ADITIONS TO THE METHOD OF ELECTRON BEAM ENERGY MEASUREMENT USING RESONANT ABSORPTION OF LASER LIGHT IN A MAGNETIC FIELD.

Melikian R.A. (YerPhI, Yerevan)

1. NEW CONDITION OF RESONANT ABSORPTION

Below we can see that the resonant condition can be presented in more convenient form, allowing to consider process of absorption in details and to estimate the accuracy of measurement of electron beam energy. Electrons and photons are injected in a magnetic field B under, small angles φ and θ to the z-axis, accordingly (Fig.1).

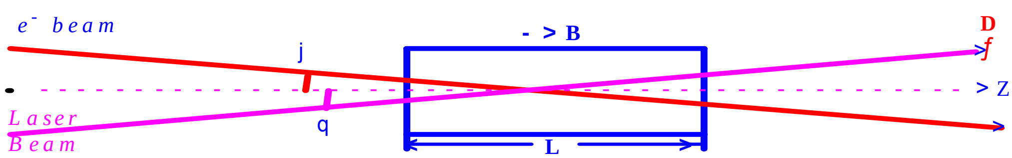

Fig.1

- In a magnetic field the electrons have a discrete spectrum of energy [2] shown on Fig.2:

$$\varepsilon_{n,\zeta} = [m^2 + P_z^2 + eB(2n+1+\zeta)]^{1/2}, \quad \textbf{(1)}$$

where n=0,1,2,… labels the electron energy levels, P_Z is the z-component of the electron momentum.

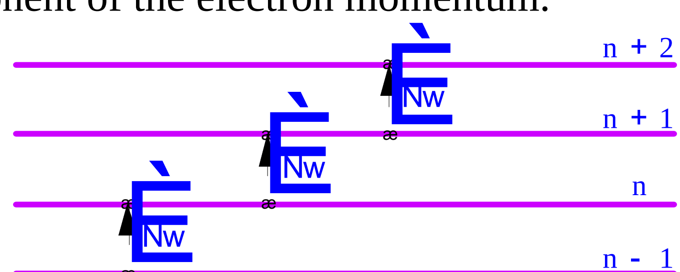

Fig.2

After incoming of electrons in a magnetic field the components of their momentum P_Z and $P_\perp$ are redistributed, and electrons occupy the quantum levels $\varepsilon_{n,\varsigma}$. Then photons of frequency ω can be resonantly absorbed at transitions of electrons between levels of energy. Using the law of energy-momentum conservation for photon absorption:

$$\varepsilon_{n,\zeta} + \omega = \varepsilon_{n',\zeta}, \qquad P_{z,0} + \omega\cos\theta = P_z \qquad \textbf{(2)}$$

and spectrum of energy $\varepsilon_{n,\varsigma}$ from (1) we find the condition of resonant absorption:

$$\omega\left[\gamma(1 - V_z\cos\theta) + \frac{\omega}{m}\frac{(\sin\theta)^2}{2}\right] = \omega_c(n' - n) \qquad \textbf{(3)}$$

We consider transitions between of electron energy levels without of change of electrons spin direction and transitions on the main harmonic n' - n = 1. The second member in the relation (3) can be neglected because

$$\frac{\hbar\omega}{2mc^2}(\sin\theta)^2 < 10^{-10}\frac{\omega_c}{\omega}.$$

Then from (3):

$$\omega = \frac{\omega_c}{\gamma(1 - V_z\cos\theta)} = \frac{\omega_c}{\gamma - \frac{P_z}{m}\cos\theta} \qquad \textbf{(4)}$$

In (4) the factor $(1 - V_Z\cos\theta)^{-1}$ describes Doppler's effect which in case of relativistic electrons has essential influence on a spectrum of absorption of photons.

- Substituting in (4) P_Z found from (1) we receive the new resonance absorption condition :

$$\gamma - \cos\theta\cdot\sqrt{\gamma^2 - 1 - \Omega\frac{\hbar\omega}{mc^2}(2n + 1 + \varsigma)} = \Omega \qquad (5)$$

where γ depends from $\Omega = \omega_c/\omega$, θ and n.

