SOME REMARKS ON THE ENERGY-MOMENTUM CONSERVATION LAWS USED IN LASER LIGHT RESONANCE ABSORPTION METHOD FOR MEASUREMENT OF ELECTRON BEAM ENERGY .

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Abstract

An existance of opened problems, connected with the use of some energy-momentum conservation laws, proposed earlier to describe photon intereaction with electron beam is discussed.

1 Introduction.

An application of the resonance absorption (RA) of photon by electron [1], [2], which moves in the homeogenious magnetic field, for the task of electron beam energy measurements was discussed in a number of talks given at different conferences [3], [4].

The idea of this method is based on the theoretical formula which was obtained for the first time in the framework of the non-relativistic quantum mechanics (see [5]) and latter on was generalized for the relativistic case basing upon the Dirac [6] and the Klien-Gordon equations (see, for instance , [7]). These formulae give the energy eigenvalues and the wave functions of the particle which moves in the homogeneous magnetic field H.

The energy of the electron p^0, which is the fourth component of the electron 4-momentum vector $P_\mu = (p^0, p^x, p^y, p^z)$, is defined in this case by formula ([6]):

$$c^2(p^0)^2 \equiv E^2_\lambda(n, p_z) = E^2_z + E^2_{T,\lambda}(n), \tag{1}$$

where the first term

$$E_z^2 = m_e^2 c^4 + p_z^2 c^2 \tag{2}$$

is the square of the relativistic energy of a free electron that moves in a beam (taken as the z-axis). The second term (in what follows by h we denote the value of Plank constan devided by 2π)

$$E_{T,\lambda}^2(n) = h(\frac{eH}{m_e c})(m_e c^2)(2n + 1 + 2\lambda) \tag{3}$$

is the square of the relativistic energy of the electron transversal motion, which depends on the strength of the magnetic field H as well as on the spin projection "λ" on z-axis. Hence, the transversal energy of the electron has the quantized values, numerated by the main quantum number "n" ($n = 0, 1, 2, 3...$). Also m_e is the electron mass, e is its charge and $\frac{eH}{m_e c} = \omega_c$ is the cyclotron frequency.

In what follows we shall use the expression for the difference of $E_{T,\lambda}^2(n)$ for the two neighboring values of energy levels (for example, with n+1 and n quantum numbers) which stems from the formula (3):

$$\Delta E_{T,\lambda}^2 = E_{T,\lambda}^2(n+1) - E_{T,\lambda}^2(n) = 2h(\frac{eH}{m_e c})(m_e c^2) = 2h\omega_c(m_e c^2). \tag{4}$$

Now, let us note that in the case of spin projection $\lambda = -\frac{1}{2}$ the ground state ($n = 0$) would have, as it follows from (3), the value of the transversal energy been equal to zero: [1]

$$E_{T,\lambda=-\frac{1}{2}}^2(0) = 0. \tag{5}$$

Herefrom comes an important note (which would be used in what follows) that for the ground state with $n = 0$ and the spin projection $\lambda = -\frac{1}{2}$ the expression for the *total energy of the electron in magnetic field* do coincides with *the energy of a free electron*:

$$E_{\lambda=-\frac{1}{2}}(n = 0, p_z) = E_z = \sqrt{m_e^2 c^4 + p_z^2 c^2}. \tag{6}$$

In the nonrelativistic limit the expression for the relativistic energy takes the form from which one may find its connection with the nonrelativistic expression for the energy levels of Schroedinger equation:

$$E_\lambda(n, p_z) = \sqrt{E_z^2 + E_{T,\lambda}^2(n)} = \sqrt{m_e^2 c^4 + p_z^2 c^2 + h(\frac{eH}{m_e c})(m_e c^2)(2n + 1 + 2\lambda)} \approx$$

$$\approx m_e c^2 + (\tfrac{p_z^2}{2m_e}) + (\mu_B H)(2n + 1 + 2\lambda) = m_e c^2 + E^{Schr}.$$

(7)

[1] The next one level also having $n = 0$, but with the spin projection $\lambda = +\frac{1}{2}$, would have the energy $E_{T,\lambda=+\frac{1}{2}}^2(0) = 2h\omega_c(m_e c^2)$ in accordance with formulae (3), (4).

