mu < 2$   
Apply (or not) phase space condition:  
 $2/3 < x_L < 3/2, p = 1; 2$   
Exchange  $\log R$  vs.  $R$  matching
- Envelope defines uncertainty band
- allowed  $\alpha_s(M_Z)$  variation in the band determines  $\pm \Delta\alpha_s$
- Determine  $\langle \alpha_s(M_Z) \rangle$  and iterate



# $\alpha_s$ Combination of the LEP QCD Working Group



	LEP I	LEP II
$\alpha_s$	0.1197	0.1196
$\pm$ stat.	0.0002	0.0005
$\pm$ sys. ex.	0.0008	0.0010
$\pm$ had.	0.0010	0.0007
$\pm$ theo.	0.0048	0.0044
$\pm$ tot.	0.0049	0.0046
$\pm$ tot.rel	$\sim 4\%$	



# Power Corrections – The Dokshitzer-Webber-Ansatz

Parameterise **unknown** (analytical) behaviour of the **physical** strong coupling below the IR-matching scale  $\mu_I$  by its **universal** mean value.

$$\alpha_0(\mu_I) = \frac{1}{\mu_I} \int_0^{\mu_I} dk \alpha_s(k)$$

Leads to a power term  $\mathcal{P} \propto 1/E_{\text{cm}}$

increasing the **mean values**

$$\langle y \rangle = \langle y_{\text{pert}} \rangle + c_y \mathcal{P}$$

shifting the **distributions**

$$\mathcal{D}_y(y) = \mathcal{D}_{\text{pert}}(y - c_y \mathcal{P})$$

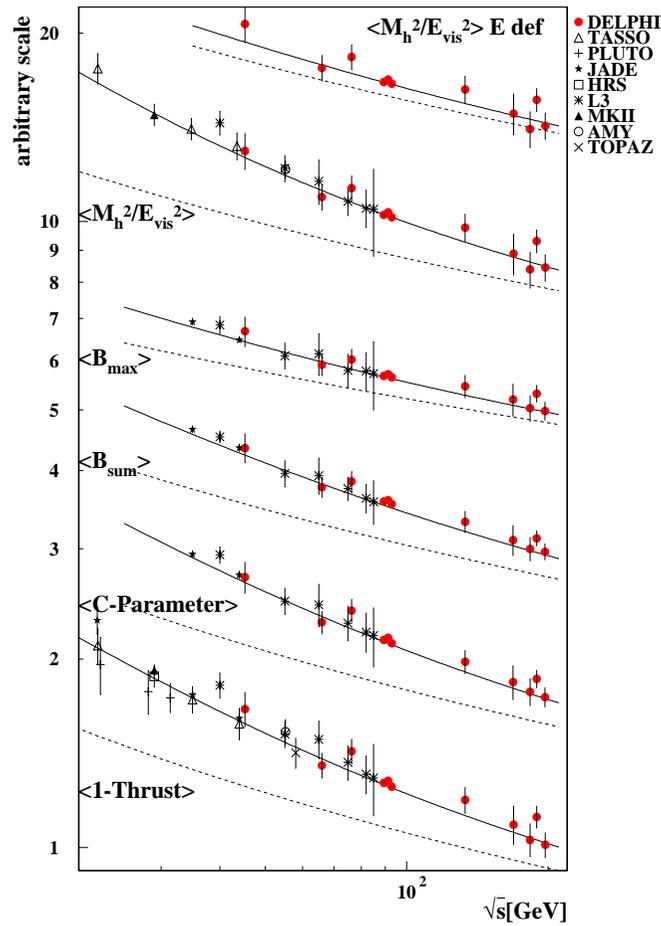
$$\mathcal{P} = \frac{4C_F}{\pi^2} \mathcal{M} \frac{\mu_I}{E_{\text{cm}}} \left[ \alpha_0(\mu_I) - \alpha_s(\mu) - \left( b_0 \cdot \log \frac{\mu^2}{\mu_I^2} + \frac{K}{2\pi} + 2b_0 \right) \alpha_s^2(\mu) \right]$$

Observable specifics are absorbed in  $c_y$ :  $c_{1-T} = 2$   $c_{M_h^2/E_{\text{vis}}^2} = 1$  ...

