New Aspects on "Electron Beam Energy Measurement Using Resonance Absorption of Laser Light by Electrons in a Magnetic Field"

Melikian R.A.

Principles of the "Resonant Absorption" method is published in Proc. of 7th Conference EPAC 2000 and is reported with some additions at meeting of "Energy Spectrometer" in Zeuthen, March, 2004г [1], [2].
Now I would like to discuss some new additions and corrections.

1.The probability of synchrotron radiation in the region of the magnetic field B.

It is known, that intensity of synchrotron radiation in field $\vec{B}$ is distinct from zero, if $\vec{B}$ has a transverse component $B_\perp = B\sin\varphi \approx B\varphi$ to velocity of electron $\vec{V}_e$, where φ - is the angle between vectors $\vec{B}$ and $\vec{V}_e$ (Fig.1).

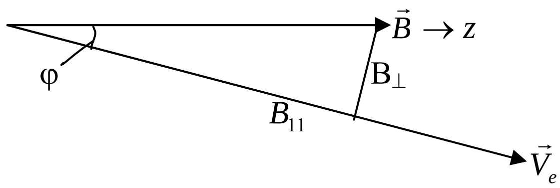

Fig.1

It is known, that synchrotron radiation is formed on a length Δl of electron trajectory, on which vector $\vec{V}_e$ turn on angle $\theta \approx 1/\gamma$, where γ - is the relativistic factor of electron. From geometry of movement of electron follows that:

$$\Delta l = R\cdot\theta, \quad \text{where:} \quad R = V_e/\omega_H \approx c/\omega_H, \quad \omega_H = eB_\perp/mc\gamma .$$

So:

$$\Delta l = R\cdot\theta \approx \frac{c}{\omega_H}\frac{1}{\gamma} = \frac{c}{\left(\frac{eB_\perp}{mc}\right)}. \qquad (1)$$

For example if $B = 2\cdot 10^4 Gs$ and $\varphi = 2\cdot 10^{-4} rad$, then $B_\perp = B\varphi = 4Gs$, and $\Delta l \approx 4.26$ m.

It means, that on length of magnet interest to us $L \le \Delta l$, the trajectory of electrons will not be bent because of synchrotron radiation

2. Optimum choice of parameters of a laser and electron beam

For optimum choice of parameters of laser and electron beams we consider various dispositions (i.e. various angles φ and θ_0) of electron and laser beam concerning direction of axis of magnetic field $\vec{B}$ for different: Nd:YAG and CO_2 lasers.

We can choose parameters of a laser and electron beams only in a complex.

Really, for the given γ-factor of electrons and for angles $\varphi \le \pm 0.5\cdot 10^{-4}$, $\theta_{\min} = \theta_0 - \lambda/D < \theta < \theta_{\max} = \theta_0 + \lambda/D$ because of the relation

$$\varphi^2 + \theta^2 \approx \frac{2\Omega}{\gamma}\ , \qquad (2)$$

the resonant absorption of photons by electrons is possible only at certain values of Ω in interval: $\Omega_{\min} < \Omega < \Omega_{\max}$ (see Fig.2, Fig.3 - $D = 1cm$, $\lambda = 1\mu k$, Nd:YAG laser).

