

Beam tracking through ESA spectrometer chicane

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$$\frac{d}{dt}(m\dot{\vec{r}}) = e[\dot{\vec{r}}, \vec{B}] + e\vec{E}$$

- equation of electron's motion in the magnetic and electrical field

$$\frac{d}{dt}(m\dot{\vec{r}}) = e[\dot{\vec{r}}, \vec{B}] + e\vec{E} - \frac{P}{c^2}\dot{\vec{r}} \quad \text{where} \quad P = \frac{2e^4 B^2 \gamma^2}{3m^2 c} \quad \begin{array}{l} P\text{-instantaneous} \\ \text{power of the} \\ \text{synchrotron radiation} \end{array}$$

- equation of electron's motion in the magnetic and electrical field with presence of the synchrotron radiation

$$\ddot{x} = \frac{e}{m_0 + \frac{W}{c^2}}(\dot{y}B_z - \dot{z}B_y + E_x) - \frac{2e^4 B_x}{3(m_0 + \frac{W}{c^2})c^3} \left(1 + \frac{W}{E_0}\right)^2 \frac{\dot{x}}{c^2}$$

- projection of the equation of electron's motion to x-axis

$$m = \frac{m_0 c^2 + W}{c^2} = m_0 + \frac{W}{c^2} \quad \begin{array}{l} W\text{-kinetic energy of electron} \end{array}$$

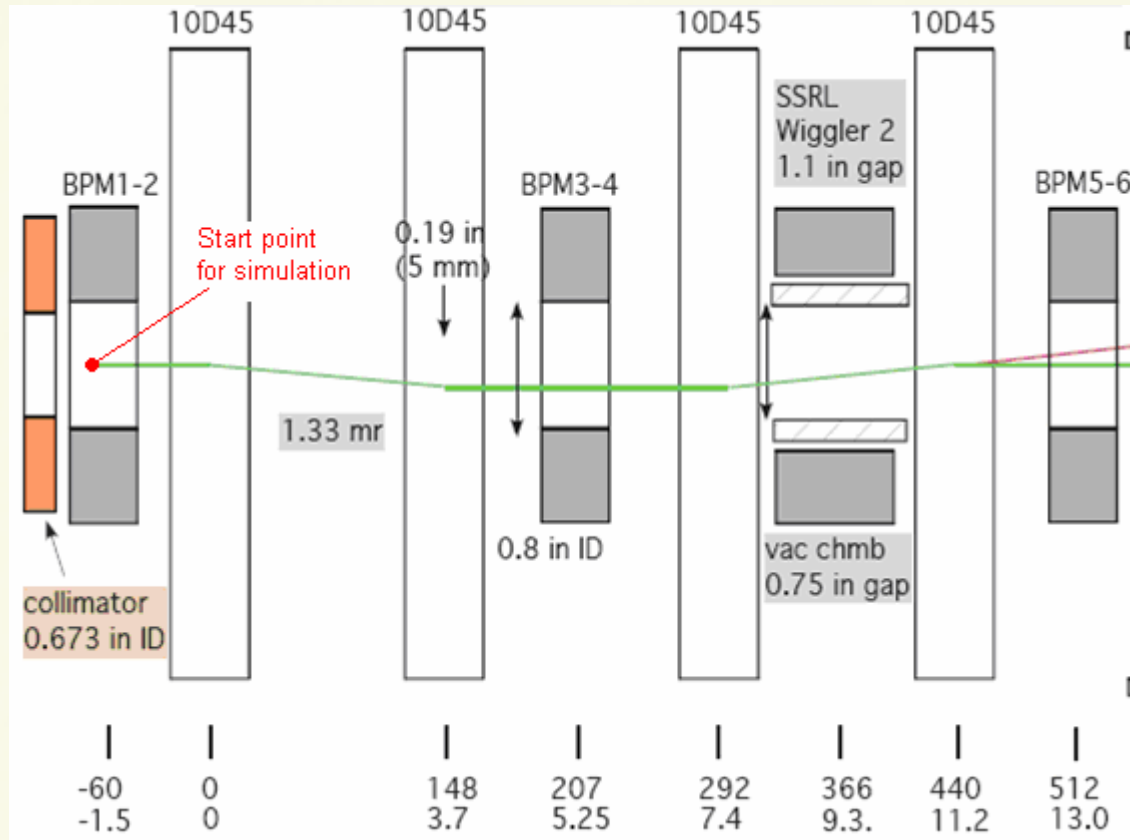
$$\begin{aligned}
\ddot{x} &= a(\dot{y}B_z - \dot{z}B_y + E_x) - bB_x^2\dot{x} \\
\ddot{y} &= a(\dot{z}B_x - \dot{x}B_z + E_y) - bB_y^2\dot{y} \\
\ddot{z} &= a(\dot{x}B_y - \dot{y}B_x + E_z) - bB_z^2\dot{z}
\end{aligned}
\quad \text{where} \quad
a = \frac{e}{m_0 + \frac{W}{c^2}} \quad
b = \frac{2e^4(1 + \frac{W}{E_0})^2}{3(m_0 + \frac{W}{c^2})^3 c^3}$$

