Resonance Absorption and Experimental Conditions

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All fundamental formulas can be found in the paper of Melikian. Using energy and momentum conservation one obtains the following condition for resonance absorption:

$$\hbar\omega(\gamma - \sqrt{\gamma^2 - 1}cos\phi cos\Theta + \hbar\omega sin^2\Theta/2mc^2) = \hbar\omega_c \tag{1}$$

In the following we assume $\phi = 0$ and use for small angles and large γ the approaches: $sin\Theta = \Theta$, $\sqrt{\gamma^2 - 1} = \gamma - 1/2\gamma$. Now we have:

$$\hbar\omega(\frac{1 + \gamma^2\Theta^2}{2\gamma} + \frac{\hbar\omega\Theta^2}{2mc^2}) = \hbar\omega_c \tag{2}$$

We transform this equation to have an expression for Θ^2, the angle between laser and beam directions:

$$\Theta^2 = \frac{mc^2(2\gamma\hbar\omega_c - \hbar\omega)}{\hbar\omega(\gamma\hbar\omega + \gamma^2 mc^2)} \tag{3}$$

To obtain some Θ^2 we have to use the following expressions and constants:

1) The electron mass energy equivalent is: $mc^2 = 0.5MeV$.
2) The relativistic factor $\gamma = E/mc^2$, where E is the electron beam energy.
3) The electron cyclotron frequency in dependence on magnetic field B is: $\omega_c/B = 1.758 \cdot 10^{11}s^{-1}T^{-1}$. Together with $\hbar = 6.582 \cdot 10^{-22}MeVs$ one obtains: $\hbar\omega_c = B[T] \cdot 1.157 \cdot 10^{-4}eV$, with B in units of Tesla.
4) The laser energy $\hbar\omega$ we assume can vary in the range: $\hbar\omega = 0.1...10eV$.

Now we have all to calculate Θ^2. Lets repeat the example from Melikian's paper with the following input: $\gamma = 10^5$, for $\lambda = 10.6\mu m$ we get: $\hbar\omega = 0.12eV$, and $B = 2.02T$. This gives us:

$$\Theta^2 = \frac{0.5MeV(2 \cdot 10^5 \cdot 2.02 \cdot 1.157 \cdot 10^{-4}eV - 0.12eV)}{0.12eV(10^5 \cdot 0.12eV + 10^{10}0.5MeV)} \tag{4}$$

$$\Theta^2 = 4.2 \cdot 10^6 \frac{41.6eV - 0.12eV}{0.5 \cdot 10^{16}eV} = 3.5 \cdot 10^{-8}rad^2 \tag{5}$$

Hence, the result agrees with the angle of $2 \cdot 10^{-4} rad$ from Melikian.

Now we discuss briefly the essential consequences of formula (3). The first plot in fig.1 shows the dependence of Θ^2 on the magnetic field in the range up to 10 Tesla. The electron beam energy was set to 250 GeV and for the laser energy 3 values were assumed: 0.1, 1.0 and 10 eV. The CO2 laser has roughly 0.1 eV photon energy. For this case and field strength between 1 and 5 Tesla one concludes from fig.1 that the resonance condition is fulfilled in a small angular range between about 0.1 and 0.2 mrad.

Fig.2 shows the resonance condition for different electron beam energies in dependence on the magnetic field strength for a laser energy of 0.1 eV (CO2 laser). The smaller the beam energy the larger is the angle of the laser beam in respect to the electron beam if resonance absorption happens. For a beam energy of 45 GeV resonance absorption happens at angles above 0.2 mrad, what can be helpful to built a test set-up.

Fig.3 shows the resonance condition for a laser energy of 1 eV. Clearly the angle Θ for resonance absorption decreases what makes a practical realization more difficult.