- Let's estimate the value of n for the given γ , Ω and θ. From (5) we find:

$$n = \frac{1}{\left(\frac{2\Omega \cdot \hbar\omega}{mc^2}\right)} \cdot \left[\gamma^2 - 1 - \left(\frac{\gamma - \Omega}{\cos\theta}\right)^2 - \frac{\Omega \cdot \hbar\omega}{mc^2}(1+\varsigma)\right] \qquad (6)$$

Using (6), from condition $n \geq 0$ we find the restriction on angle θ:

$$\theta \leq \sqrt{\frac{2\Omega}{\gamma} - \frac{1}{\gamma^2}} \qquad (7)$$

From (6) we can find $n_{\max}$, taking into account that $\partial n / \partial\theta = 0$ at $\theta = 0$. Neglecting in (6) by small member $\frac{\Omega \cdot \hbar\omega}{mc^2}(1+\varsigma) << 1$ for $n_{\max}$ we find approximately:

$$n_{\max} = \frac{1}{\frac{\hbar\omega}{mc^2}} \cdot \left(\gamma - \frac{1}{2\Omega} - \frac{\Omega}{2}\right) \qquad (8)$$

According to (6) we can find the dependence n from θ for fixed γ and Ω (Fig.3).

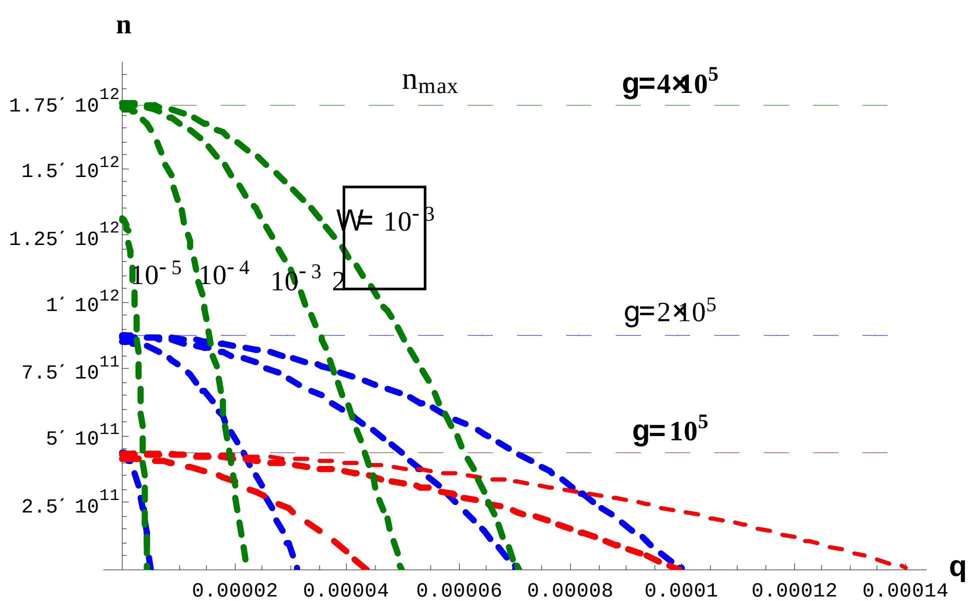

Fig.3

We assume, that N_e electrons are allocated on n quantum levels according to the Gaussian distribution law (Fig.4).