- equations of electron's motion in Cartesian coordinate system (point means differentiation respect to time (time – independent variable))

Used coordinate system was adjusted with those one which Nikolay Morozov have used in his simulations of the spectrometer magnets' field.

This system of the equations was integrated by means of Runge - Cutter method of 4-th order.

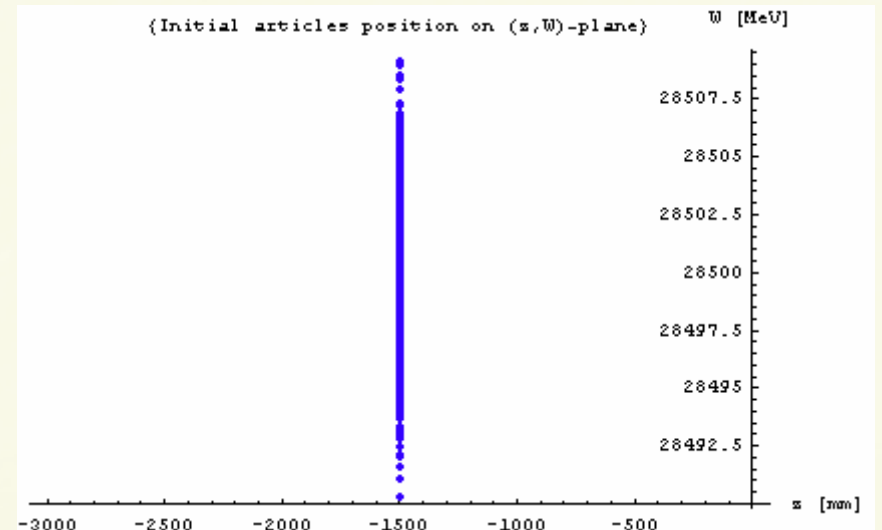
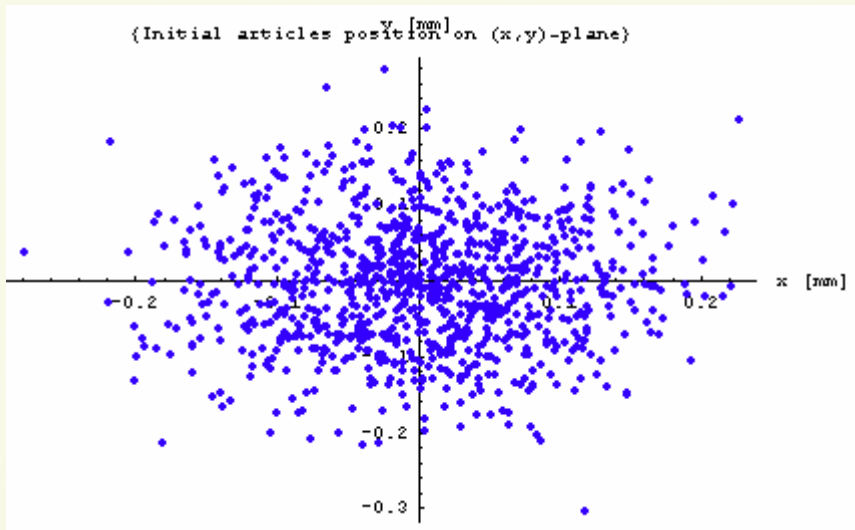
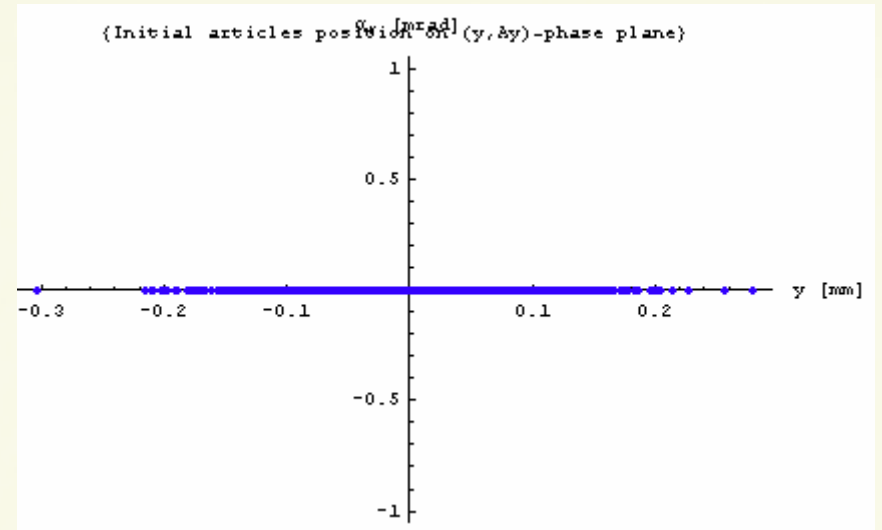
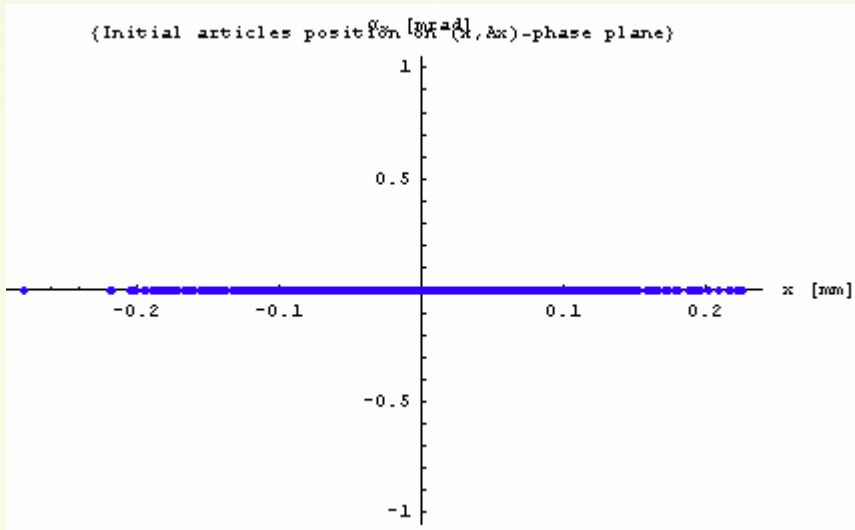
Initial data at the start point ($z=-1.5\text{m}$)