The next fig.4 illustrates the interplay between electron beam energy and angle for resonance absorption. The magnetic field was set to 2 Tesla and the laser energy to 0.12 eV (CO2). The electron beam energy is given in terms of $\gamma = E/mc^2$. The larger the beam energy the smaller the angle of resonance absorption what is a bad condition for practical cases. For electron energies above 500 GeV the resonance angle is below 0.1 mrad, what in turn means a long set-up for the crossing of the laser and electron beam.

Next we deal with the errors of the resonance absorption. The angle of resonance absorption depends on the electron beam energy E, the laser energy $\hbar\omega$, and the magnetic field B. Correspondingly, from (3) one can derive by error propagation the following formula for the error of the beam energy.

$$\Delta E = mc^2 \Delta\gamma \tag{6}$$

$$(\Delta\gamma)^2 = (f_1 \Delta B)^2 + (f_2 \Delta\hbar\omega)^2 + (f_3 \Delta\Theta)^2 \tag{7}$$

with

$$f_1 = 2\hbar\omega_c/(\hbar\omega\Theta^2) + \hbar\omega/(2\hbar\omega_c) \tag{8}$$

$$f_2 = 2\hbar\omega_c/(\hbar\omega^2\Theta^2) + 1/(2\hbar\omega_c) \tag{9}$$

$$f_3 = 4\hbar\omega_c/(\hbar\omega\Theta^3) \tag{10}$$

Fig.5 shows the relative beam energy error $\Delta E/E$ in dependence on the relative magnetic field error $\Delta B/B$ neglecting other error sources as angles and laser energy. We observe a clear linear relation - the relative energy error is as good as the relative magnetic field error, other parameters as electron beam energy, the angle Θ and the magnetic field strength B do not influence the relative energy error.

The most critical parameter for the energy error is the error or jitter of the resonance angle Θ. The influence of the angular error on $\Delta E/E$ is shown in fig.6 for 4 angles $\Theta = 0.06, 0.1, 0.2, 1.0$ mrad. The relative magnetic field error was fixed at $\Delta B/B = 2 \cdot 10^{-5}$. The lower angles from 0.06 mrad to 0.2 mrad fulfill the resonance condition for a 250 GeV electron beam with a CO2-laser and magnetic fields in the range from 1 to 5 Tesla. To fufill the resonance condition for the angle of 1 mrad either the laer wave length has to be much longer then the 10.6 μm of the CO2-laser and/or the magnetic field strength much larger then 5 Tesla. In any case to reach an energy resolution below 10^{-4} the angular error $\Delta\Theta$ must be in the range of 10^{-5} mrad. This means practically if all devices are placed on a commom girder of 50 m length to reach a position stability of the optical devices of at least 500 nm.

At fixed electron energy, the wave length of the laser and the strength of the magnetic field define the angle of resonance absorption. This angle should be as large as possible since it determines together with the beam tube diameter (what means the minimum distance where any intrumentation is possible) the necessary length of the common girder where all magnets, position measuring and monitoring devices, and optical devices has to be placed.

Other, not yet clear input is missing for the absolute rate decrease by resonance absorption. Also detection problems as resolution, acceptance etc. are not taken into account.