2 The laser photon interaction with the electron.

In one of the most yearly papers [1] the authors considered the process of *photon absorbtion by an electron which moves in the homogenious magnetic field*, basing upon the formulae (1-3) for quantized energy spectrum given in [6]. The energy-momentum conservation law was written in [1] only for two components of momenta. For the energy:

$$p_1^0 + k_1^0 = p_2^0, \tag{8}$$

and for the longitudinal (along z-axis, i.e. along the direction of the magnetic field) component: [2]

$$p_1^z + k_1^z = p_2^z, \tag{9}$$

It was emphasized by authors that "the particle in a magnetic field has no a definite transverse momentum". The energy components of electron 4 -momenta are defined according to formulae (1)-(3) what means that $cp_1^0 = \sqrt{E_\lambda^2(n, p_2^z)}$, if the beam electron was initially "on the n-th orbit" and

$$cp_2^0 = \sqrt{E_\lambda^2(n+1, p_2^z)}, \tag{10}$$

if it has moved to the " n+1 orbit",

Taking the square of equation (8) and with an account of formula (1) one comes to the equation for a non spin-flip case

$$E_\lambda^2(n, p_1^z) + 2ck_1^0 E_\lambda(n, p_1^z) + c^2(k_1^0)^2 = E_\lambda^2(n+1, p_2^z), \tag{11}$$

which can be rewritten (in what follows we shall use the notation $k_1^0 = E_{las}^\gamma/c$) like

$$2E_{las}^\gamma E_\lambda(n, p_1^z) + (E_{las}^\gamma)^2 = E_\lambda^2(n+1, p_2^z) - E_\lambda^2(n, p_1^z). \tag{12}$$

This is a general form of equation (8) for any values of p_1^z and p_2^z. For our further purposes let us express the components of the photon momentum $\vec{k} = (\vec{k_T}, k^z)$ through the angle θ between the direction of laser photon momentum and the momentum of beam electron: $ck^z = c|\vec{k}|cos\theta = (E_{las}^\gamma)cos\theta; \quad ck_T = c|\vec{k}|sin\theta = (E_{las}^\gamma)sin\theta$.

The right hand side of equation (12) may be rewritten with an acount of (1)-(3) as follows

$$E_\lambda^2(n+1, p_2^z) - E_\lambda^2(n, p_1^z) = (cp_2^z)^2 - (cp_1^z)^2 + \Delta E_{T,\lambda}^2 . (13)$$

With a help of (9) we may present the right hand side of equation (12) in the following way

$$(cp_2^z)^2 - (cp_1^z)^2 + \Delta E_{T,\lambda}^2 = 2cp_1^z(E_{las}^\gamma)cos\theta + {E_{las}^\gamma}^2 cos\theta^2 + \Delta E_{T,\lambda}^2 . (14)$$

[2]In these formulae the laser photon momentum 4-vector has the following components $k_1^\mu = (k_1^0, \vec{k_1})$ with $k_1^0 = |\vec{k_1}|$, what is valid for real massless photon.

Now let us substitute the right hand side of equation (12) by the expression given in (14). At the same time let us group the terms with ${E^{\gamma}_{las}}^2$ into the right hand side of (12) and also move the terms which are proportional to E^{γ}_{las} to the left side. We shall also introduce a new notation

$$(E^{\gamma}_{las})^2(cos\theta^2 - 1) = (E^{\gamma}_{las})^2 sin\theta^2 \equiv (E^{\gamma}_{T})^2 \tag{15}$$

in the right hand side of (12), where $(E^{\gamma}_{T})^2$ has the physical sense of the square of the photon transvers energy, or (it is the same in a case of photon) of the square of the photon transvers momenta.