Power terms (i.e.  $\alpha_0(\mu_I)$ ) depend on renormalisation scheme (in general  $\overline{\text{MS}}$ ).

Fit  $\alpha_0(\mu_I)$  and  $\alpha_s$  to **mean values** and **distributions** of event shapes

# Power Corrections à la Dokshitzer–Webber



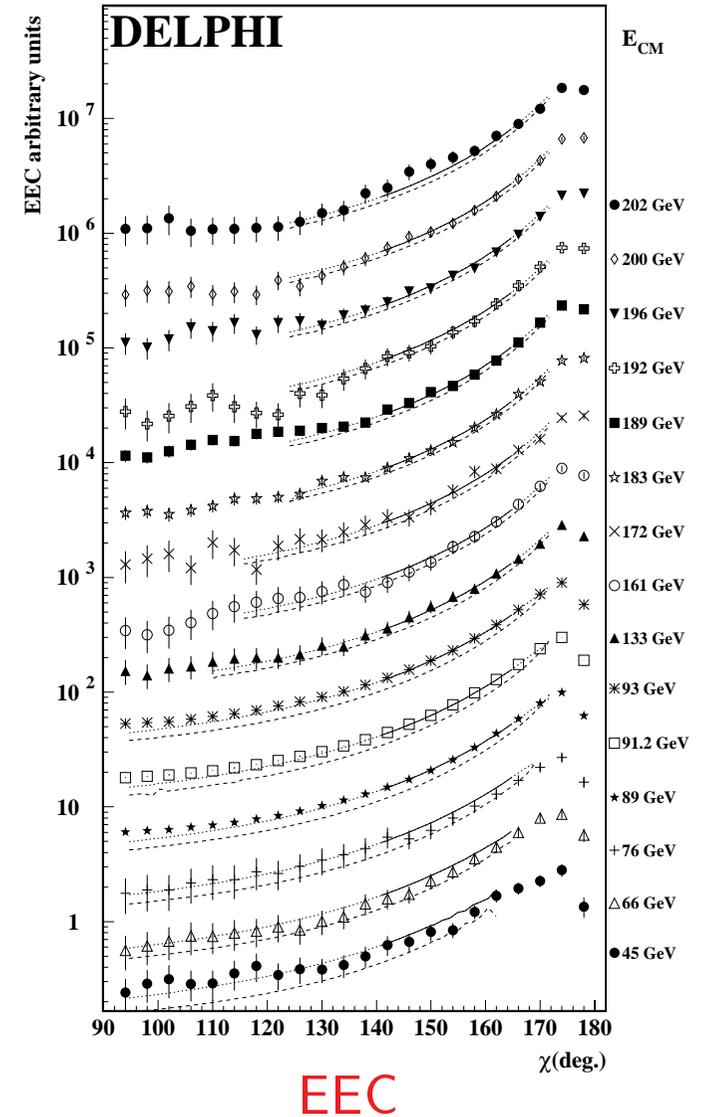
Typically 10% power correction

Successful description of the energy dependence of

← mean values

and

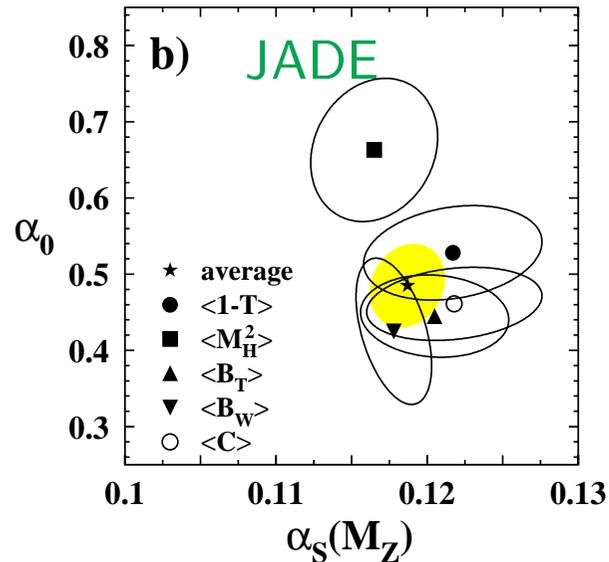