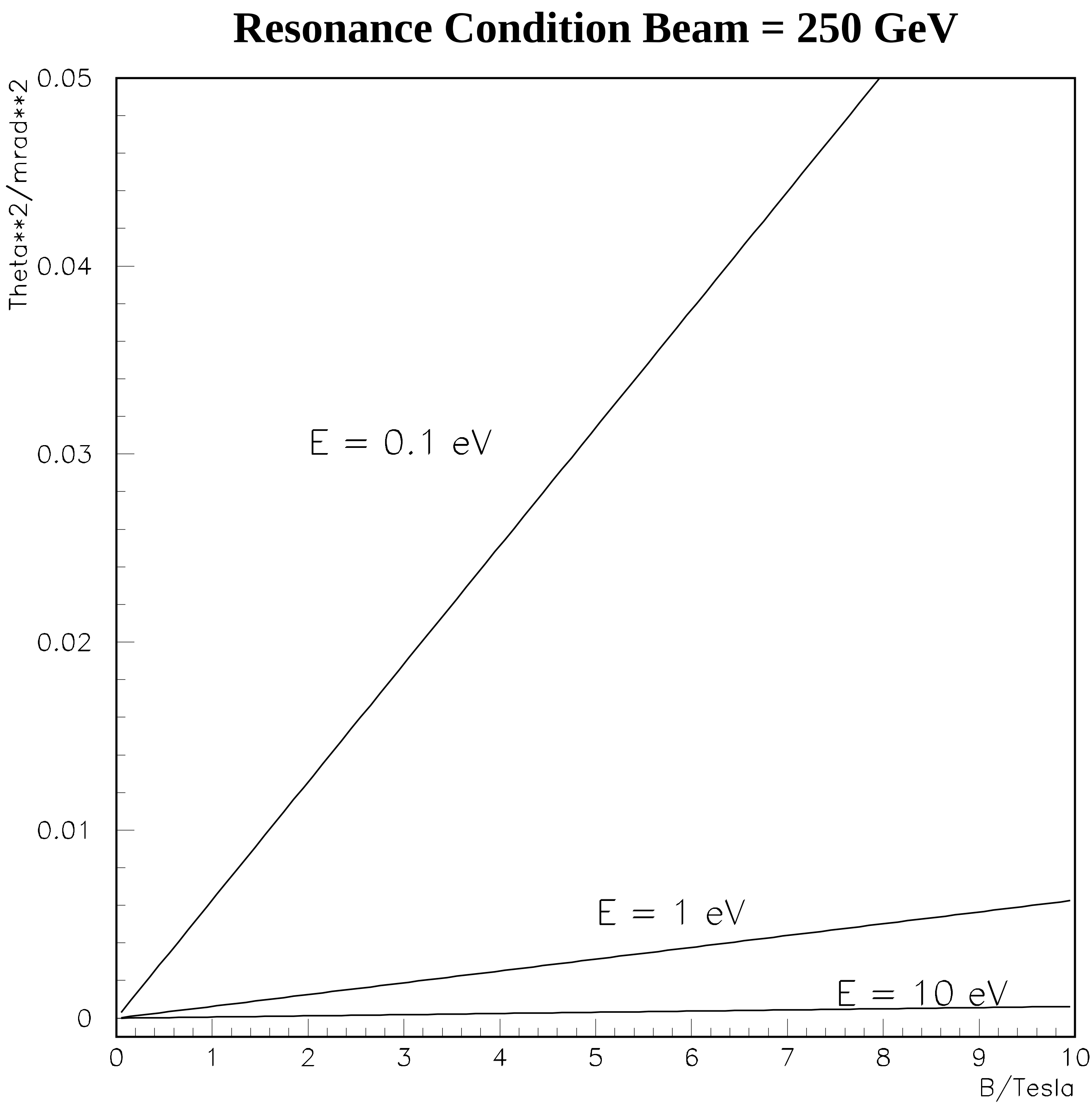

Figure 1: The squared angle Θ^2 in dependence on the magnetic field B for laser energies of 0.1, 1 and 10 eV