In the result of this algebraic manipulations the equation (12) takes the form of the following equality

$$E_{\lambda}(n, p^z_1) - cp^z_1 cos\theta = \frac{\Delta E^2_{T,\lambda} - (E^{\gamma}_{T})^2}{2E^{\gamma}_{las}}. \tag{16}$$

which exactly coinsides with the analogous equation written in [1] (see formula (4) there) for determining the frequencey of the absorbed photon.

One may see that in the right hand side of equation (16) there has appeared the difference of two transversal variables. One of them $(E^{\gamma}_{T})^2$ is the square of the transversal energy of incoming laser photon which has nothing to do with the external magnetic field. The second one is the difference of two values which are nothing else but the squares of the tranversal energies of the electrons which sit on two neibouring "$n + 1''$ and "n" quantized levels. The exact expression of this difference is given by formula (4).

After this fixing of the physical meaning of these two terms in the right hand side let us recall the statement done in [1] that *the particle which moves in a magnetic field does not have any definite transversal momentum (in our case it may be only the transversal component of interacting electron because E^{γ}_{T} is the initial state variable of the laser photon).*

If this statement of the quoted authors is a right one *then the right hand part of equation (16) seems to have no any practical use at all!* [3]

The second feature of this equation may also produce some doubtes in its physical self-consistence. The reason for these doubts is quite evident. The right hand side of (16) contains the physical values of order of the square of laser photon energy E^{γ}_{las} value, which stands in the denominator. To make it easier, let us remark that equation (16) was obtained without a use of any special restrictions, imposed onto the value of the angle θ (between the electron beam and the laser photon). Thus, we may put it to be equal to the value $\theta = 90^0$, i.e. to consider a case of

orthogonal orientation of laser beam to the electron beam. In such a case the equation

[3]Please, note that we have not introduced the electron PT component into this euation by hand or in any other way. It has appeared there only because of the use of the equation (8) of energy conservation and of its square (12). The transverse component already containes in the electron energy formula, see (1) and (7) in an implicite form and it follows from the relativistic definition of energy.

(16) takes a more simple form for physical analysis, i.e.

$$E_\lambda(n, p_1^z) = \frac{\Delta E_{T,\lambda}^2 - (E_T^\gamma)^2}{2E_{las}^\gamma}. \tag{17}$$

It directly expresses the beam energy $E_\lambda(n, p_1^z)$ through the energy of the laser beam. Let us take for example a typical value $E_{las}^\gamma = 1eV$. The square of the laser photon transverse energy $(E_T^\gamma)^2$, defined by (35), might be of $1eV^2$ order for the not very small values of $sin\theta$. Let us also emphasize that the idea of application of equation (16) to the process of "resonance absorbtion" was based on the supposition that the difference of two neibouring electron levels "n+1" and "n" assumed to be close to the energy of the absorbed photon. So, without going to any details, we may see, that, the values in the right hand side are assumed (withing the "resonance absorbtion " approch) to be of the order of laser photon energy $E_{las}^\gamma = 1eV$. At the same time (and withing the same approach) the left hand side may contain the electron beam energy, like, for instance, of $E_\lambda(n, p_1^z) = 500GeV$ order. This example clearly demonstraites the inconsistency of the equation (17). Taking of another values of the angle θ, i.e. passing to equation (16), does not change drastically the situation if the energy of the beam belongs to GeV region.