distributions ⇒



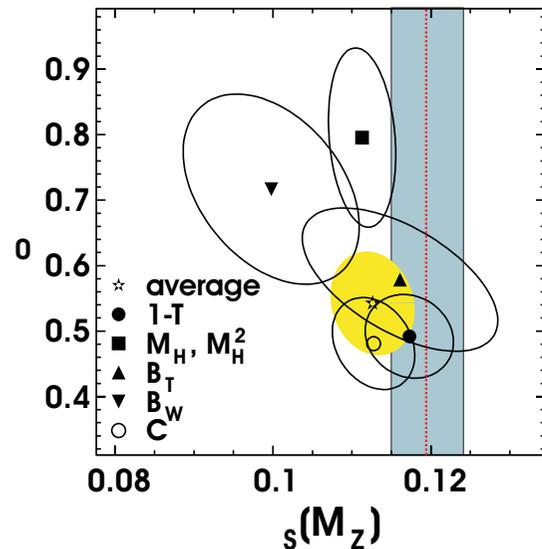
EEC

# Power Corrections à la Dokshitzer–Webber ( $\alpha_s$ & $\alpha_0$ Results)

Mean Values



Distributions



- JADE fits use data from  $E_{\text{cm}} = 14$  to 189 GeV:

- Means:

$$\alpha_s(M_Z) = 0.1187 \pm 0.0014 \pm 0.0001^{+0.0028}_{-0.0015}$$

$$\alpha_0(2 \text{ GeV}) = 0.485 \pm 0.013_{\text{fit}} \pm 0.001_{\text{sys}}^{+0.065}_{-0.043_{\text{th}}}$$

- Distributions:

$$\alpha_s(M_Z) = 0.1126 \pm 0.0005 \pm 0.0037^{+0.0044}_{-0.0030}$$

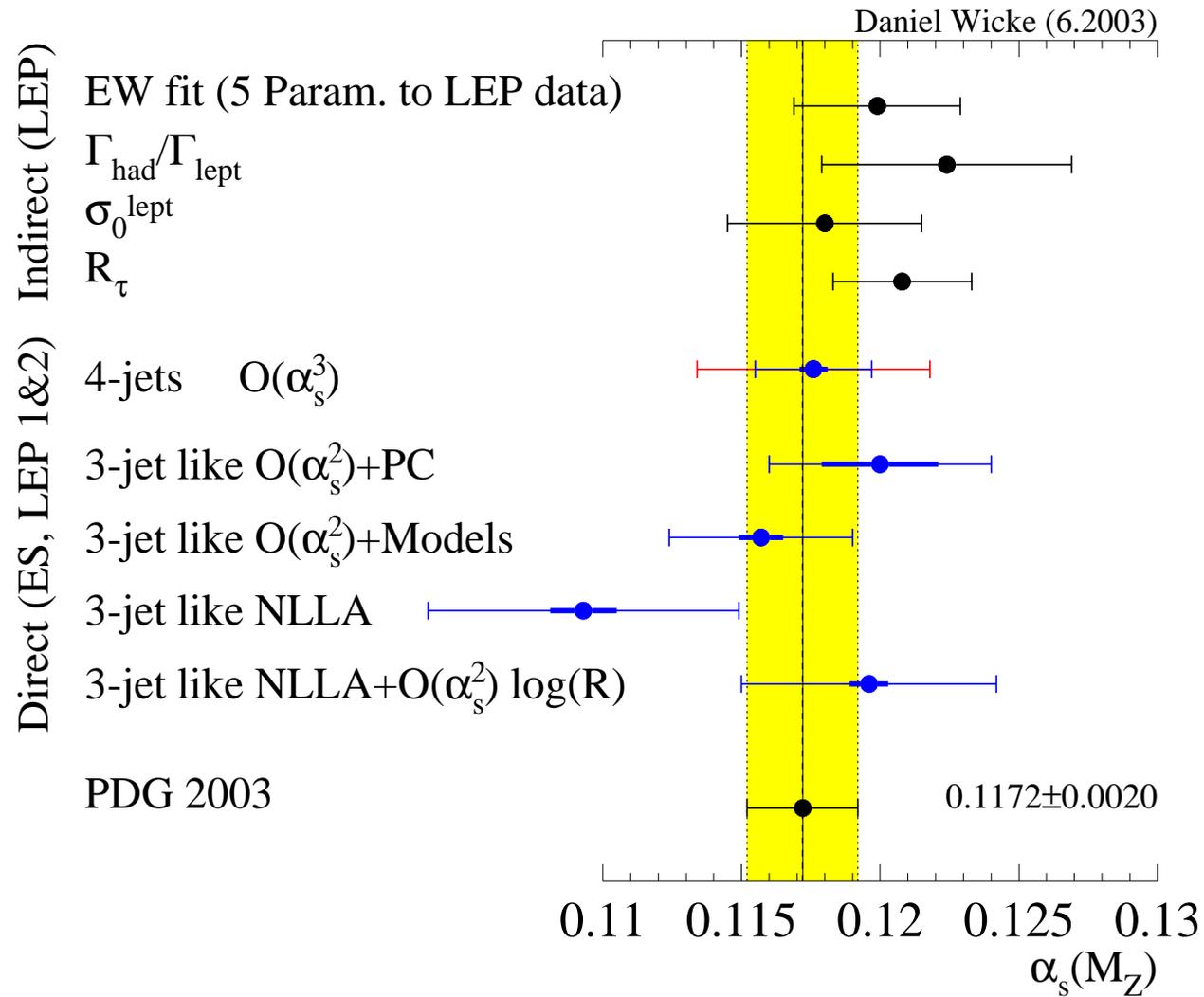
$$\alpha_0(2 \text{ GeV}) = 0.542 \pm 0.005_{\text{fit}} \pm 0.032_{\text{sys}}^{+0.084}_{-0.060_{\text{th}}}$$

- $\alpha_0$  results from means and distributions only consistent within **total** error.

- $\alpha_s^{(\text{PowerCorrections})} < \alpha_s^{(\text{MonteCarlo})}$  !

← light blue band: current LEP average

# Comparison of $\alpha_s$ Results from LEP



# Running of $\alpha_s$

Asymptotic freedom & confinement  $\iff$  running of the QCD coupling  $\alpha_s(E)$

Renormalisation Group Equation controls the running:

$$\frac{\partial \alpha_s}{\partial \ln Q^2} = \beta(\alpha_s) = -\alpha_s^2 \cdot \frac{\beta_0}{4\pi} \left( 1 + \frac{\beta_1}{2\beta_0} \alpha_s + \dots \right)$$

$$\frac{\partial \alpha_s^{-1}}{\partial \ln Q^2} = \frac{\beta_0}{4\pi} \left( 1 + \frac{\beta_1}{2\beta_0} \alpha_s + \dots \right)$$

$\beta$  function  $\iff$    $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f$

$\implies$  is influenced by all strongly interacting particles.

A measurement of  $\beta$  function implies **model independent** limits on hypothetical particles.

# Renormalisation group invariant perturbation theory

Use the observable itself as expansion parameter

→ No dependence on renormalisation scale.