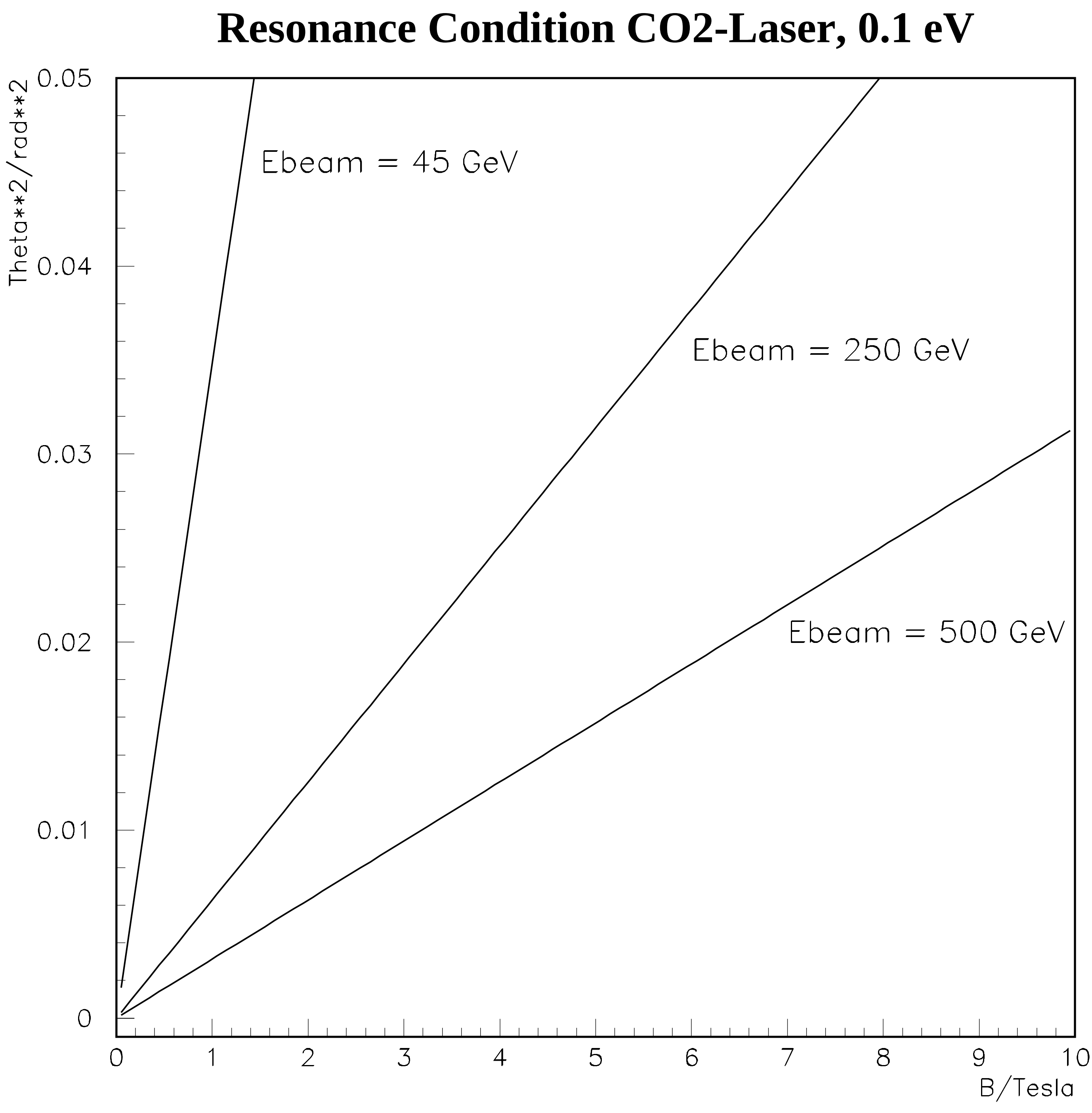

Figure 2: The squared angle Θ^2 in dependence on the magnetic field B for beam energies of 45, 250 and 500 GeV

