**MEASUREMENT OF ELECTRON BEAM ENERGY USING RADIATION IN THE SUPERPOSITION OF LASER WAVE AND MAGNETIC FIELD**

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**We discuss a method for measurement of energy of a relativistic electron beam using radiation in the combined fields of a circularly polarized laser beam and a magnetic field along direction of electron beam.**

## 1. Spectrum of electrons radiation.

**Earlier it has been shown that the rate of electrons energy change at movement in a field of a laser wave and a magnetic field is given by expression [1, 2]:**

$$\frac{d\gamma}{dt} = \xi\omega_l\sqrt{\frac{2\Omega}{\gamma}} \qquad (1)$$

**where** $\gamma = \varepsilon / mc^2$ **is the relativistic factor of electron,** $\xi = eE/mc\omega_l$ **- parameter of laser intensity,** $E$ **- amplitude of electric field of a wave,** $\omega_l$ **- frequency of a laser,** $\Omega = \omega_c / \omega_l$ ,

$\omega_c = eB/mc$ **and** $B$ **- value of a magnetic field.**

**Expression (1) describes both acceleration of electrons [1,3,4] and generation of radiation by electrons [5, 6] depending on their initial phase.**

According to (1) the loss of energy $\hbar\omega_r$ of electrons at radiation on length $\ell$ is equal (Fig.1):

$$\hbar\omega_r = eE\cdot\ell\cdot\frac{c}{V}\sqrt{\frac{2\Omega}{\gamma}} \qquad (2)$$

and the wavelength of radiation will be

$$\lambda_r = \frac{2\pi\cdot\hbar c}{eE\cdot\ell}\frac{V}{c}\sqrt{\frac{\gamma}{2\Omega}}\ , \qquad (3)$$

where $V$ - velocity of electron.

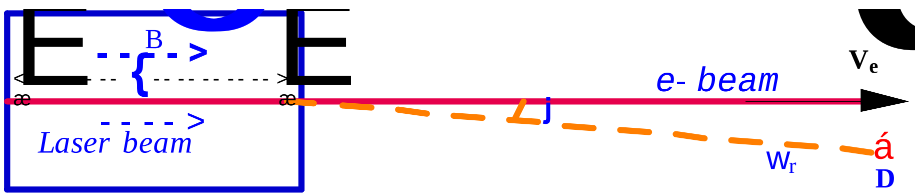


Fig.1

To find the radiation spectrum of electron in a field of a laser wave and in magnetic field we notice that radiation is formed onto the some length of trajectory $\ell$ which electron passes during time $\tau = \ell/V$ [7,8]. Owing to delay of a wave in a point of observation $P$ the duration of radiation time in laboratory system of coordinates will be

$$\Delta t = (1-\vec{k}\vec{V})\tau \qquad (4)$$

where $\vec{k}$ - unit vector in a direction of radiation.

**Because $\Delta t$ has the small value then according to the general properties of wave packages , in a point of observation will be registered a set of harmonics with frequencies [7,8]:**

$$\Delta\omega_r \geq \frac{1}{\Delta t} = \frac{1}{(1-\vec{k}\vec{V})\tau} \qquad \textbf{(5)}$$

**So, the frequency of radiation will be determined according to expression**

$$\omega_r = \frac{V}{\ell(1-\frac{V}{c}\cos\varphi)} , \qquad \textbf{(6)}$$

**and the wavelength of radiation according to (6) will be**

$$\lambda_r = 2\pi\cdot\frac{c}{V}\ell\cdot(1-\frac{V}{c}\cos\varphi) , \qquad \textbf{(7)}$$

**where $\varphi$ is angle between vectors $\vec{V}$ and $\vec{k}$ .**

**For the given energy of electron $\gamma$ , the length of a magnet $L \geq \ell$ should be chosen taking into account that detectors of radiation with high spectral sensitivity and high-speed response exist for lengths of waves $\lambda \leq 12\mu m$ (or for frequencies of radiation $\omega_r \geq 2\cdot 10^{14}$ ).**