For a mean event shape  $R \propto \langle y \rangle / A_f \sim \alpha_s$  require RGE:

$$\frac{\partial R^{-1}}{\partial \ln Q^2} = \frac{\beta_0}{4\pi} \left( 1 + \frac{\beta_1}{2\pi\beta_0} R + \rho_2 R^2 + \dots \right)$$

→ Allows to measure  $\beta_R \approx \beta$  directly from a mean event shape, e.g.  $\langle 1 - T \rangle$ ;

Solve this RGE like the one for  $\alpha_s$ , determine integration const.  $\Lambda_R \iff \Lambda_{\overline{\text{MS}}}^{\text{QCD}}$

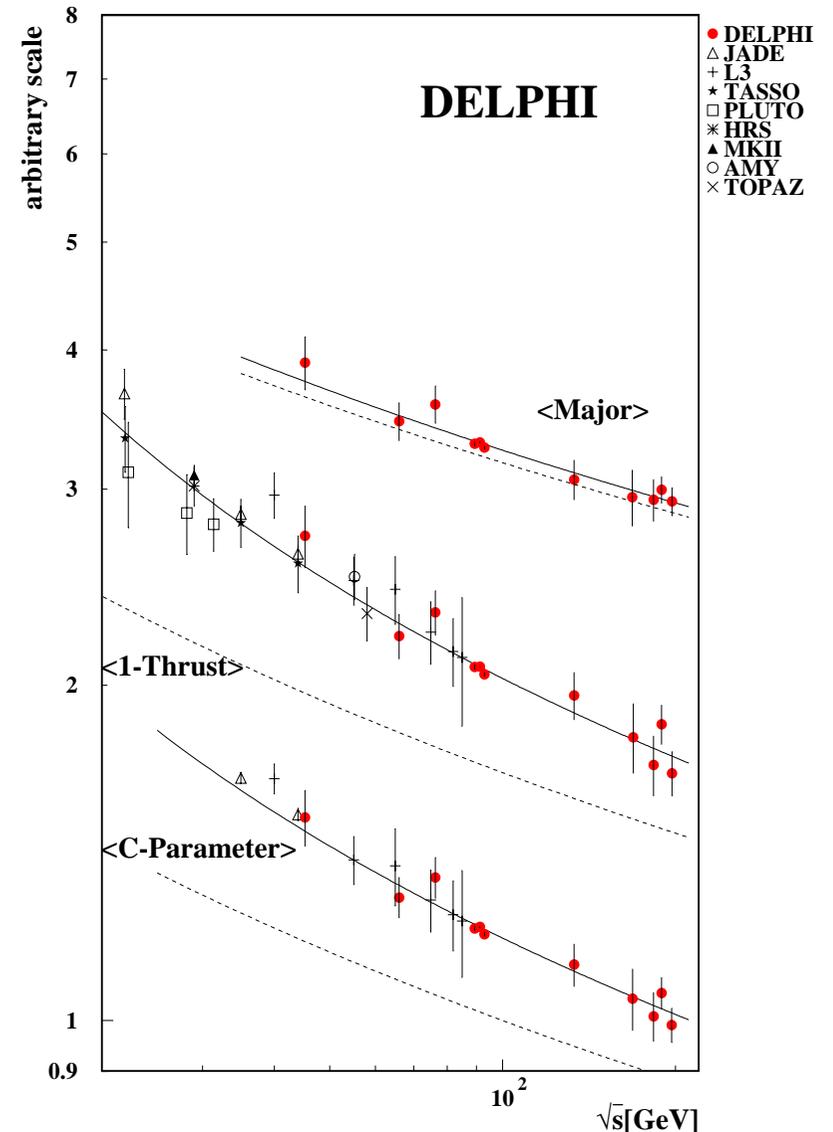
- Allows measurements of  $\Lambda_{\overline{\text{MS}}}^{\text{QCD}}$  without freedom of renormalisation scale.
- RGI is numerically equivalent to ECH; **RGI** resums **UV** divergencies.
- Power corrections can be included in RGI theory.

