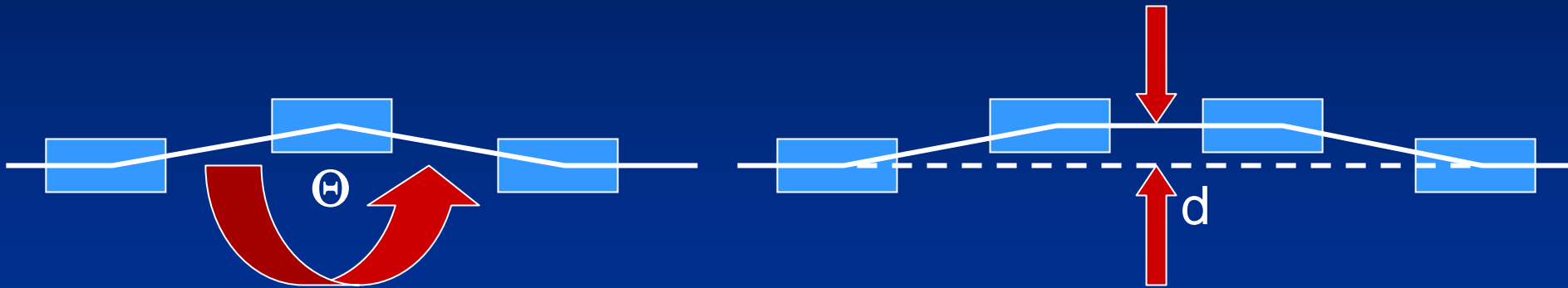


# Comparison of 3- and 4-magnet-versions of the ILC energy spectrometers

K.Hiller, DESY Zeuthen, Yerevan meeting 10/06



Measure the kink angle  $\theta$   
of the electron trajectory

Measure the offset  $d$  in  
respect to no magnetic field

Use Beam Position Monitors  
→ Compare E-resolution

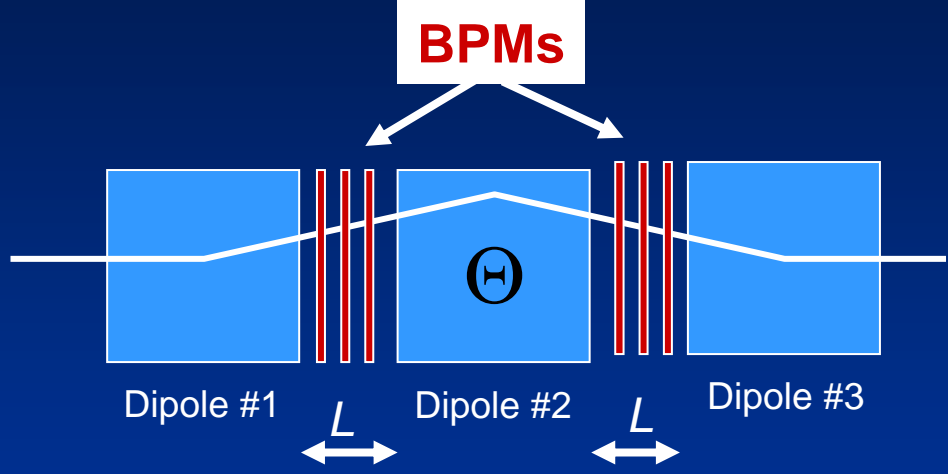
# To remind: 3-Magnet-Spectrometer

$$E = c \cdot Bl / \Theta$$



$$\left(\frac{\Delta E}{E}\right)^2 = \left(\frac{\Delta Bl}{Bl}\right)^2 + 2f(N) \cdot \left(\frac{E \cdot \Delta x}{c \cdot L \cdot Bl}\right)^2$$

$Bl$  ... analyzing field integral  
 $f(N)$  ... factor for N BPMs (2.0 ... 1.0)  
 $\Delta x$  ... BPM resolution  
 $L$  ... length of the measuring range



## Instrumentation:

- 1) Analyzing magnet dipole #2 with integral field  $Bl$
- 2) BPMs with resolution  $\Delta x$  to measure slopes of e-trajectory

Dipoles #2 and #3 not relevant for  $\Theta$ -measurement !!!