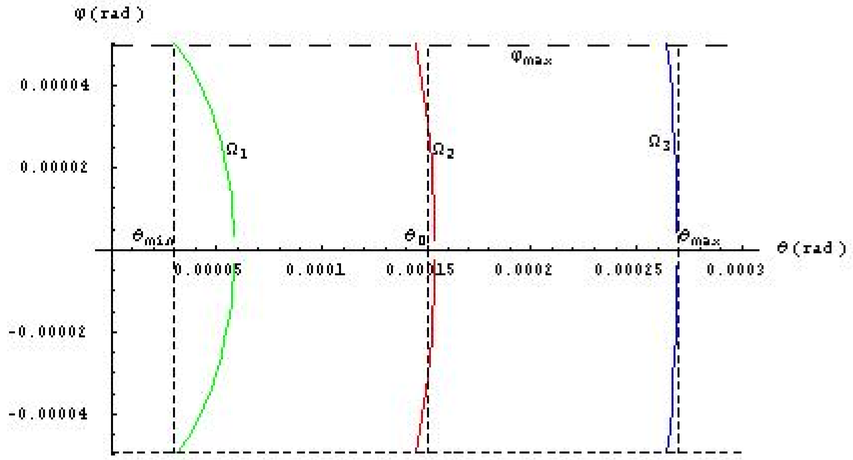

Fig.2. Limits of change of angles φ and θ for: $\theta_0 = 1.5\cdot 10^{-4}$ (rad), Nd:YAG laser ($\lambda = 1\mu k$), $D = 1cm$, $\gamma = 10^5$, $\Omega_1 = 1.75\cdot 10^{-4}$, $\Omega_2 = 1.18\cdot 10^{-3}$, $\Omega_3 = 3.63\cdot 10^{-3}$.

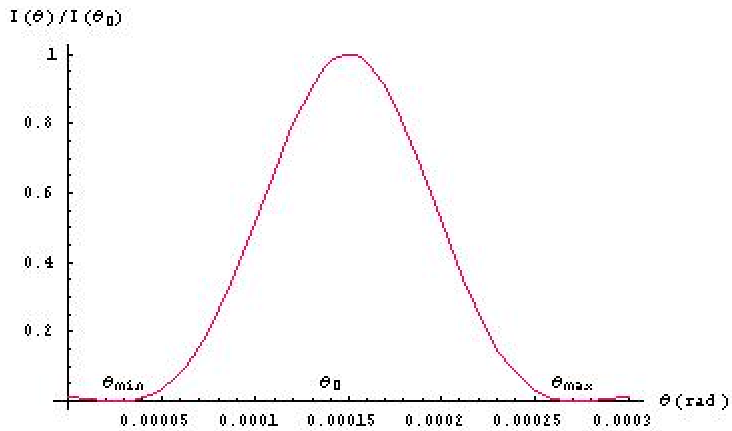

Fig.3. Distribution of light intensity as a function of diffraction angle θ.

We assume , that concerning to axis of magnetic field $\vec{B}$ the laser beam falls under a small angle θ_0. Then distribution of light intensity depending on angle of diffraction is determined by expression:

$$I(\theta) = I_0\left[\frac{2J_1(\alpha)}{\alpha}\right]^2, \qquad \alpha = \frac{\pi D}{\lambda}(\theta - \theta_0), \qquad (4)$$

and has the shape, schematically represented on Fig.3.

On the other hand from concept that electron can absorb only an integer $\nu \ge 1$ photon we have condition (Fig.4):

$$\xi \geq \frac{1}{\psi} \frac{\lambda_c}{L}, \qquad (3)$$

where $\lambda_c = \frac{\hbar}{mc} \approx 3.861 \cdot 10^{-11} cm$ - is the Compton length of wave of electron, L - is the length of magnet, $\psi = \varphi + \theta$ - is the angle between electron and laser beam. $\xi = \frac{eE}{mc\omega}$ - is non-dimensional parameter of intensity of the laser. Intensity of the laser W is connected to amplitude of electric field of a wave by the following relation :

$$W\left[w/cm^2\right] = \left(\frac{E\left[V/cm\right]}{19.4}\right)^2. \qquad (4)$$

For example if $\xi = 3.861 \cdot 10^{-9}$, $L = 1m$, then from (3) we have: $\psi = 10^{-4} rad$ and if $\lambda = 10.6\mu k$ then: $eE = \xi \cdot mc^2 \cdot \frac{2\pi}{\lambda} \approx 11{,}7\, eV/cm$ and from (4): $W \approx 0.6\, w/cm^2$.