# RGI with power corrections

- Gives good description of the data  
small spread from 7 observables
- $\alpha_s(m_Z) = 0.1179$ ; spread 0.0020
- Power terms are compatible with zero!

## Comparing Pure RGI to data

- Gives still a good description of the data
- $\alpha_s(m_Z) = 0.1201$ ; spread 0.0020
- Mean values can be described  
**without hadronisation correction** at 2% level  
using a theory without renormalisation scale  
freedom.



Power corrections are to large part missing higher order terms in  $\overline{\text{MS}}$  scheme

# The measurement of the $\beta$ -Function

$\beta_R$  is slope of  $1/R$  vs  $\log s$

From DELPHI  $\langle 1 - T \rangle$ :

$$\frac{\partial R^{-1}}{\partial \ln Q} = 1.38 \pm 0.05 \quad n_f = 4.7 \pm 1.2$$

Uncertainty due to power terms **small!**

Include **low energy** data, extract  $\beta_0$ :

$$\beta_0 = 7.86 \pm 0.32$$

$$n_f = 4.75 \pm 0.44 \text{ (using QCD expression)}$$

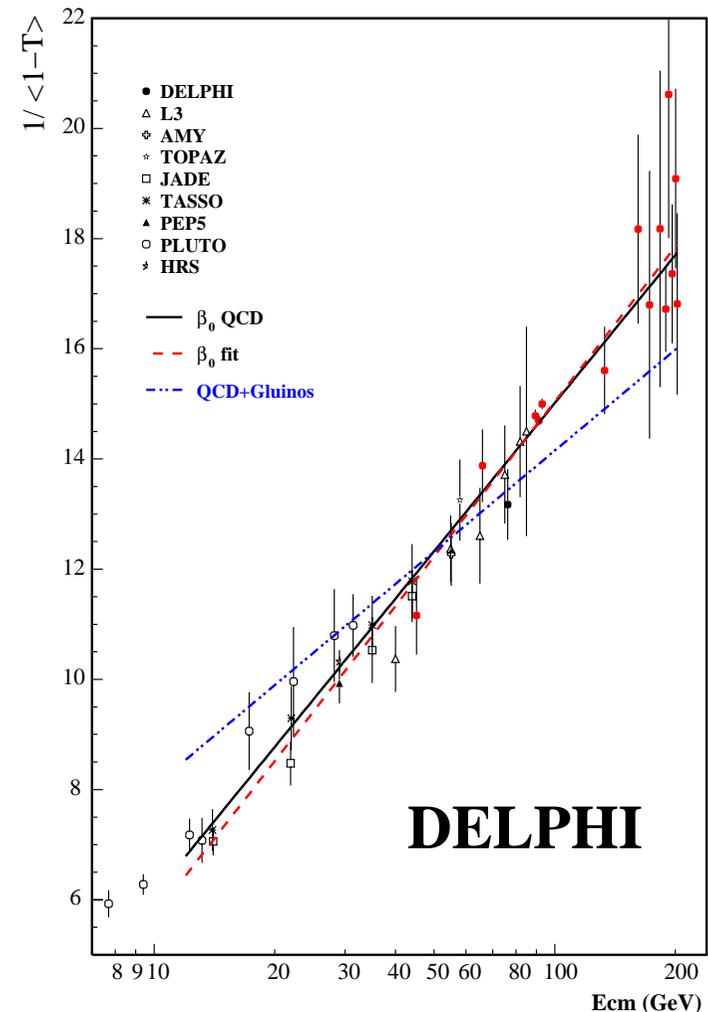
Compare indirect measurement (via  $\alpha_s$ ):