# To remind: 3-Magnet-Spectrometer

For standard settings:

$$L = 10\text{m}$$

$$\Delta B/B = 2 \cdot 10^{-5}$$

$$\Delta x = 200\text{nm}$$

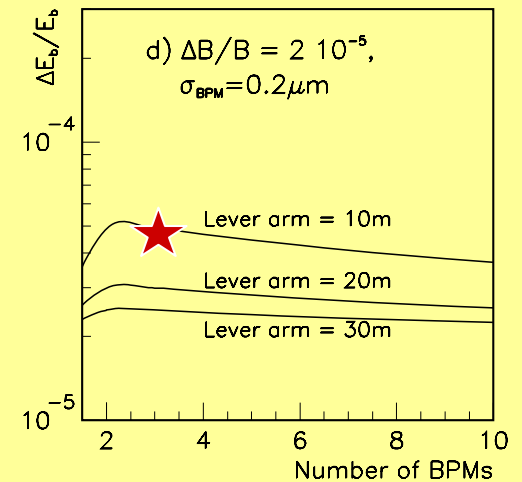
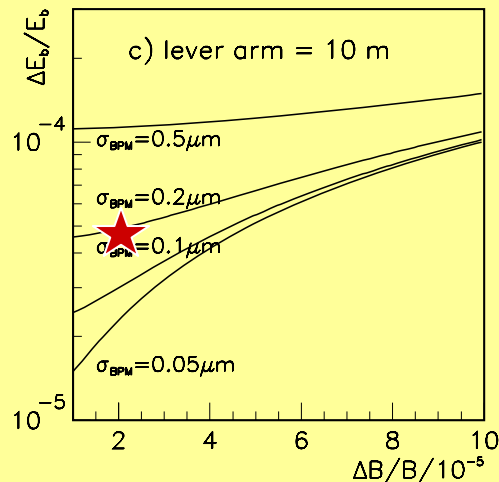
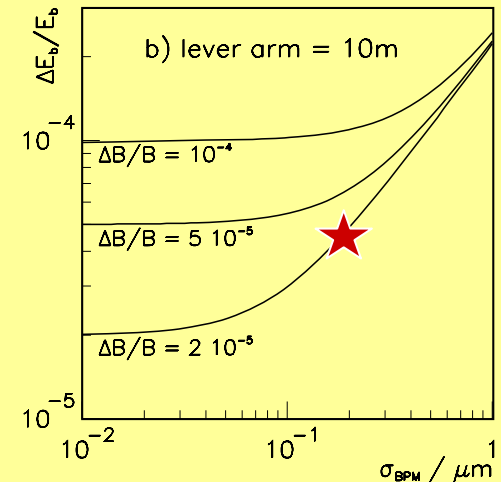
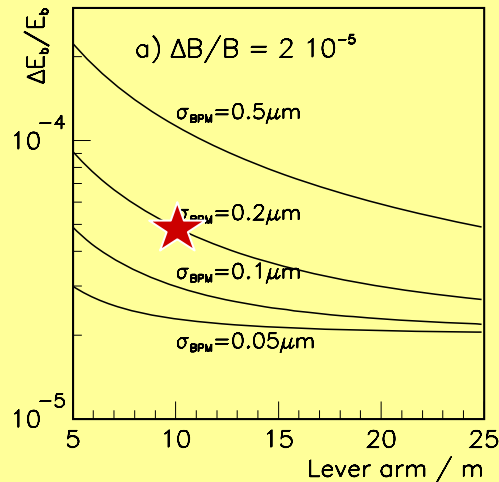
$$\# \text{ BPMs} = 3 \dots 4$$

one obtains:

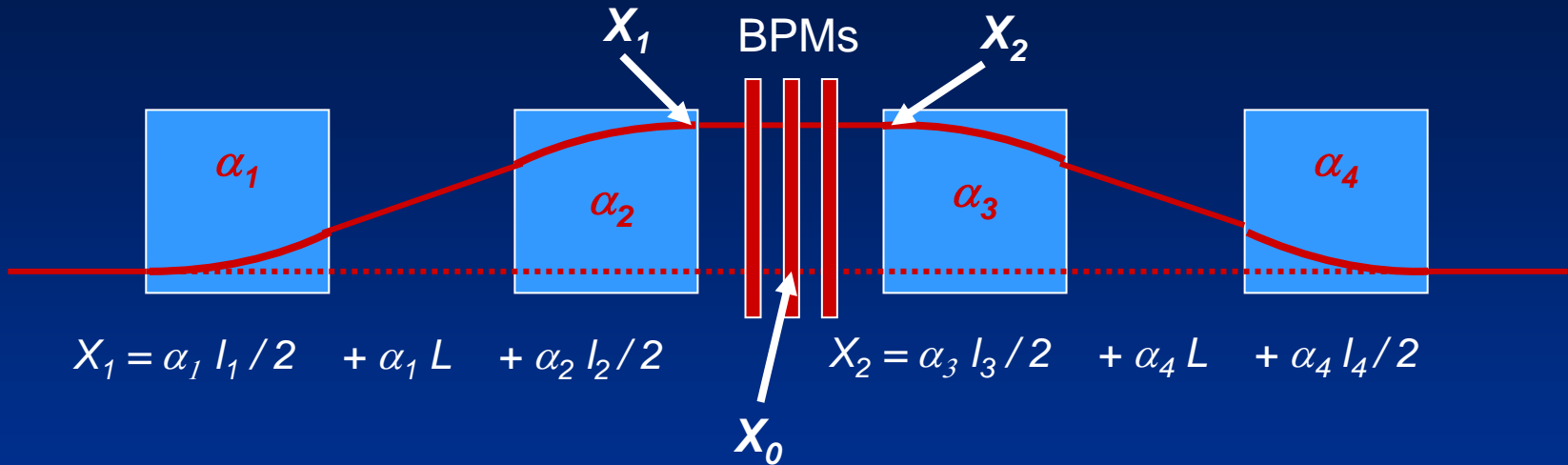
$$\rightarrow \Delta E/E \sim 5 \cdot 10^{-5} \quad \star$$

For more details see  
published ILC note  
LC-DET-2004-029

## Energy Spectrometer Resolution - 3 Magnets



# 4 - Magnet - Spectrometer



$$X = (X_1 + X_2) / 2 = (\alpha_1 l_1 + \alpha_2 l_2 + \alpha_3 l_3 + \alpha_4 l_4) / 4 + (\alpha_1 L + \alpha_4 L) / 2$$

Using  $a = c B l / E$  gives

$$E = 1/X \ c/4 \ {B_1 l_1 (l_1 + L) + B_2 l_2^2 + B_3 l_3^2 + B_4 l_4 (l_4 + L)}$$

- all 4 magnets contribute to offset X and to E-measurement error
- Assumption BPM range > 5 mm !!!

# 4 - Magnet - Spectrometer

... the partial derivatives gives the error of the E-measurement:

factor related to 2 measurements averaged over N BPMs

$$\left(\frac{\Delta E}{E}\right)^2 = \sum \left(\frac{\Delta Bl_i}{Bl_i}\right)^2 + 2/N \cdot \left(\frac{E \cdot \Delta x}{c \cdot (l + L/2) \cdot Bl}\right)^2$$

$Bl_i$  ... field integrals

$N$  ... number of BPMs

$\Delta x$  ... BPM resolution

$L$  ... distance between magnets

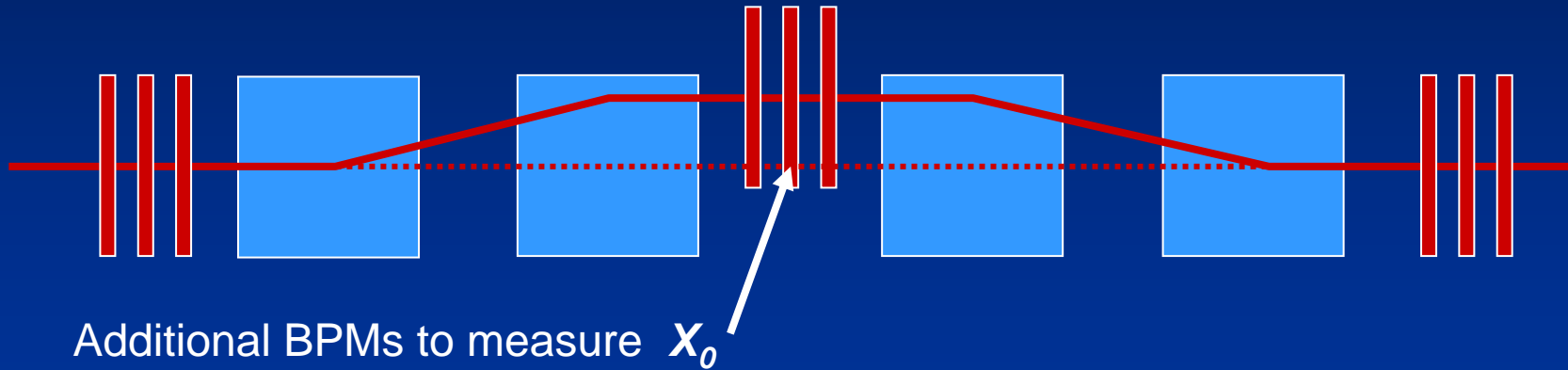
$l$  ... magnet length  $l_1 = l_2 = l_3 = l_4$