On a top of this let us mention also some mathematical contradiction contained in this equation. Recalling the equation (7) one may see that the left hand side of equation (16) may be of the positive sign only:

$$E_\lambda(n, p_z) - cp_1^z cos\theta =$$

$$\sqrt{m_e^2c^4 + p_z^2c^2 + h(\frac{eH}{m_ec})(m_ec^2)(2n+1+2\lambda)} - cp_1^z cos\theta > 0.$$

(18).

while the right hand side includes the value which is obtained by substruction of the incoming transversal photon energy square $E_T^{\gamma 2}$ (it has a sense of the origin of appearing of the transverse motion of electron to a higher orbit) from the square of the difference of two neibouring energetic levels . Due to the physical process of photon action (by this transverse energy) onto the electron, the square of the electron momentum gets the additional part to his transversal component, equal to $(\Delta E_{T,\lambda}^2)^2$. [4] From the view point of the physical sense the obtained addition to the square of the electron transversal component $(\Delta E_{T,\lambda}^2)^2$ cannot be larger than the value of the input $(E_T^\gamma)^2$. In other words: *due to the physical common sense, which suppose that a physical body could not get more energy than another physical oject (which acts on it) has, the right hand side of equation(16) may be equal to zero or have a negative value.*

So, here we see the contradiction with the physical meaning of two terms that appear in the right hand side of equation (16). [5]

[4]Recall the already disussed independence of the motionc along the transversal direction from the motion along the longitudinal one, i.e. along the z-axis.

[5]Let us emphasize that equation (16) has appeared as the result of algebraic transformations of two energy-momentum conservation equations of [1] and it coinsides with the analogous equation (4) form the same article.

The situation we meet here reminds the already mentioned situation with the leading order Feynman diagrams which are forbidden within QED [8], [9] also due to the contradiction of the kinematic relations for $2 \to 1$ (photon absorbtion by electron or e+e- annihilation into a real photon) or $1 \to 2$ (e+e- production by a real photon) processes with the energy-momentum conservation laws for the processes in which 3 particles do participate.

To our mind the possible way out is just to follow the analogy with the QED case, discussed in Section 3, and to consider an analog of the Compton effect but for a case when the initial state electron, which was sitting on the level "n" and having the 4-momentum p_1 (i.e. his energy $E_\lambda(n, p_{1z}) = \sqrt{E^2_{p_{1z}} + E^2_{T,\lambda}(n)}$ is defined by formula (7)), after the interaction with the photon, which has the 4-momentum k_2, moves to another orbit "m", where it has the 4-momentum p_2 and the energy $E_\lambda(m, p_{1z}) = \sqrt{E^2_{p_{1z}} + E^2_{T,\lambda}(m)}$ and this transition is accompanied by emission of the final state photon of k_2 4-momentum. Such a modification of the conservation law, may allow to avoid the contradictions mentioned above.

References

[1] I.A.Gilinski and K.A.Ryasantsev; Izv.Vuz. Radiophysics, **5** (1964) 838;

[2] R.A.Melikian and D.P.Barber, DESY report 98-015 (1998), hep-ph/9903007;

[3] R.A.Melikian and D.P.Barber Proc. 13 Intern. Symposium on High Energy Spin Physics (Spin98), p. 495,Protvino, Russia, 8-12 Sept. 1998;

[4] D.P.Barber and R.A..Melikian, Procc. of EPAC 2000, Vienna, Austria,p.996, http://accelconf.web.cern.ch/accelconf/e00/PAPERS/MOP1B01.pdf;

[5] L.D.Landau and E.M.Lifshitz, "Quantum Mechanics", Nauka, Moscow, 1989.

[6] A.I.Akhiezer,V.B.Berestetskii "Quantum Electrodynamics, Fizmatgiz, Moscow , 1957. Interscience Publishes, New York, 1965;

[7] A.A.Sokolov, I.M.Ternov and C.W.Kilminster "Radiation from Relativistic Electrons", American Institute of Physics(translation series), 1986 ; Naika, Moscow,1983.

[8] V.B.Berestetskii, E.M.Lifshitz, L.P.Pitaevskii, "Quantum Electrodynamics", Nauka, Moscow, 1980; Pergamon press (1982),Addison Wesley Publ.,1971;

[9] N.N.Bogoluibov, D.V.Shirkov "Introduction to the Theory of Quantized Fields", Fizmatgiz, Moscow , 1957; A Wiley-Interscience Publication; John Wiley and Sons, New York,1979;