LEP event shapes:

$$\beta_0 = 7.67 \pm 1.63$$

$R_\tau, F_2, F_3, R_Z$ , event shapes ... world data:

$$\beta_0 = 7.76 \pm 0.44$$



QCD + light gluinos excluded

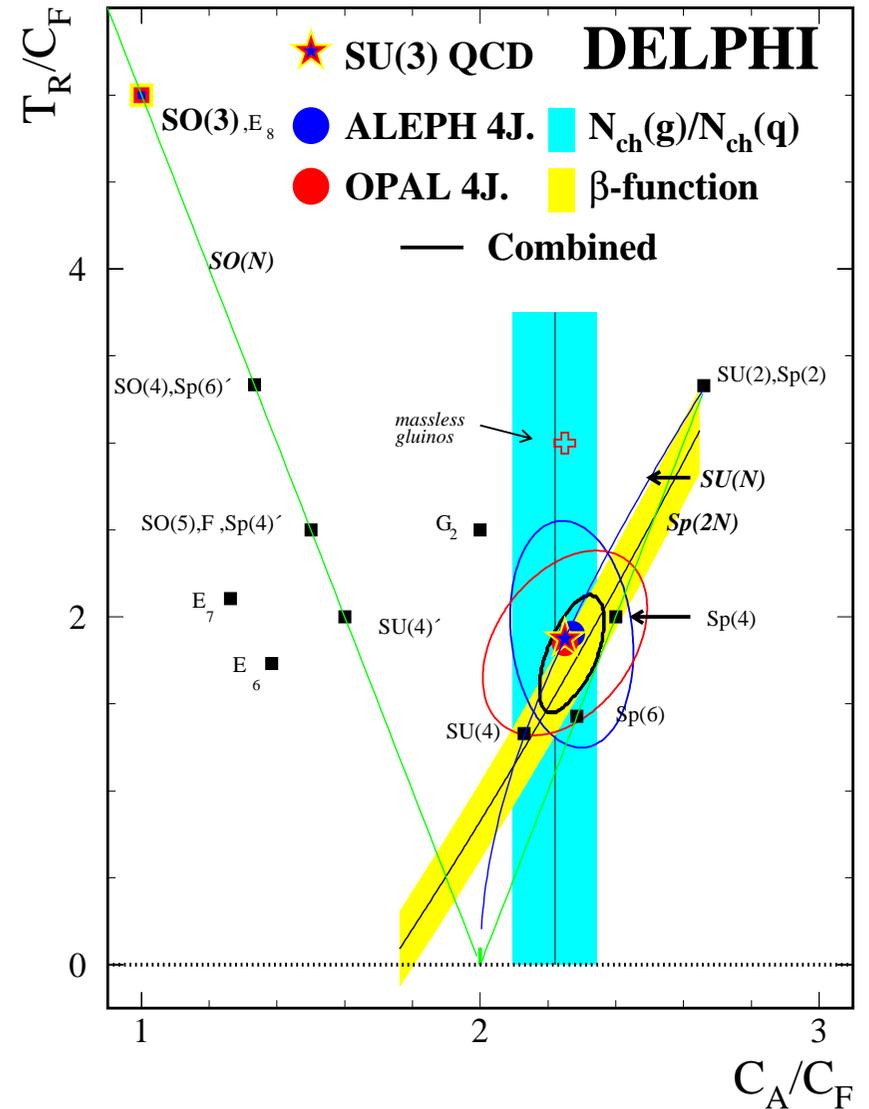
# The Gauge Group of Strong Interactions

the measurements of

- the QCD  $\beta$ -function
- the multiplicity of gluon and quark jets
- the 4-jet angular distributions
- ...

strongly restrict the gauge group of the strong interaction to

$$SU(3)_c$$



# Summary

$e^+e^-$  experiments provide extensive precision tests of QCD

- Heavy quark fragmentation  $\rightarrow$  input for hadron colliders
- Consistent picture of the energy evolution from b factories to LEP II  
 $\alpha_s$  &  $\beta$ -function,  $m_b$ , inclusive distributions
- Absolute measurements of  $\alpha_s$  to be improved  $\rightarrow$  NNLO calculation
- $SU(3)_c$  is experimentally identified as the gauge group of strong interactions