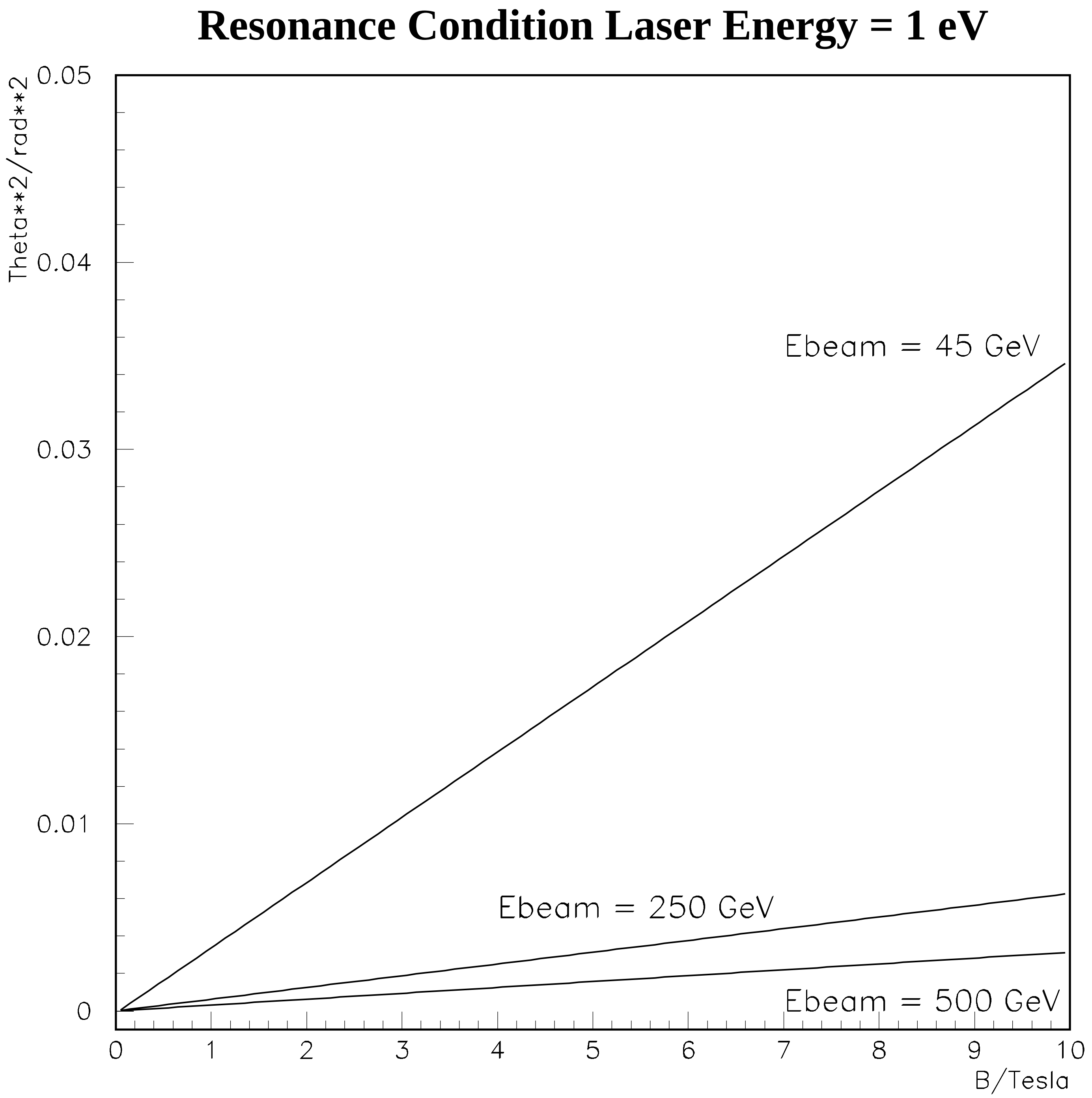

Figure 3: The squared angle Θ^2 in dependence on the magnetic field B for laser energies of 1 eV