**Excluding $\ell$ from (2) and (6) we receive**

$$\left(\hbar\omega_r\right)^2 = \frac{eE\cdot\hbar c}{1-\frac{V}{c}\cos\varphi}\sqrt{\frac{2\Omega}{\gamma}} \qquad \textbf{(8)}$$

**From (8) follows, that at invariable parameters $\gamma$ , $\Omega$ and $E$ the frequency of radiation $\omega_r$ depends on radiation angle $\varphi$ . The radiation frequency $\omega_r$ has the maximal value for $\varphi = 0$ and decreases with growth of angle $\varphi$ .**

**If to exclude $\omega_r$ from (2) and (6) then the length of radiation formation will be**

$$\ell^2 = \frac{\hbar c}{eE}\left(\frac{V}{c}\right)^2 \sqrt{\frac{\gamma}{2\Omega}} \frac{1}{1 - \frac{V}{c}\cos\varphi} \qquad \textbf{(9)}$$

**From (9) follows, that at invariable parameters $\gamma$ , $\Omega$ and $E$ the length of radiation formation $\ell$ depends on radiation angle $\varphi$ . The length $\ell$ has the maximal value for $\varphi = 0$ and decreases with growth of angle $\varphi$ .**

**On the other hand we can take into account that electrons in a magnetic field have discrete spectrum of energy**

$$\varepsilon_{n,\zeta} = [m^2 + P_z^2 + eB(2n + 1 + \zeta)]^{1/2} \ , \qquad \textbf{(10)}$$

**then electrons can radiate only at transitions between levels of energy $\varepsilon_n \to \varepsilon_{n'}$ .**

**Using the spectrum of electron energy (10) and the law of energy-momentum conservation at the photon radiation:**

$$\varepsilon_{n,\zeta} = \varepsilon_{n',\zeta} + \omega_r \, , \qquad P_{z,0} = P_z + \omega\cos\varphi \qquad \textbf{(11)}$$

**for frequency of radiation we have expression:**

$$\omega_r = \frac{(n-n')\omega_c}{\gamma\left(1-\frac{V_z}{c}\cos\theta\right)}, \qquad \textbf{(12)}$$

**where** $\theta$ **- is angle between axis** $Z$ **and direction of radiation** $\vec{k}$ **,** $n-n' = 1,2,3,\ldots$

**From numerical estimations of radiation frequency** $\omega_r$ **on expression (6) and (12) follows, that for the radiation transitions the big change of quantum numbers** $n-n' >> 1$ **are possible (Fig.2). Because** $\omega_r$ **is proportional to** $n-n'$ **and** $\omega_c$ **then use of magnetic fields of relatively small value is possible** $(B \le 1kGs)$ **.**

**But, according to (2) use of a magnetic field of small value is limited, because it demands application of the laser with the greater intensity.**

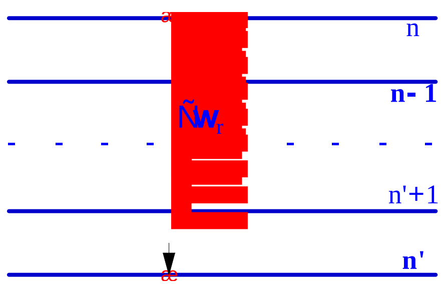


**Fig.2**

**Let's consider numerical estimations.**

**I. If** $\gamma = 2\cdot 10^{5}$ , $\ell = 100cm$ **and** $\varphi = 0$ **then according to (6) we have:** $\omega_r = 2.4\cdot 10^{19}\sec^{-1}$**.**

**According to (12) in a case of** $\theta = 0$ **and** $n - n' = 6\cdot 10^{3}$ **we have:** $\omega_c = 10^{10}\sec^{-1}$ **what corresponds to magnetic field** $B = 1kGs$ **.**