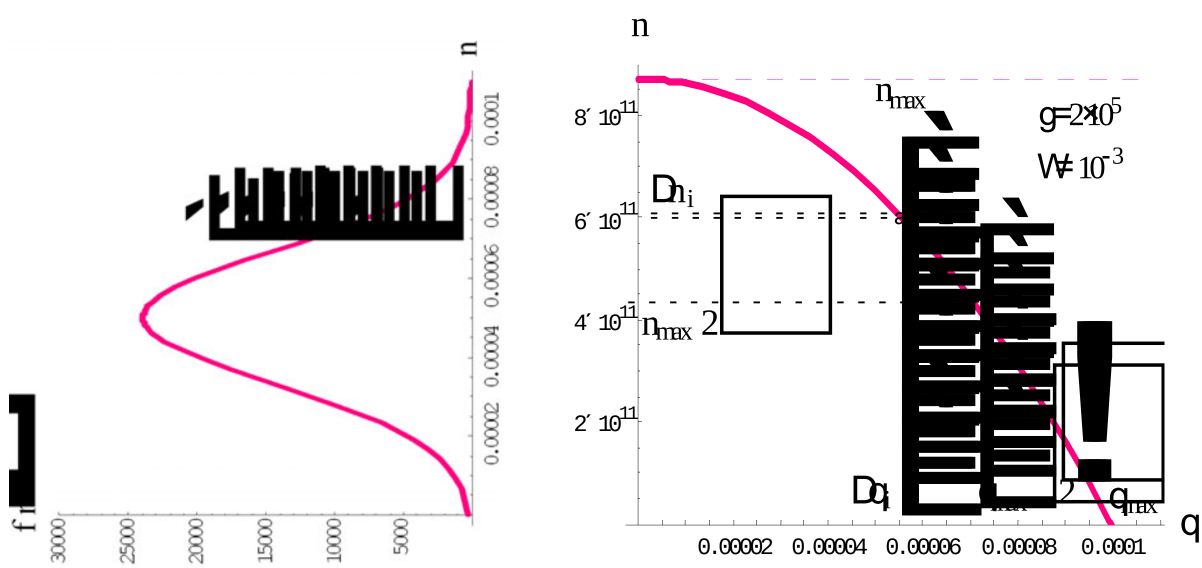

Fig.4

From Fig.4 it is clear that electrons with fixed γ and Ω can resonantly absorb photons in interval of angles q®0¸ qmax , where according to (7)

$$\theta_{\max} = \sqrt{\frac{2\Omega}{\gamma} - \frac{1}{\gamma^2}} \qquad (9)$$

Let's notice, that the light beam of diameter D because of diffraction diverges in limits of angle $\theta_d \cong \lambda / D$ (λ - length of wave) around of a wave vector direction and $\theta_{\max} < \theta_d$.

For example, electrons at levels Δn_i can resonantly absorb photons in interval of angles $\Delta\theta_i$ Fig.4. If intensity of absorption of photons by electrons from quantum levels Δn_i is equal to $\Delta I_{abs,i}$ then intensity of absorption by all electrons will be:

$$I_{abs} = \sum_{i=0}^{n_{\max}} \Delta I_{abs,i} \qquad (10)$$

2. DETERMINATION OF THE ELECTRON BEAM ENERGY. THE RELATIVE ACCURACY OF MEASUREMENT.

The γ-factor of electron beam can be found from resonant condition (5)

$$\gamma = \frac{\Omega \pm \cos\theta\sqrt{\Omega^2 - (\sin\theta)^2(1+\dfrac{\hbar\omega}{mc^2}(2n+1+\varsigma))}}{(\sin\theta)^2} \qquad (11)$$

Dependence of γ from Ω according to (11) for different angles θ and quantum numbers n is shown on Fig.5.