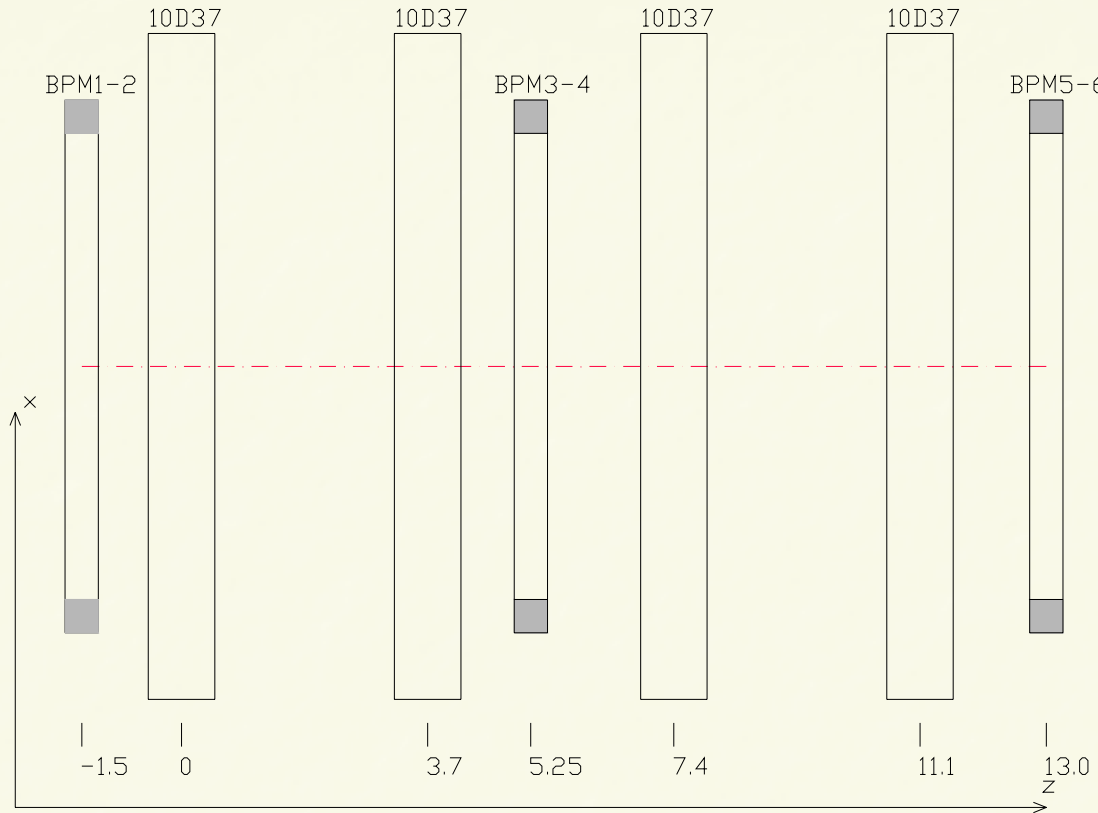
Spectrometer chicane scheme (taken from ChicaneSketch1.16.pdf)