**II. If** $\gamma = 10^{2}$ **,** $\ell = 3cm$ **and** $\varphi = 0$ **then according to (6) we have:** $\omega_r = 2\cdot 10^{14}\sec^{-1}$ **.**

**According to (12) in a case of** $\theta = 0$ **and** $n - n' = 10^{2}$ **we have:** $\omega_c = 10^{10}\sec^{-1}$ **what corresponds to magnetic field** $B = 1kGs$ **.**

## 2. Determination of electron beam energy

**For determination of electron energy we use expression (8) what can be written as**

$$\gamma - \cos\varphi\cdot\sqrt{\gamma^{2} - 1} = \sqrt{\gamma}\cdot G \qquad \textbf{(13)}$$

**where**

$$G = \frac{eE\cdot\hbar c}{(\hbar\omega_r)^{2}}\sqrt{2\Omega} \qquad \textbf{(14)}$$

**The relation (13) is equation of the fourth degree relative to** $\gamma$ **. Two of four solutions** $\gamma$ **of the equation (13) are imaginary and have not the physical sense. Other two solutions of** $\gamma$ **are real and have the physical sense. Expressions of these solutions are bulky and consequently we do not represent here. Dependence** $\gamma$ **from** $G$ **is shown on Fig.3. The upper and the lower branch on the diagram correspond to these two solutions in case of** $\varphi = 5\cdot 10^{-4}$

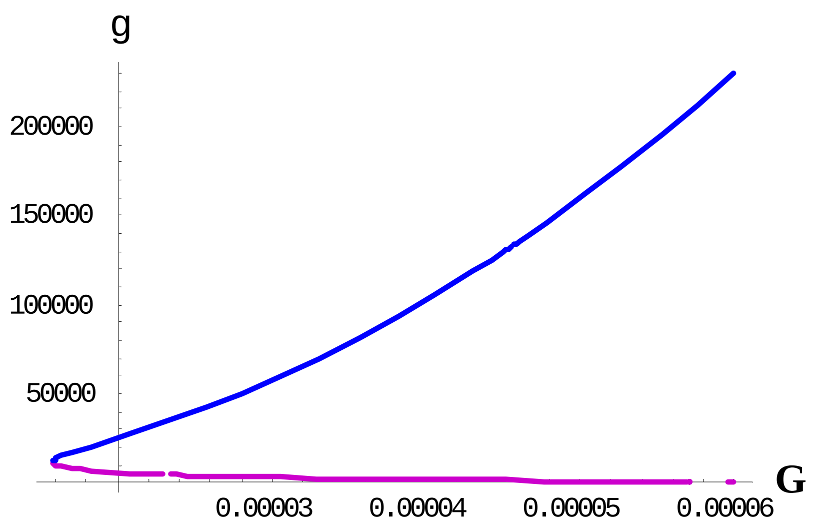


**Fig.3**

**The solution of the equation (13) in case of $\varphi = 0$ should be considered separately. In this case the (13) represents the equation of thirds of degree. Two of these solutions are imaginary and have no physical sense. The real solution can be written as:**

$$\gamma = \frac{G^2}{12} + \frac{\left(24 + G^4\right) G^{2/3}}{12\left(216 + 36G^4 + G^8 + 24\sqrt{3}\sqrt{27 + G^4}\right)^{1/3}} + \frac{\left(216 + 36G^4 + G^8 + 24\sqrt{3}\sqrt{27 + G^4}\right)^{1/3}}{12G^{2/3}} \qquad \textbf{(15)}$$