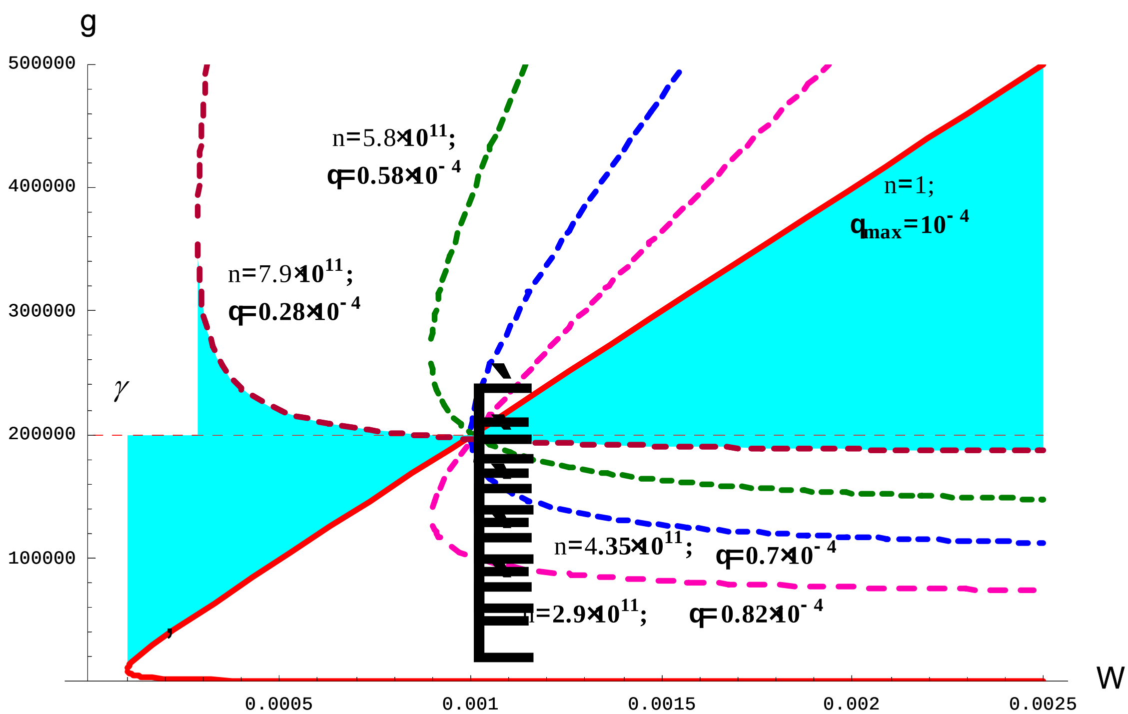

Fig.5

From Fig. 5 we see that the behavior of dependence γ from Ω for parameters n and θ essentially differs from behavior of resonant curves when parameters had been angles φ and θ [1]. In case of parameters n and θ the resonant curves are crossed in a point (γ_0, Ω_0) (whereas in case of φ and θ these curves are not crossed).

From Fig. 5 we see when the angle θ varies in limits from $\theta_{\max}$ up to $\theta_{\max}/\sqrt{2}$ the resonant point is on the upper branch of curve, whereas in interval of angles $\theta_{\max}/\sqrt{2}$ up to $\theta = 0$ the resonant point is on the lower branch.

- The choice of parameters n and θ allows to understand details of absorption and the possibility of measurement of energy with accuracy 10^{-4} near to centre of distribution over energy of electron beam, when the energy spread electron beam is 10^{-3}.

Resonant lines near to the point γ_0, Ω_0 of Fig.5 are shown on Fig.6, when $\Omega = \Omega_0 \pm \Delta\Omega$ and the electron beam energy spread is $\gamma_0 \pm \Delta\gamma$.