... for comparison  
we assume :

$$Bl_1 = Bl_2 = Bl_3 = Bl_4$$

# Beam jitter measurement

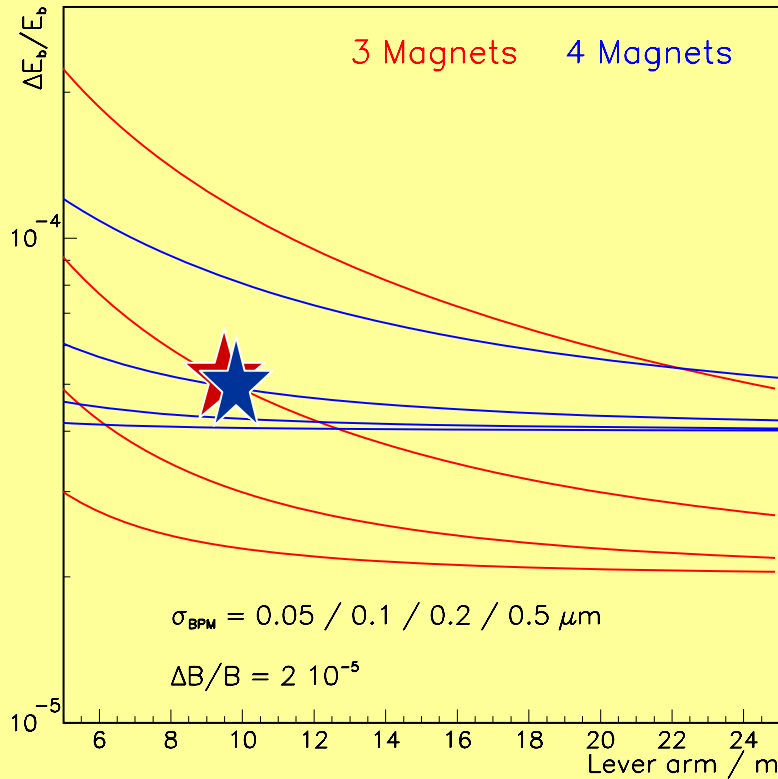
... version to take into account beam position jitters:



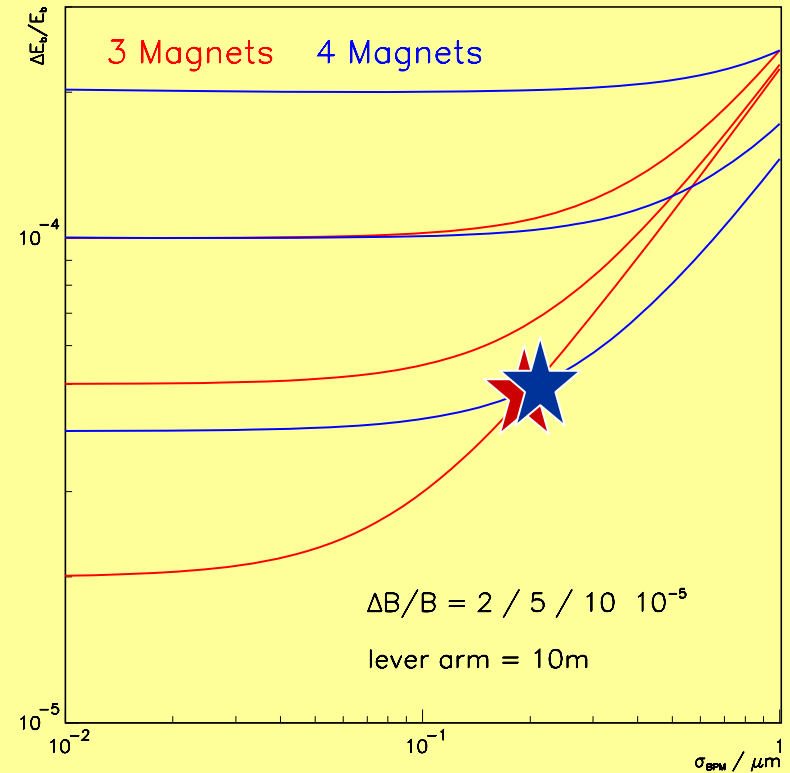
Error of  $X_0$  measurement taken into account – same error as position measurement by the middle BPM triplet  
→ E-resolution plots will not be influenced (strongly)