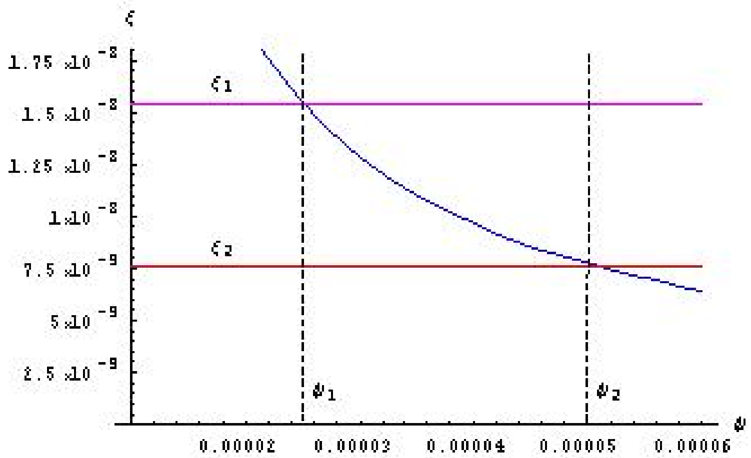

Fig.4. The dependence of laser intensity parameter ξ from angle $\psi = \varphi + \theta$ for fixed length of magnet $L = 1m$, $\xi_1 \approx 1.54 \cdot 10^{-8}$, $\xi_2 \approx 7.72 \cdot 10^{-9}$.

From (3) follows, that intensity of the laser will be reasonable, if the angle ψ is: $\psi_{\max} > \psi = \varphi + \theta \geq \psi_{\min}$ and then according to it should be: $\xi_{\max} > \xi > \xi_{\min}$ (see Fig.4). From (3) follows an additional condition

$$\varphi \geq \frac{1}{\xi} \frac{\lambda_c}{L} - \theta \qquad (5)$$

to relation (2) between angles φ and θ for given values of ξ, L.
For CO_2 laser for angles $\theta_0 = 0$ and $\theta_0 = 0.5 \cdot 10^{-3} rad$ the distribution of light intensity has the shape, schematically represented on Fig.5.

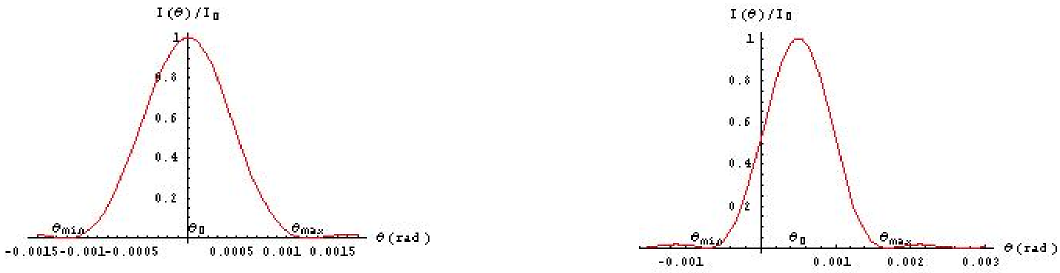

Fig.5. Distribution of light intensity for CO_2 laser for $\theta_0 = 0$ and $\theta_0 = 0.5 \cdot 10^{-3} rad$.

For CO_2 laser for $\theta_0 = 0$ the limits of change of angles φ and θ are schematically shown on Fig.6. The resonant curves are shown at $\gamma = 10^5$ for various values of Ω.