Bunch of 1000 particles was generated at $z=-1.5\text{m}$

Portraits of generated bunch on different phase planes

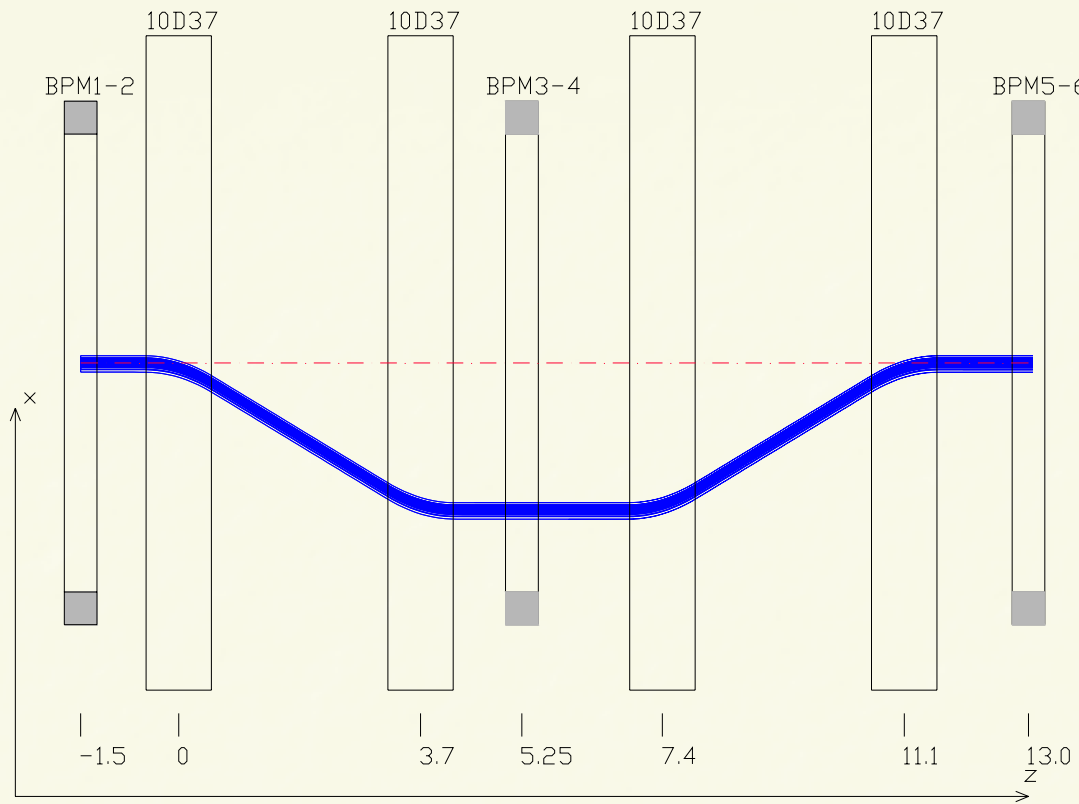


$\epsilon_x=0$ $\epsilon_y=0$ $W_{\text{average}}=28500.1$ MeV $dW/W_{\text{average}} = \pm 0.02\%$