**Dependence $\gamma$ from $G$ in a case of $\varphi = 0$ for some interval of $G$ it is shown on Fig.4.**

**It is obvious, that the case $\varphi = 0$ is of interest for measurement of low energy of electrons.**

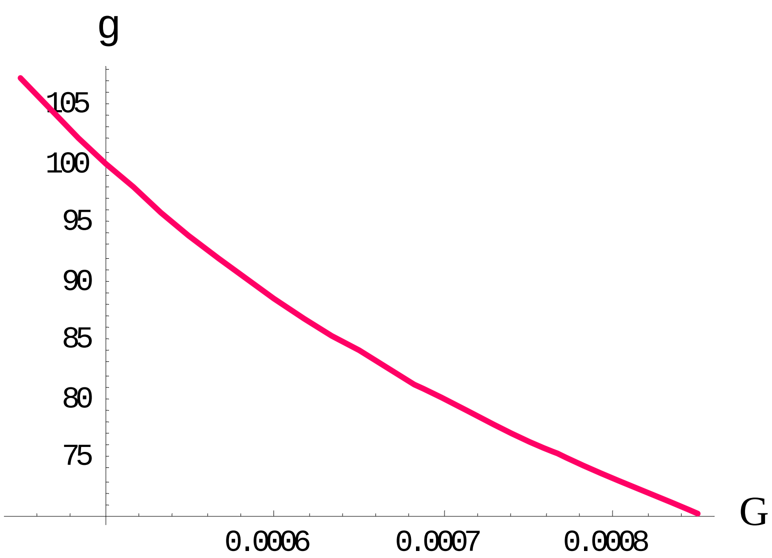


**Fig.4**

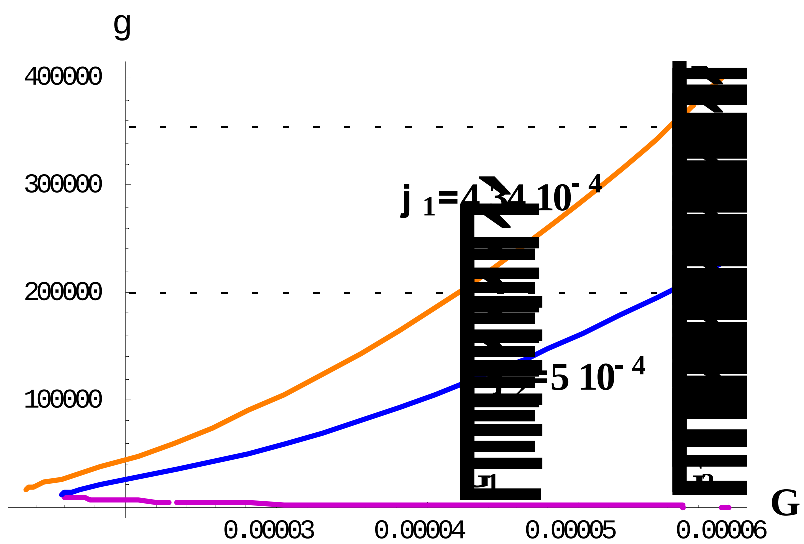


**Fig.5**

**From Fig.3 follows that for measuring the high energy of electrons should be use the upper branch of a curve. Dependence** $\gamma$ **from** $G$ **for radiation angles** $\varphi_1 = 4.34 \cdot 10^{-4}$ **and** $\varphi_2 = 5 \cdot 10^{-4}$ **are shown on Fig.5. As it is seen from Fig. 5 the value of** $\gamma$ **can be found with various** $G_1$ **and** $G_2$ **corresponding to different angles** $\varphi_1$ **and** $\varphi_2$ **. This property of dependence** $\gamma$ **from** $G$ **we shall use to find** $\gamma$ **.**

## 2.1. The first version of determination of electron energy.

**Using the relation between amplitude of electric field $E$ of a wave and intensity $I_l$ of laser:**

$$E[V/cm] \cong 19.4 \cdot \sqrt{I_l [W/cm^2]} \qquad \textbf{(16)}$$

**the expression for $G$ (14) can be written as:**

$$G = \frac{19.4\sqrt{I_l} \cdot \hbar c}{(\hbar\omega_r)^2}\sqrt{2\Omega} \qquad \textbf{(17)}$$