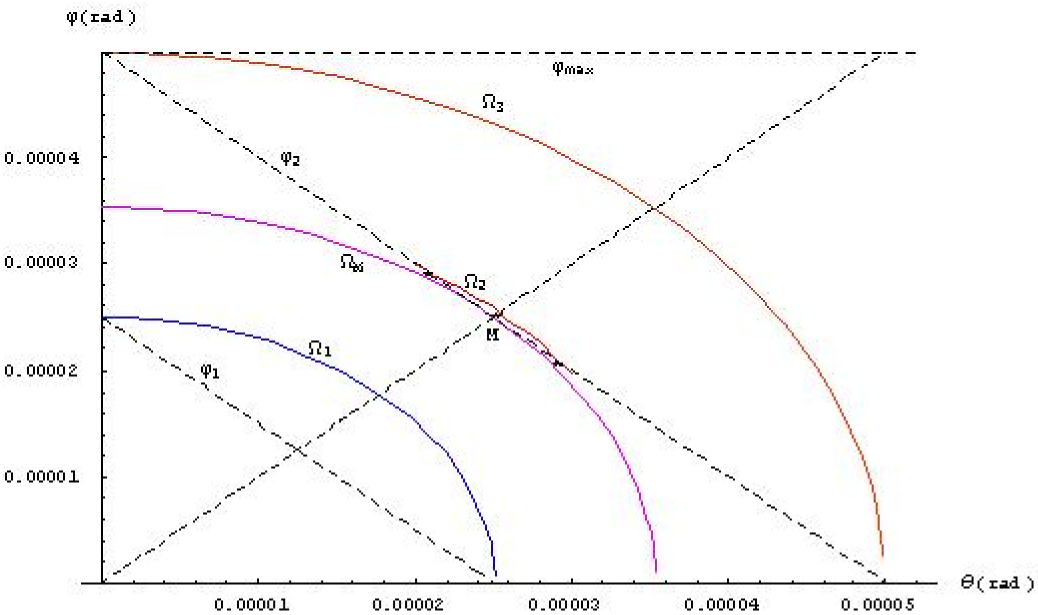

Fig.6. The limits of change of angles φ and θ for: $\gamma = 10^5$, $\Omega_1 = 0.36 \cdot 10^{-4}$, $\Omega_M = 0.675 \cdot 10^{-4}$, $\Omega_2 = 0.695 \cdot 10^{-4}$, $\Omega_3 = 1.25 \cdot 10^{-4}$, $\theta_0 = 0$, $\lambda = 10.6 \mu k$, $\theta_{\max} \approx 0.0012$ rad, $L = 1m$, $\xi_1 \approx 15.44 \cdot 10^{-9}$, $\xi_2 \approx 7.72 \cdot 10^{-9}$, $\xi_3 \approx 2.76 \cdot 10^{-9}$, $W_1 \approx 5.56\, w/cm^2$, $W_2 \approx 1.39\, w/cm^2$, $W_3 \approx 0.18\, w/cm^2$.

Note, that for considered small angles φ and θ, for example when $\Omega_3 = 1.25 \cdot 10^{-4}$, the value of a magnetic field is equal: $B = 1.25$ kGs.

On Fig. 6 the threshold lines (dotted lines) φ_1, φ_2, φ_3 found according to a condition (5) for different values of ξ_1, ξ_2, ξ_3 also are shown. From condition (3) and Fig. 6 follows, that for example for the laser with parameter of intensity ξ_2 photons can be absorbed by electrons only in a case when resonant curves lay on right-hand side from a line φ_2, i.e. when

$$\varphi_2 \geq \frac{1}{\xi_2} \frac{\lambda_c}{L} - \theta .$$

Note that we can reach change of position of a line φ both by change of value ξ and change of length of a magnet L .

In a case of the Nd:YAG laser with length of a wave $\lambda = 10.6\mu k$ for $\gamma = 10^5$, $\theta_0 = 0$ and $\xi_2 \approx 7.72 \cdot 10^{-9}$ the required intensity of the laser will be:

$$W_2 = \left(\frac{\xi \cdot mc^2 \cdot 2\pi}{19.4 \cdot \lambda} \right)^2 \approx 139 w/cm^2,$$

while in a case of CO_2 laser the required intensity $W_2 \approx 1.39\, w/cm^2$ is less in 100 times.

Now we can estimate the intensity of resonant absorption $I_{abs}(\Omega)$ of photons by electrons and the width of this resonance in a case of $\varphi \geq \varphi_2$.

For this purpose we should take into account, that the value of $I_{abs}(\Omega)$ is proportional to number N_e of electrons absorbing photons and to intensity of the laser $I(\theta)$ for those angles where the resonant curve passes. Besides we assume that electrons on angles φ have the Gaussian distribution (Fig.7).