# Comparison 3 versus 4 magnets (1)

## E-Resolution vs Distance of Magnets



## E-Resolution vs BPM Resolution

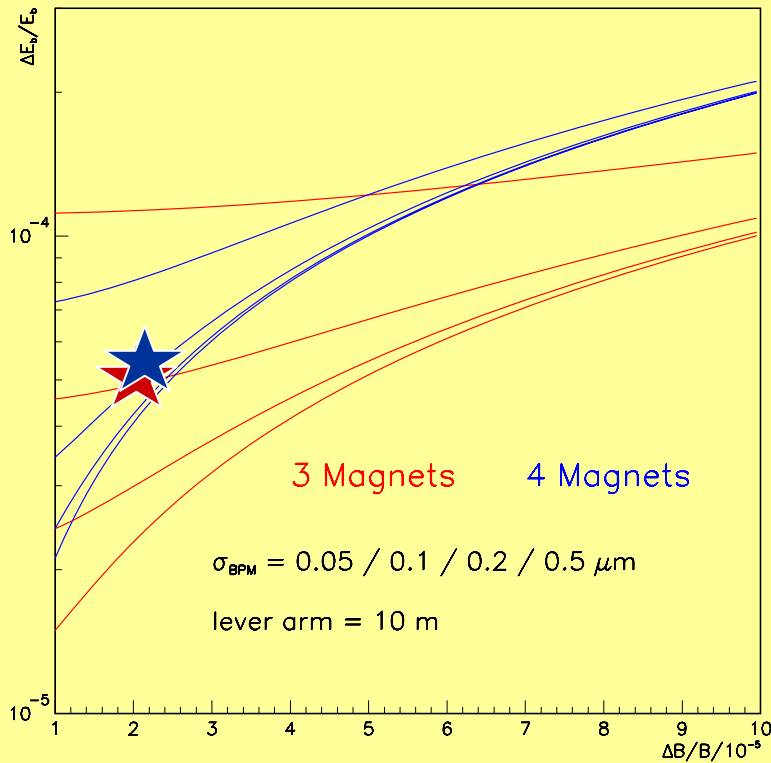


→ for standard settings very similar E-resolution

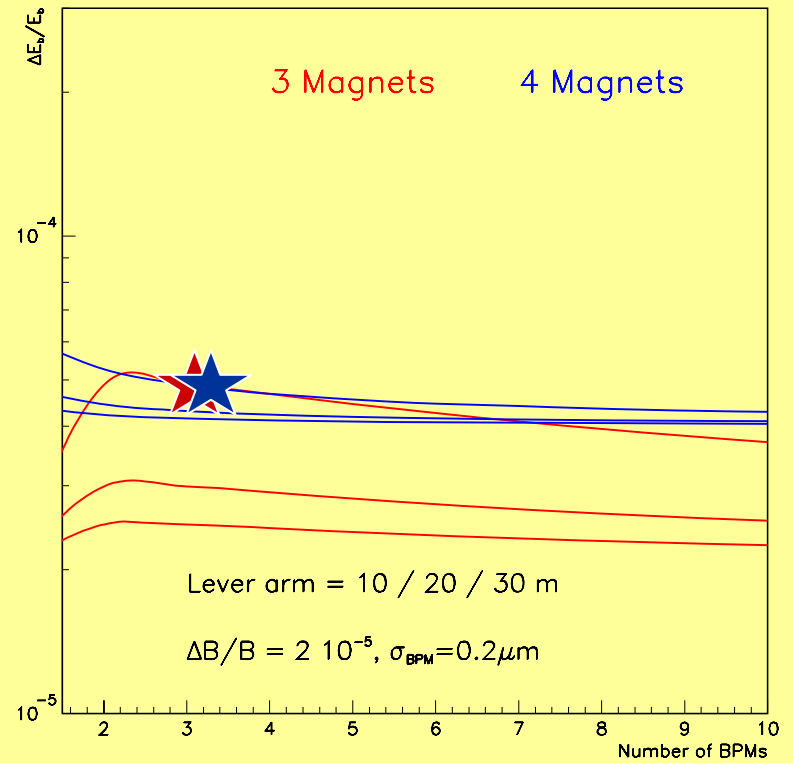
→ 3 magnets better with larger lever arm or smaller BPM resolution