**Now, according to (13) and (17) we have expression:**

$$\gamma - \cos\varphi \cdot \sqrt{\gamma^2 - 1} = \sqrt{\gamma}\,\frac{19.4\sqrt{I_l} \cdot \hbar c}{(\hbar\omega_r)^2}\sqrt{2\Omega} \;, \qquad \textbf{(18)}$$

**and energy of electrons can be found if to measure four parameters:** $\varphi$ **,** $\omega_r$ **,** $I_l$**,** $\Omega$ **.**

**If** $I_l$ **and** $\Omega$ **are known and constant, then it is enough to measure** $\varphi$ **and** $\omega_r$ **to find energy of electrons.**

## 2.2. The second version of determination of electron energy.

**We assume that two detectors simultaneously register radiation of electrons under angle** $\varphi_1$ **with frequency** $\omega_{r,1}$ **and under angle** $\varphi_2$ **with frequency** $\omega_{r,2}$ **(Fig.5).**

**Taking into account that in this case in expression (18) $I_l$ and $\Omega$ are identical for both angles $\varphi_1$ and $\varphi_2$ , we have**

$$\left(\gamma - \cos\varphi_1 \cdot \sqrt{\gamma^2 - 1}\right)\omega_{r,1}^2 = \left(\gamma - \cos\varphi_2 \cdot \sqrt{\gamma^2 - 1}\right)\omega_{r,2}^2 \qquad \textbf{(19)}$$

**whence it follows that**

$$\gamma = \frac{\omega_{r,1}^2 \cos\varphi_1 - \omega_{r,2}^2 \cos\varphi_2}{\sqrt{\left(\omega_{r,1}^2 \cos\varphi_1 - \omega_{r,2}^2 \cos\varphi_2\right)^2 - \left(\omega_{r,1}^2 - \omega_{r,2}^2\right)^2}} \;. \qquad \textbf{(20)}$$

**From (20) follows, in order to find energy of electrons it is necessary to measure the following four parameters:** $\varphi_1$ **,** $\varphi_2$ **,** $\omega_{r,1}$ **,** $\omega_{r,2}$ **.**

## 3. Estimation of the laser intensity, frequency and lengths of radiation formation

**3.1. Frequency of radiation** $\omega_r$ **under various angles of radiation** $\varphi$ **for the given** $\gamma$ **and** $\ell$ **can be found according to the formula (6). In particular for** $\gamma = 2 \cdot 10^5$ **and** $\ell = 100cm$ **the frequency of radiation under various angles** $\varphi$ **is shown on Fig.6. The frequency of radiation in case of** $\varphi = 0$ **is equal** $\omega_r = 2.4 \cdot 10^{19} \sec^{-1}$ **(or energy of photons will be** $\hbar\omega_r \approx 15$ **KeV).**