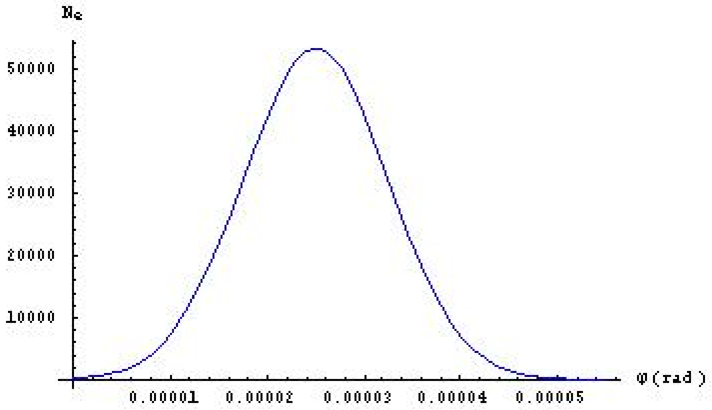

Fig.7. Gaussian distribution of electrons on angles φ.

Taking into account it, from Fig.6 is obvious that $I_{abs}(\Omega)$ will be maximal for $\Omega = \Omega_3$. From Fig.6 it is clear, that the minimal value $I_{abs}(\Omega) = 0$ the intensity of absorption has in point M for $\Omega = \Omega_M$.

Dependence of absorption intensity $I_{abs}(\Omega)$ from Ω have shape, shown on Fig.8.

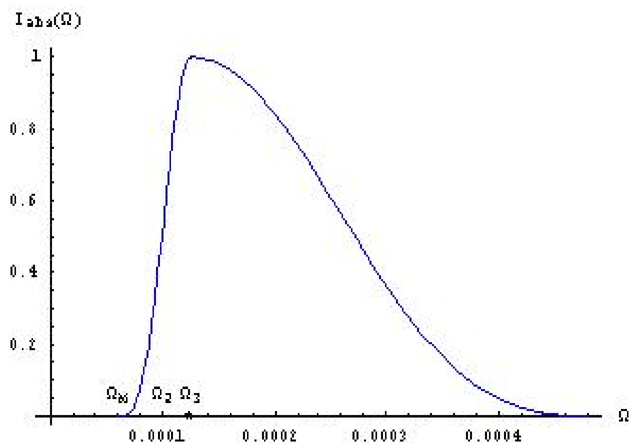

Fig.8. Dependence of absorption intensity $I_{abs}(\Omega)$ from Ω.

Using Fig.6 it is easy to find a mathematical relation: $\frac{1}{\sqrt{2}}\sqrt{\frac{2\Omega_3}{\gamma}} = \sqrt{\frac{2\Omega_M}{\gamma}}$ or $\Omega_3 = 2 \cdot \Omega_M$. Then, if approximately to consider that for $\Omega = \Omega_{1/2} = \frac{\Omega_3 + \Omega_M}{2}$ the intensity of absorption is equal to $\frac{1}{2} I_{abs,\max}$, then the width of resonance is equal: $\sigma \approx \Omega_3 - \Omega_{1/2} = \frac{1}{4}\Omega_3 \approx 0.3 \cdot 10^{-4}$. If to take $\Omega = 1.03 \cdot \Omega_M$ then in process of absorption will give the contribution approximately 1/4 part from the general number N_e of electrons. In this case the width of resonance will be: $\sigma \approx 10^{-6}$. Certainly we assume in this case that the intensity of the laser (or $\xi = \frac{eE}{mc\omega} \approx \frac{19.4\sqrt{I}}{mc\omega}$) is supported with accuracy better than 1 %.

The left sharp edge of a curve $I_{abs}(\Omega)$ from Ω_M to Ω_3 (Fig.8) can be used as threshold discriminator (detection threshold) for fact of absorption of photons by electrons at measurement of electron beam energy (see Fig.9).