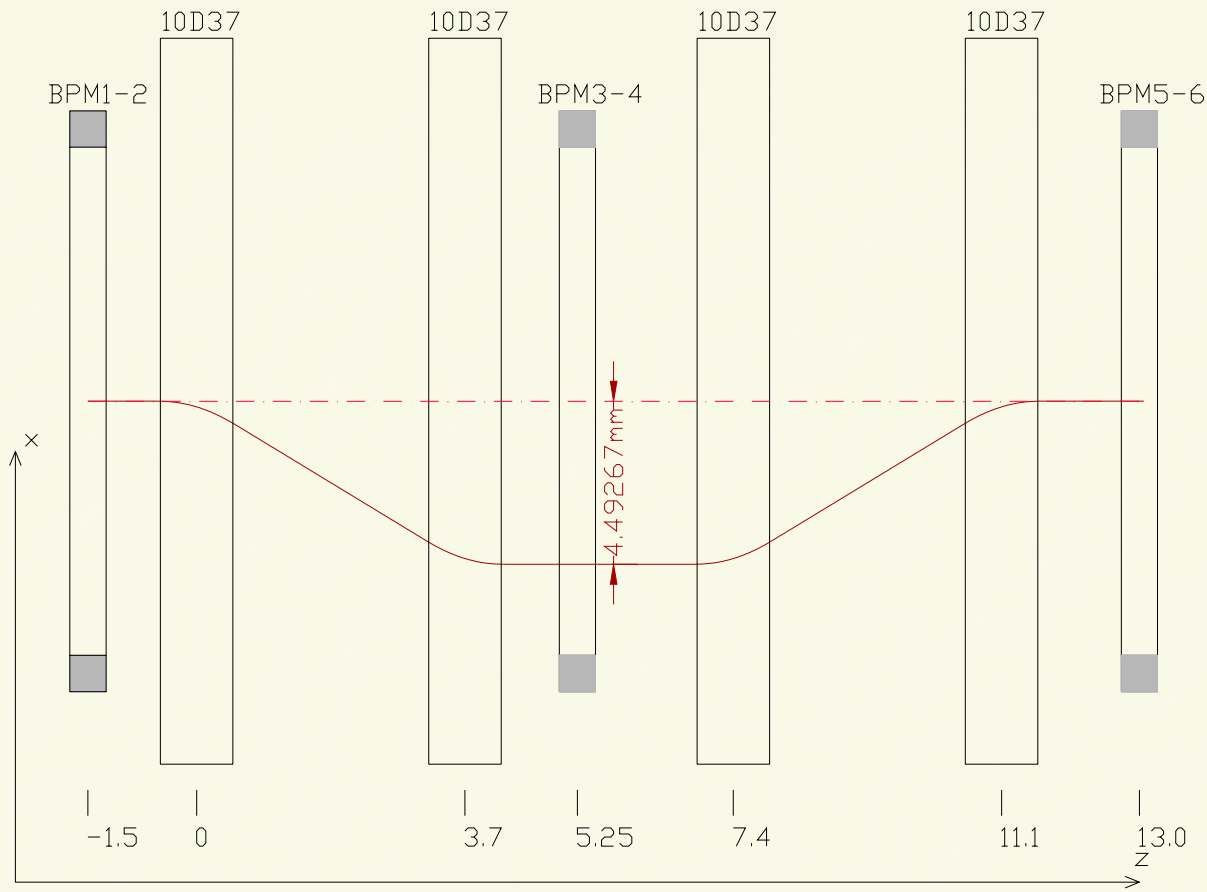
Chicane scheme used in simulations of beam dynamics (in the locations of BPM bunch position was measured)

Magnetic field map for whole chicane was constructed using map of 10D37 magnet



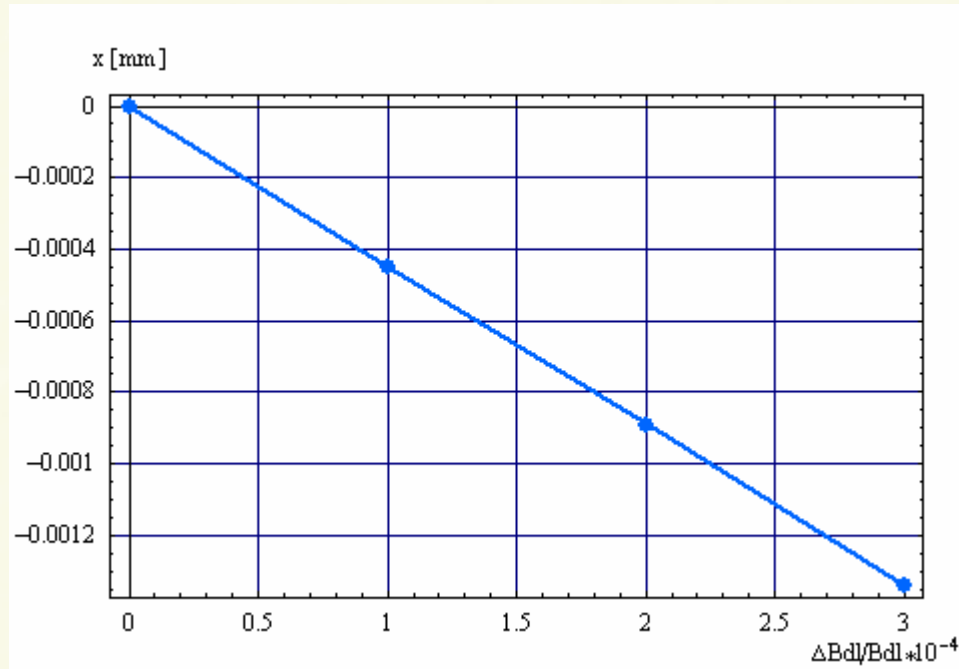
Plan view of the bunch passing through the spectrometer chicane