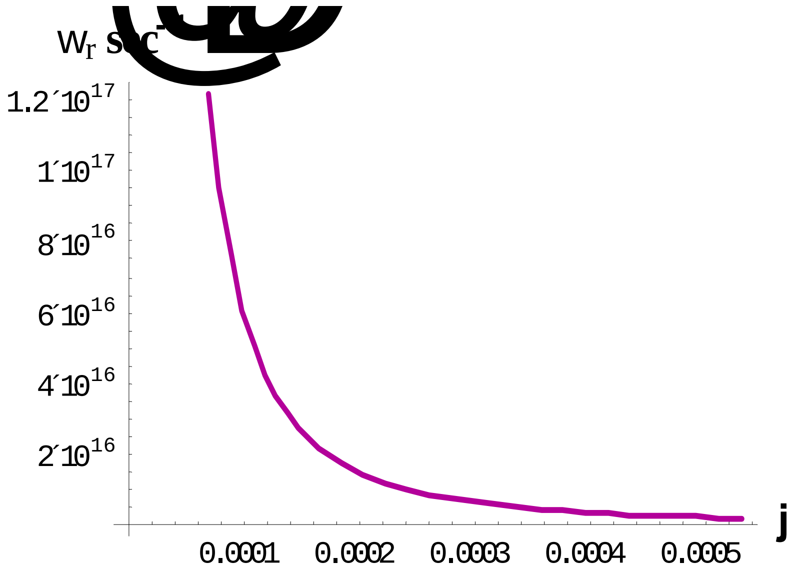


**Fig.6**

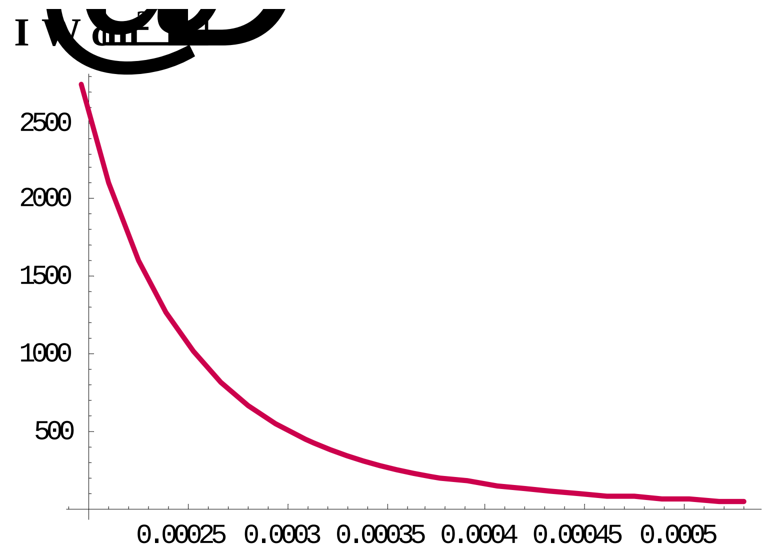


**Fig.7**

**3.2. Intensity of the laser** $I_l$ **necessary for radiation under different angles** $\varphi$ **can be found according to a relation (16). For this purpose at first we find the value of** $E$ **for the given** $\gamma$ **,** $\ell$ **,** $\Omega$ **and for various angles** $\varphi$ **from formula (9). In particular for** $\gamma = 2 \cdot 10^5$ **,** $\ell = 100cm$ **and** $\Omega = 10^{-3}$ **the intensity of the laser necessary for radiation of a electron under various angles** $\varphi$ **is shown on Fig.7. The intensity of the laser in case of** $\varphi = 0$ **is equal** $I_l = 5.6 \cdot 10^9 W / cm^2$ **.**

**As it can be seen from Fig.7, for given parameters** $\gamma = 2 \cdot 10^5$ **,** $\ell = 100cm$ **and** $\Omega = 10^{-3}$ **lower quantity for intensity of the laser turn out at angles of radiation** $\varphi = 0.4 \cdot 10^{-3} \div 0.5 \cdot 10^{-3}$ **, when accordingly we have:** $I_l = 170 \div 70 W / cm^2$ **and** $\hbar\omega_r = 2.5 \div 1.7eV$ **.**

**3.3. Taking into account relations (16) and (9) the length of radiation formation can be written as:**

$$\ell^2 = \frac{\hbar c}{19.4\sqrt{I_{las}}}\left(\frac{V}{c}\right)^2 \sqrt{\frac{\gamma}{2\Omega}} \frac{1}{1 - \frac{V}{c}\cos\varphi} \qquad \textbf{(21)}$$

**In a case of** $\varphi = 0$ **from (21) we have approximately**

$$\ell^2 \cong \frac{2\gamma^2 \hbar c}{19.4\sqrt{I_{las}}} \sqrt{\frac{\gamma}{2\Omega}} \qquad \textbf{(22)}$$