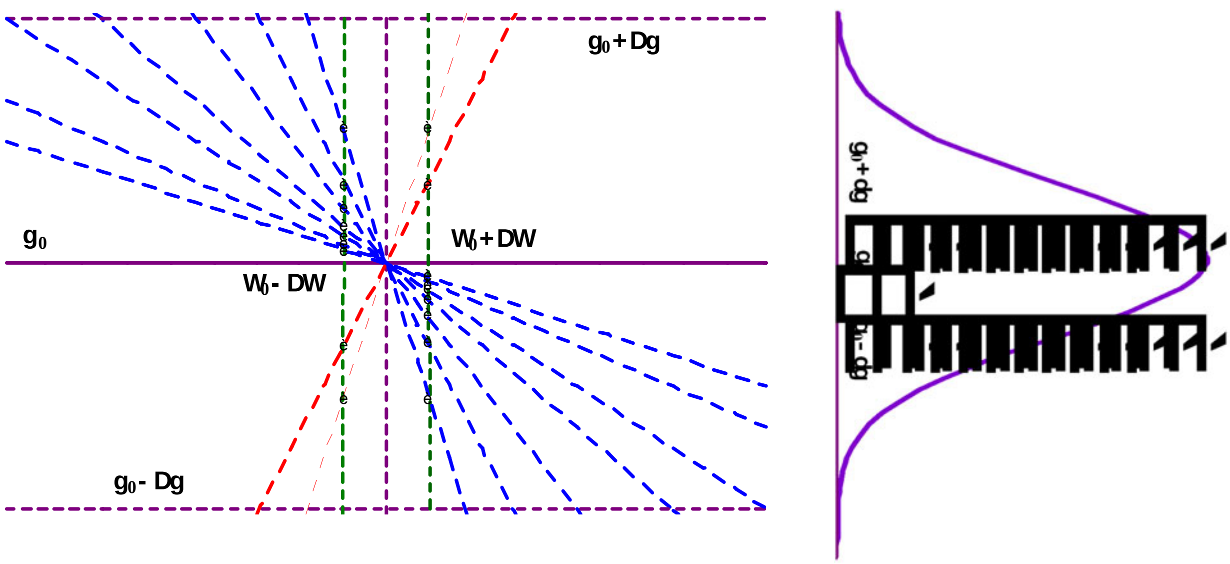

Fig.6

From Fig.6 it is clear that electrons would give the contribution in process of absorption from narrow area of energy $\gamma \pm \delta\gamma$ if to use Ω with small spread $\Delta\Omega$.

On Fig.7 it is schematically shown the dependence of intensity of absorption I_{abs} from Ω . Numerical estimations according to (11) and Fig.6 show that if $\Omega = 10^{-3}$ and $\Delta\Omega / \Omega = 10^{-4}$ then the main contribution to intensity of absorption will be given by electrons with $\Delta\gamma / \gamma = 10^{-4}$. Besides that it is clear the further decreasing of $\Delta\Omega / \Omega$ will lead to the decreasing of $\Delta\gamma / \gamma$.

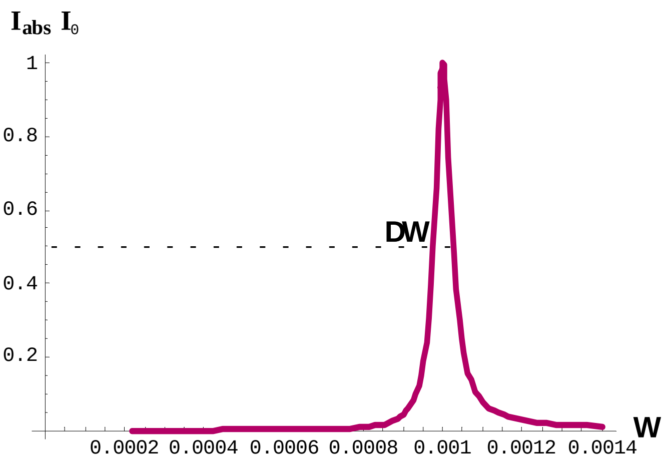

Fig.7

About the contribution to process of absorption of photons with various angles θ gives representation the Fig. 9.