Beam x-coordinate at position $z=-1.5$ m (BPM1-2) equals 0.0003 mm,
 at $z=5.25$ m (BPM3-4) equals -4.4927 mm
 at $z=13.0$ m (BPM5-6) equals 0.0003 mm.

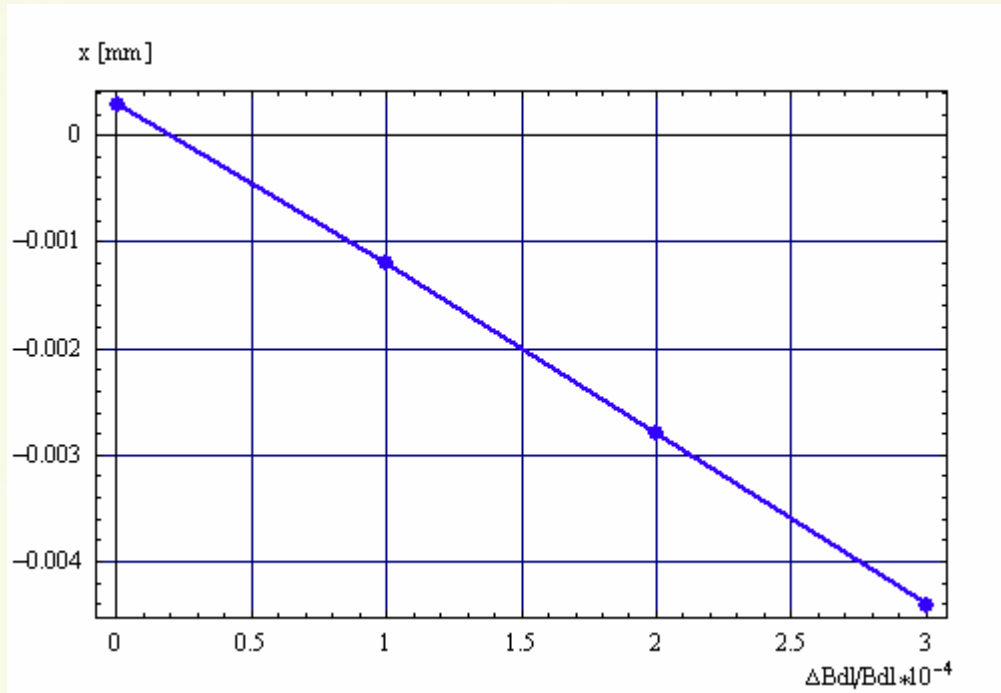


X-z-trajectory of the bunch center

Dependence of the beam center's x-position shift at z=5.25m (BPM3-4) versus the relative changing of the magnetic field in all magnets of the chicane (since the magnets will be connected in series) was obtained.



Beam center's x-position shift at z=5.25m versus relative changing of the magnetic field value in all the magnets at the same time



Beam center's x-position at $z=13\text{m}$ (BPM5-6) versus relative changing of the magnetic field value in 1-st magnet

Conclusions

Program code for beam tracking through the spectrometer chicane was created.

Test tracking of the bunch using 10D37 magnetic field map was carried out.

Dependence of beam center's position at $z=5.25\text{m}$ (BPM3-4) and $z=13\text{m}$ (BPM5-6) versus relative changing of the magnetic field were obtained. The same dependences for other variants of the chicane magnet parameters can be obtained and then used to calibrate the BPMs.