# Comparison 3 versus 4 magnets (2)

## E-Resolution vs Magnetic Field Precision



## E-Resolution vs Number of BPMs

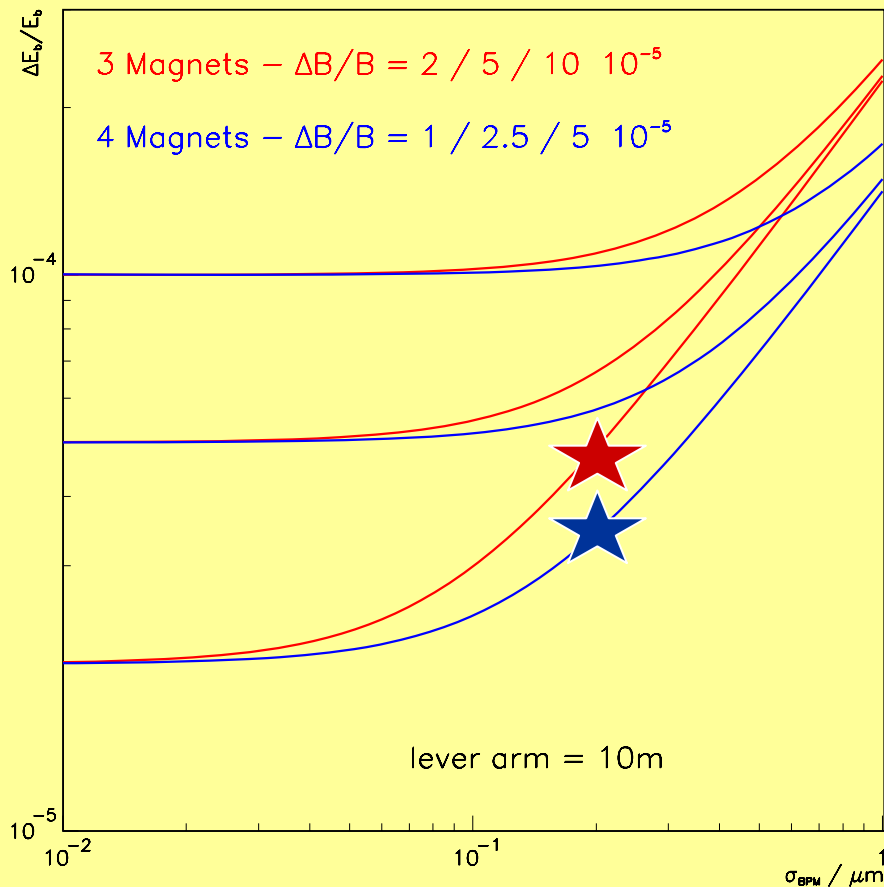


- again same E-resolution for standard setting
- 3 magnets better in case of worse field precision
- dependence on # BPMs small and similar



# Comparison 3 versus 4 magnets (3)

## E-Resolution vs BPM Resolution



Assume factor 2 smaller field integral error for 4-magnet-version ...



... same E-resolution as 3-magnet-version in case of asymptotic BPM-resolution

# Summary

- ❑ for spectrometer “standard” settings \*) the E-resolution of both versions is very similar  $\sim 5 \times 10^{-5}$
- ❑ for improved BPM resolution and/or longer lever arm 3-magnet version is better
- ❑ for worse magnet field precision 4-magnet-version suffers more
- ❑ no strong dependence on # BPMs,  
(redundancy requests 2 x 4 or 3 x 3 BPMs)
- ❑ costs are in favor of 3-magnet-version since only 1 high precision magnet is needed instead of 4

\*)  $\Delta B I = 2 \cdot 10^{-5}$ ,  $L = 10$  m,  $\Delta x = 200$ nm, 2 x 3..4 BPMs