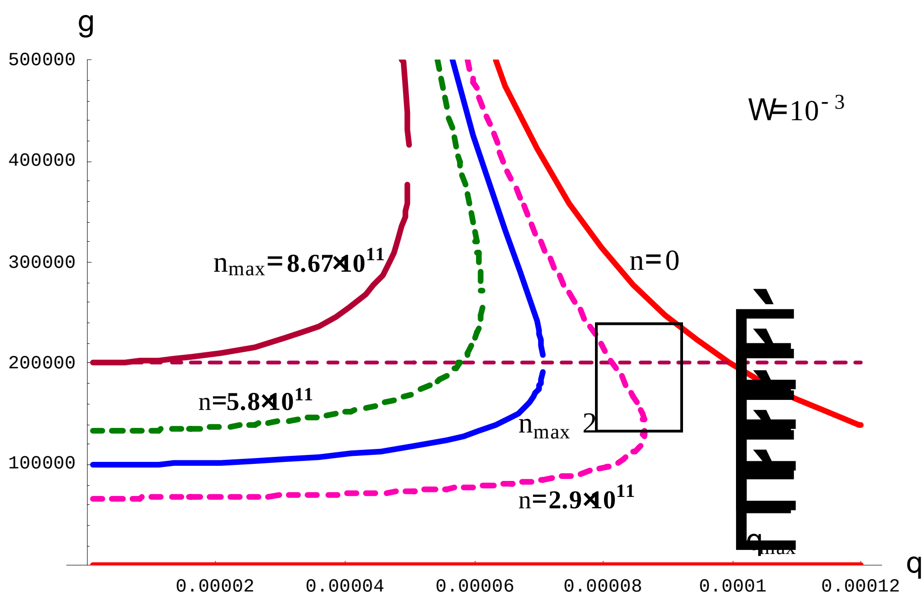

Fig.8

From Fig. 8 we see that in process of absorption give contribution photons with angles from $\theta = 0$ up to $\theta_{\max}$ (9) and electrons with $n = 0$ up to $n_{\max}$ (8).

According to (9) and Fig. 3 with growth of Ω increases n and $\theta_{\max}$, and as a result increases the intensity of photon absorption.

For the further calculations it is necessary to find the relation between angle of electron injection φ and parameters n, θ.

Using Fig.1 and (1), taking into account that $\varphi \ll 1$ and $\gamma \gg 1$ we find approximately:

$$2n\Omega \cdot \frac{\hbar\omega}{m} \approx \frac{P_\perp^2}{m^2} = (\sin\varphi)^2 \cdot \frac{P^2}{m^2} \approx \varphi^2\gamma^2 \qquad (12)$$

On the other hand from (6) we have:

$$2n\Omega \cdot \frac{\hbar\omega}{m} \approx 2\Omega\gamma - 1 - \theta^2\gamma^2. \qquad (13)$$

From (12) and (13) we receive the relation which we used earlier [1]:

$$\varphi^2 + \theta^2 \approx \frac{2\Omega}{\gamma} - \frac{1}{\gamma^2} \qquad (14)$$

We have noted [1] that intensity of photon absorption is maximal for angle $\varphi_{\max} \equiv \varphi_f$ and Ω_f, where

$$\varphi_f = \sqrt{\frac{2\Omega_f}{\gamma} - \frac{1}{\gamma^2}} \qquad (15)$$

It is important that the angle φ_f is parameter independent from electron beam energy. We can use this property of beam for determination of electron beam energy.

For this purpose, first, for known energy γ_0, over maximum of I_{abs} we find Ω_f (Fig.9) and then we can calculate:

$$\varphi_f = \sqrt{\frac{2\Omega_f}{\gamma_0} - \frac{1}{\gamma_0^2}} \qquad (16)$$

After that for unknown energy γ_U over maximum of I_{abs} we find Ω_F (Fig.9) and then we can calculate:

$$\varphi_F = \sqrt{\frac{2\Omega_F}{\gamma_U} - \frac{1}{\gamma_U^2}} \qquad (17)$$

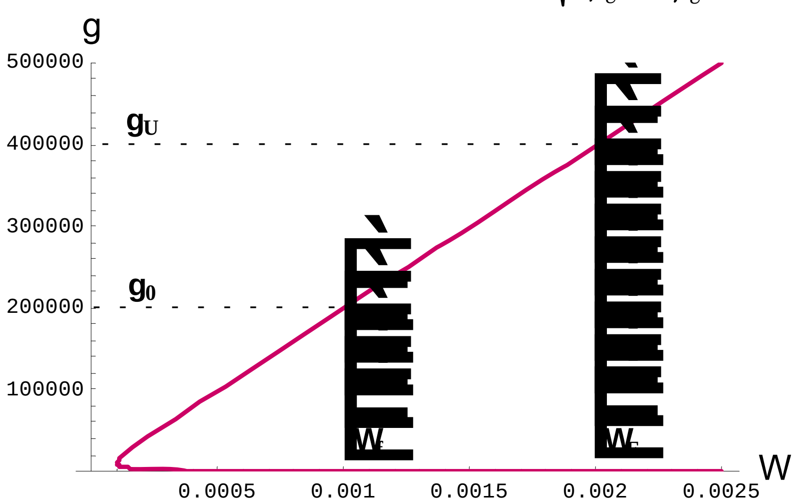

Fig.9

Taking into account that $\varphi_f = \varphi_F$, from (16) and (17) we find

$$\gamma_U = \gamma_0 \cdot \frac{\gamma_0\Omega_F + \sqrt{\gamma_0^2\Omega_F^2 - (2\gamma_0\Omega_f - 1)}}{2\gamma_0\Omega_f - 1} \qquad (18)$$