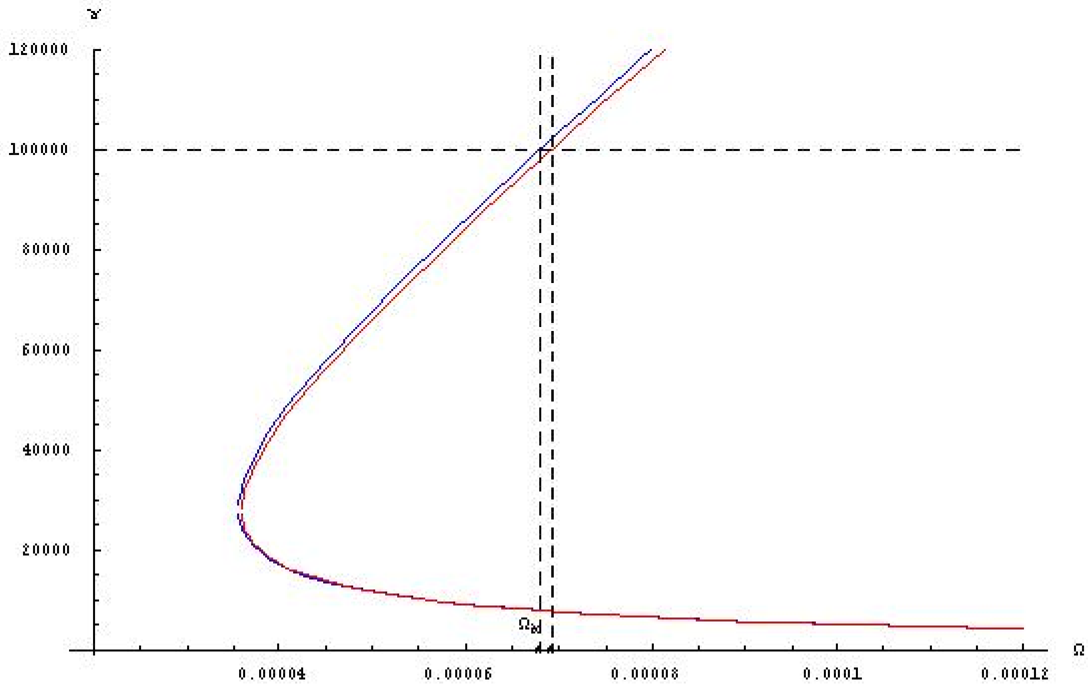

Fig.9. Dependence of γ from Ω. $\gamma = 10^5$, $\Omega_M = 0.68 \cdot 10^{-4}$, $\Omega = 1.02 \cdot \Omega_M$, $\sigma \approx 0.68 \cdot 10^{-6}$.

Really, if for some energy γ_1 of electron the intensity $I_{abs}(\Omega)$ is maximum for the some Ω_1, then:

$$\varphi_1^2 + \theta_1^2 = \frac{2\Omega_1}{\gamma_1}. \qquad (6)$$

Taking into account that for unknown energy of electrons γ_u the intensity $I_{abs}(\Omega)$ is maximal for some Ω_u at the same angles φ_1 and θ_1 we have:

$$\varphi_1^2 + \theta_1^2 = \frac{2\Omega_u}{\gamma_u} = \frac{2\Omega_1}{\gamma_1}, \qquad (7)$$

whence we can calculate

$$\gamma_u = \gamma_1 \frac{\Omega_u}{\Omega_1}. \tag{8}$$

Taking into account that for point "M" (see Fig. 6) $\varphi_M = \theta_M$ then according to (2) we have:

$$\varphi_M = \frac{1}{2} \cdot \frac{\lambda_c}{L\xi_2}, \tag{9}$$

and according to (5) we have:

$$\varphi_M^2 = \frac{\Omega_M}{\gamma} = \frac{1}{4} \cdot \left(\frac{\lambda_c}{L\xi_2} \right)^2. \tag{10}$$

From (10) follows, that if we can experimentally find Ω_M (or $\Omega_{1/2}$) then we can calculate absolute value of electron energy γ.

REFERENCE

1. R.A. Melikian, D.P. Barber. On the Possibility of Precise Measurement of Electron Beam Energy Using Resonance Absorption of Laser Light by Electrons in a Static Magnetic Field. Proc. 7th EPAC, Vienna, June 2000.

2. R.A. Melikian. Energy Measurement of Relativistric Electron Beam Using Resonance Absorption of Laser Light by Electrons in a Magnetic Field. http://www-zeuthen.desy.de/e_spec/april2004/melikian.pdf