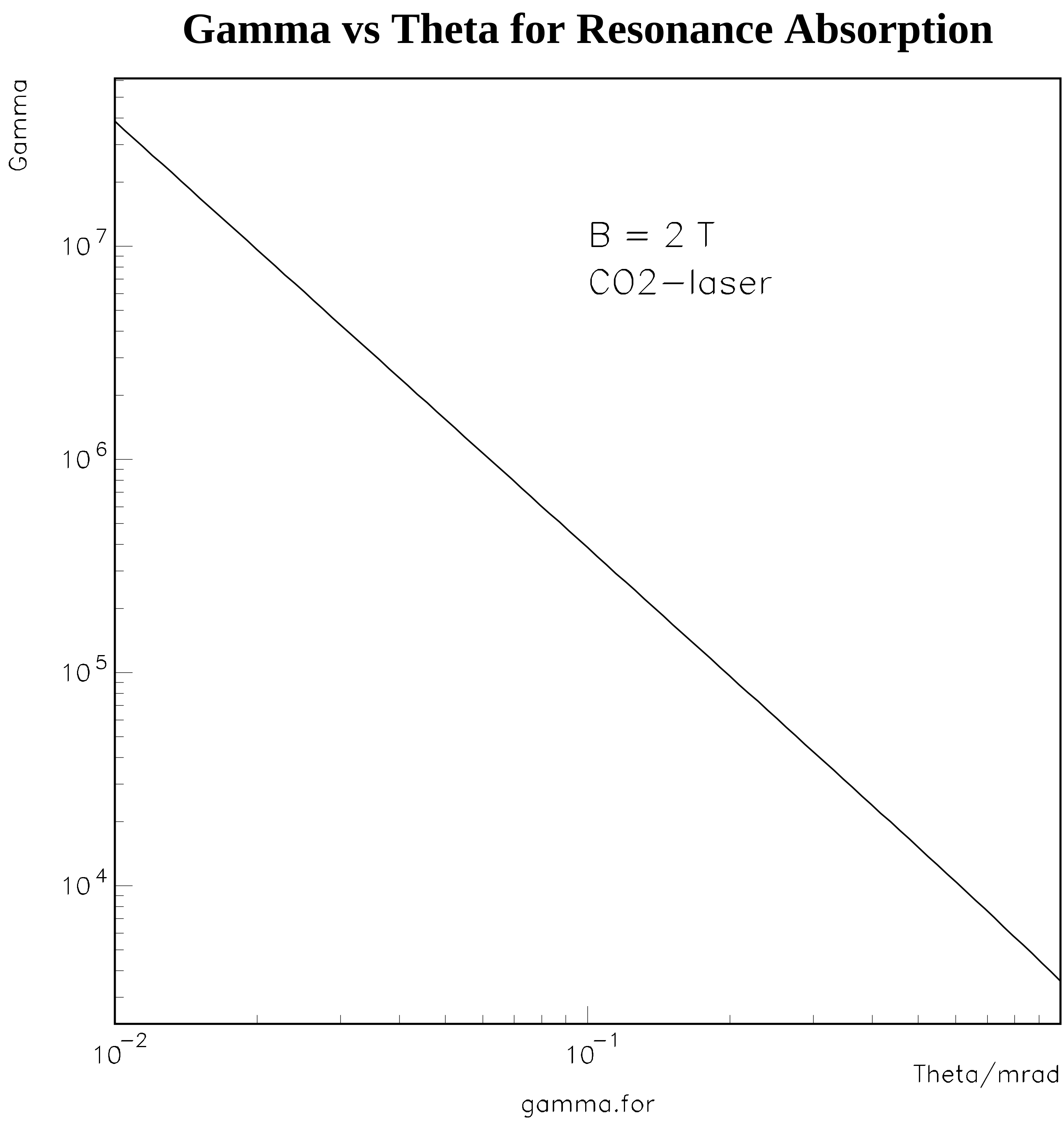

Figure 4: The relation between γ and the resonance angle Θ

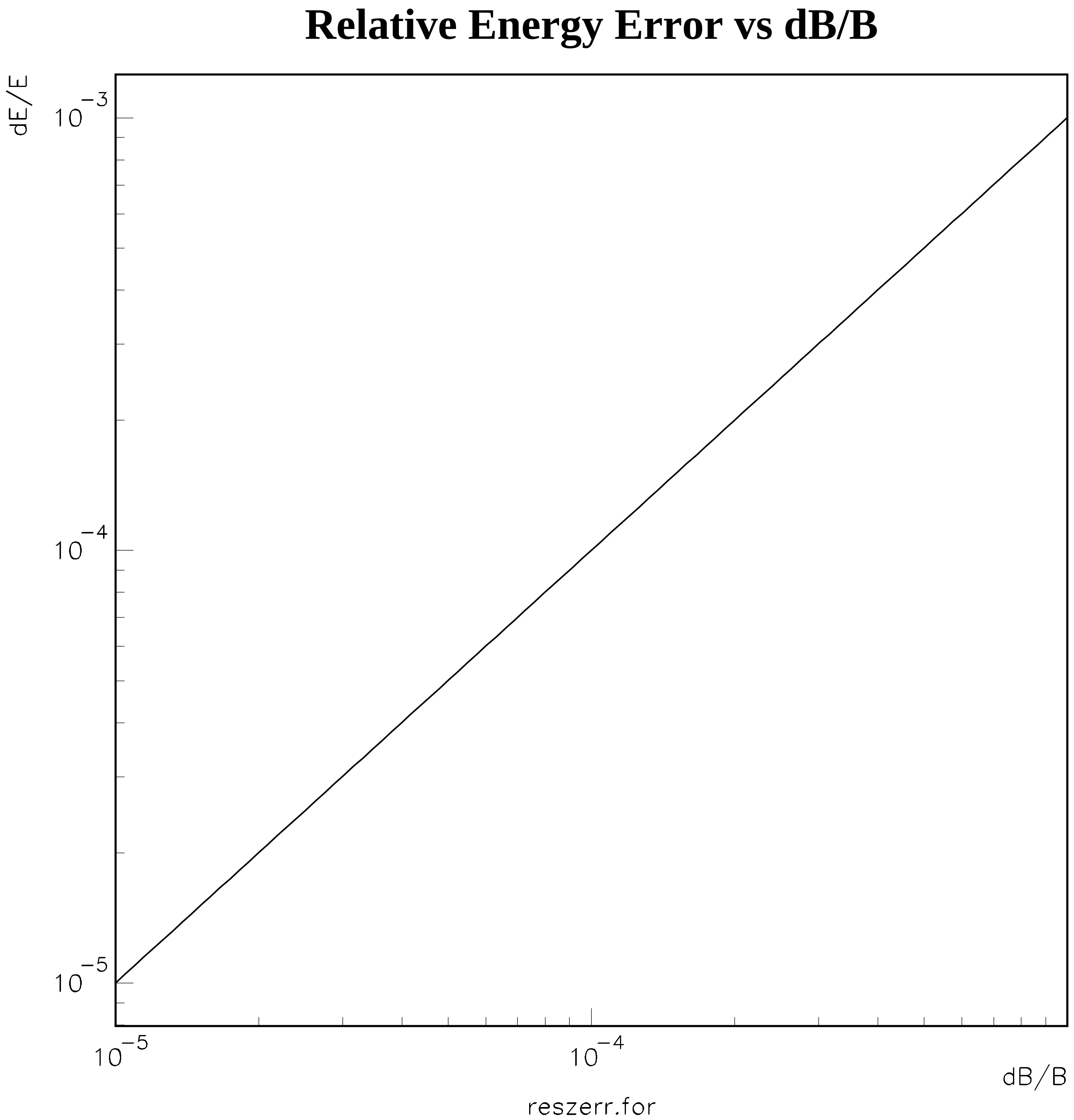

Figure 5: The relative energy error $\Delta E/E$ in dependence on the relative magnetic field error $\Delta B/B$. Other error sources like angular fluctuations or laser energy smearing are ignored.