**We assume that the electron radiates on length** $\ell_0 = L_M$ **when intensity of the laser is equal** $I_{las,0}$ **. Now, if we use the laser of intensity** $I_{las,1} < I_{las,0}$ **and at the same time** $\gamma$ **and** $\Omega$ **are constant then according to (22) we have:**

$$\frac{\ell_1^2}{\ell_0^2} = \sqrt{\frac{I_{las,0}}{I_{las,1}}} > 1 \qquad \textbf{(23)}$$

**where** $\ell_1$ **is the necessary length of radiation corresponding to** $I_{las,1}$ **.**

**So, the necessary length** $\ell_1$ **is more than** $\ell_0 = L_M$ **and therefore radiation for intensity** $I_{las,1} < I_{las,0}$ **is impossible.**

**3.4 Let's consider the case when quantityes $\gamma$ , $I_{las}$ and $\Omega$ are constant. Taking into account that $\gamma >> 1$ and $\varphi << 1$ then from (21) we have approximately**

$$\ell^2 \cong \frac{\hbar c}{19.4\sqrt{I_{las}}}\sqrt{\frac{\gamma}{2\Omega}} \cdot \frac{2\gamma^2}{1+\varphi^2\gamma^2} \qquad \textbf{(24)}$$

**Let's assume that the electron radiates under angle $\varphi_0$ on length $\ell_0$ (Fig.8). Let's find the length $\ell_1$ on which will radiate electron under angle $\varphi_1 < \varphi_0$ . According to (24) we have:**

$$\frac{1+\varphi_0^2\gamma^2}{1+\varphi_1^2\gamma^2} \cong \frac{\ell_1^2}{\ell_0^2} > 1 \qquad \textbf{(25)}$$

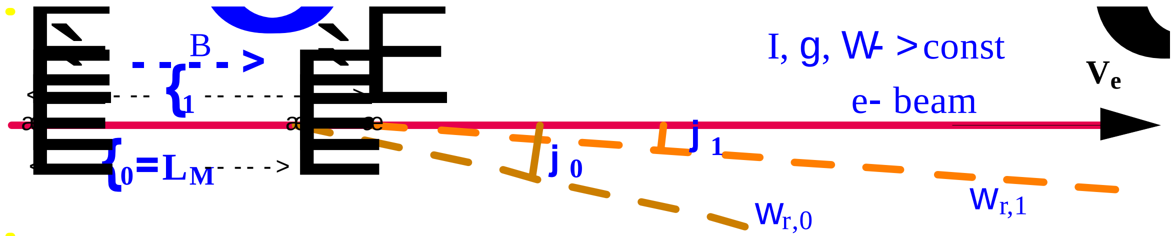


**Fig.8**

**Because $\ell_1 > \ell_0$ , therefore under angle $\varphi_1 < \varphi_0$ the electron cannot radiate.**

**3.5 Similarly to cases from above if $\gamma$, $I_{las}$, $\varphi$ are constant and we varies $\Omega$ then from (21) we receive**

$$\ell_1^2 = \ell_0^2\sqrt{\frac{\Omega_0}{\Omega_1}} \qquad \textbf{(26)}$$

Now if $\Omega_1 < \Omega_0$ then $\ell_1 > \ell_0 = L_M$ and so radiation of electrons is impossible.

So, from consideration of version 3.3, 3.4 and 3.5 follows, that the radiation of electrons depending on $I_{las}$, $\varphi$ and $\Omega$ has threshold behaviour and can be used at measurement of their energy.

## 4. Measuring of electron energy in the presence of energy spread

On Fig.9 is shown the dependence $\gamma$ from $G$ for constant $I_{las}$ , $\Omega$ and $\varphi$ according to (18). Besides that the distribution of electrons over energy is shown symbolically.