3. DIFFERENCE OF DIRECTIONS OF RADIATION AND ABSORPTION OF PHOTONS BY ELECTRONS

- Above we have seen that absorption of photons occurs within of angles $\theta = 0$ up to $\theta_{\max}$ and the main contribution is given by photons with angles $\theta_{\max}/\sqrt{2}$ (Fig.4, Fig.10).
- Radiation of electron goes under angle $\alpha \approx 1/\gamma$ concerning of electron velocity V_e.

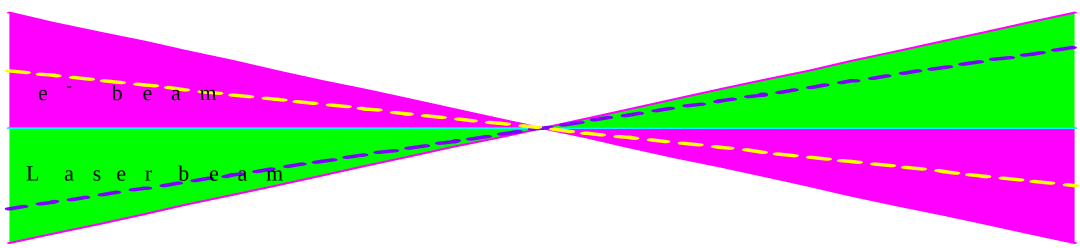

Fig.10

If the electron beam has Gaussian distribution over angles φ from $\varphi = 0$ up to $\varphi_{\max}$ then the main contribution to radiation give electrons with angles $\varphi_{\max}/2$ (Fig.4).

- Thus, overlapping of directions of radiation and absorption does not occur and detector D (Fig. 1) can register only laser photons.

4. EXPERIMENT

Acceleration of electrons in a field of electromagnetic wave propagating along a constant homogeneous magnetic field has been experimentally investigated in 1969 [2]. The experiments has confirmed opportunity of acceleration of electrons in these fields and the theoretically predicted gain of energy was in the reasonable agreement with the measured value.

- In the circular cross-section cavity resonator (with TE-111 mode) the electron beam with low energy 100-1000 eV and with a current 1-100 mA was injected (Fig.11).
  - The magnetic field was with a value of 4.2 kG, wavelength – 30cm and 3 cm.
  - It has been measured that electrons are accelerated up to 300 keV at input power of wave 25 kW and 460 keV at input power of wave 46 kW.

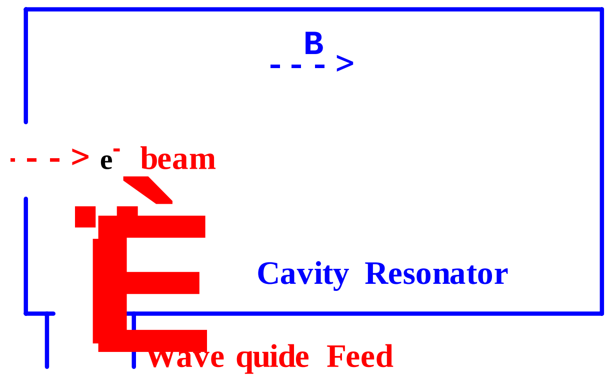

Fig.11

5. CONCLUSIONS

- The choice of parameters n and θ allows to understand details of absorption and the possibility of measurement of energy with accuracy 10^{-4} (and better) near to centre of energy distribution of electron beam.
- Directions of radiation and absorption are not overlapping and detector D registers only laser photons.

6. REFERENCES

1. http://www-zeuthen.desy.de/www_users/workshops/e_spec/may2005/new_aspects05_melikian.pdf
2. H.R. Jory and A.W. Trivelpiece, J. of Applied Physics, v.39, No.7, p. 3053, 1968.