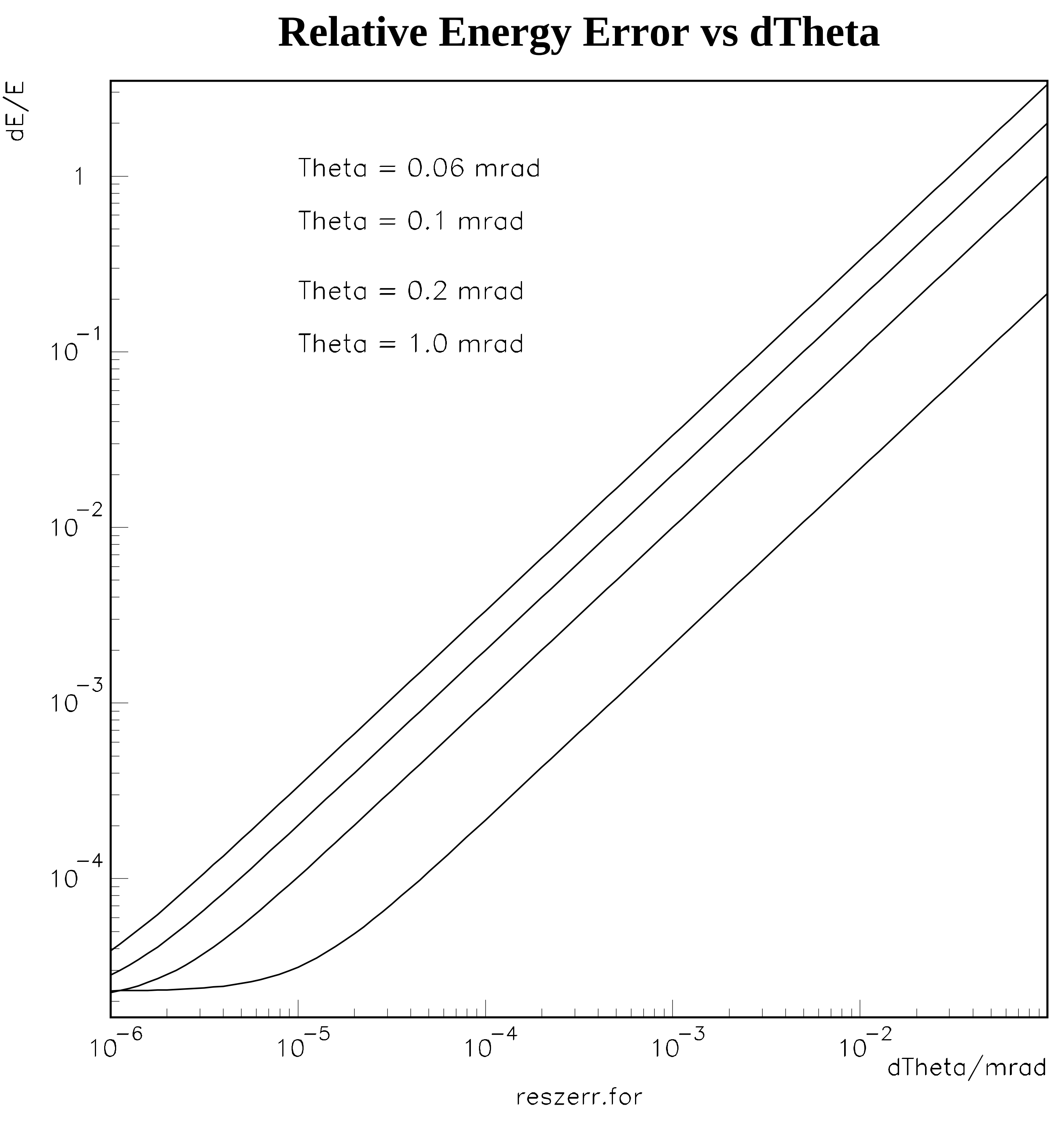

Figure 6: The relative energy error $\Delta E/E$ in dependence on the angular error $\Delta\Theta$