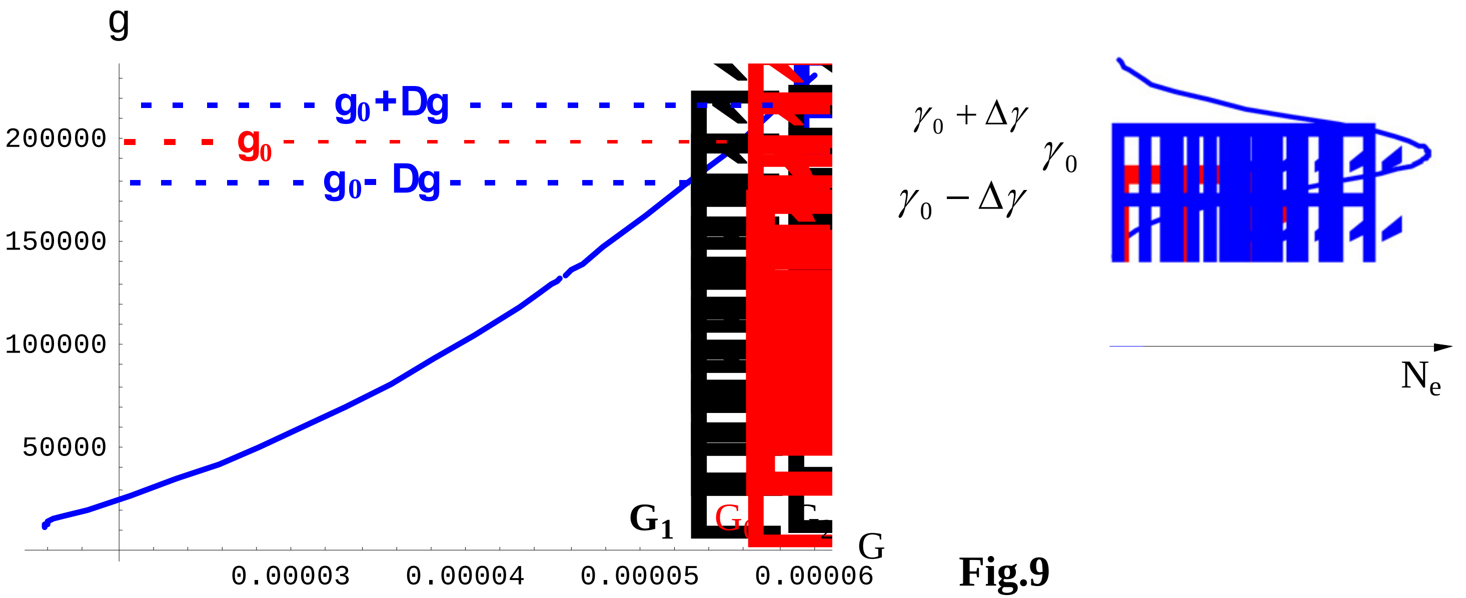


Fig.9

**From (18) follows, that to electron energies $\gamma_0 - \Delta\gamma$ and $\gamma_0 + \Delta\gamma$ corresponds different frequencies $\omega_{r,1}$ (or $G_1$ ) and $\omega_{r,2}$ (or $G_2$ ). The maximum of radiation intensity with some frequency $\omega_{r,0}$ corresponds to centre of distribution over number of electrons $N_e$ and falls on energy $\gamma_0$ . So, by maximum of intensity of radiation we can find energy $\gamma_0$ .**

## 5. Summary

- **In a range of energy interesting to us 100 - 500 GeV the frequency of radiation of electron of the discussed method is much less than in Compton effect or at synchrotron radiation. It is important because then measurement of radiation frequency by existing detectors, with high spectral sensitivity and high-speed response, becomes simpler.**
- **The big efficiency (cross-section of process) of radiation by electrons of the discussed method is an advantage of this method.**
- **This method can be applied to measurement of energy of electrons both in the range of high energy $\varepsilon = 100 \div 500$ GeV and in the range of low energy $\varepsilon \geq 50$ MeV.**
- **The possibility of transitions with the big change of quantum numbers $n - n' >> 1$ of electron energy at radiation allows to use magnetic fields concerning small value.**

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