The use of the electric field of the microbunch for the beam energy measurement.

B.Zh. Zalikhanov, E.M. Syresin

Joint Institute for Nuclear Researches

1. Introduction

In the ILC project [1] the several methods or the absolute calibration of the energy of the electron-positron beams are considered, some of them are essentially new. The application of the Compton backscattering of the laser photons on the electrons and the resonance absorption of laser light within a magnetic field are studied. The possibility of the precision measurement of the angle characteristics of the synchrotron radiation from the ILC magnetic spectrometer is investigated also.

Below one more new technique of the beam energy measurement through the electric field value from the microbunch electrons is proposed. It can be easy realised with the high sensibility the variation of the beam energy.

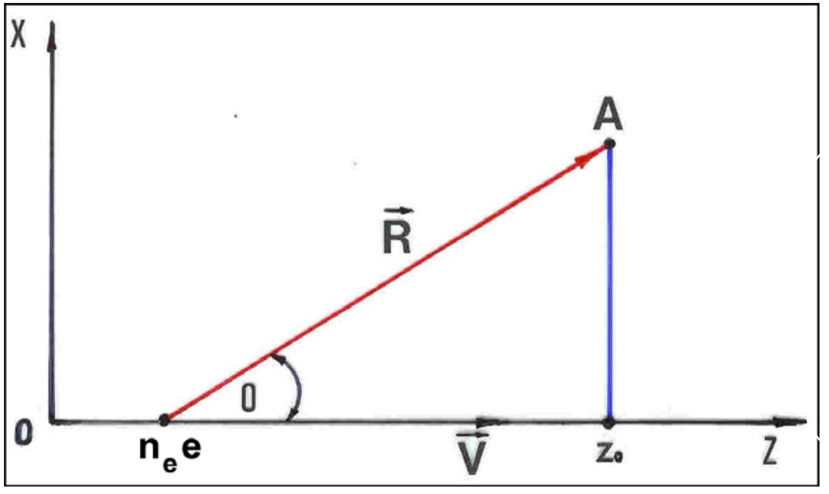


Fig.1

The electric field of the uniform charge is determined by the expression [2]

$$E = \frac{q}{4\pi\varepsilon_0 R^2} \cdot \frac{1-\beta^2}{[1-\beta^2 \sin^2\theta]^{3/2}} \qquad (1)$$

where θ is the angle between the movement direction and the radius-vector R (fig.1); β=v/c.

The value of the field is minimal along the movement direction (θ = 0, π) and is equal to

$$E_{//} = \frac{q}{4\pi\varepsilon_0 R^2}\cdot(1-\beta^2) = \frac{q}{4\pi\varepsilon_0 R^2}\cdot\frac{1}{\gamma^2} \qquad (2)$$

The electric field has maximum value in the direction perpendicular to the velocity ($\theta = \pi/2$)

$$E_{\perp} = \frac{q}{4\pi\varepsilon_0 R^2} \cdot \frac{1}{\sqrt{1-\beta^2}} = \frac{q}{4\pi\varepsilon_0 R^2} \cdot \gamma \qquad (3)$$

From the equations (2) and (3) one can see, that with the increase of the charge velocity *v* the component $E_{\parallel}$ decreases, but the value $E_{\perp}$ increases.

From the relation of the equations (2) to (3) we get

$$\frac{E_{\perp}}{E_{//}} = \gamma^3 \qquad (4)$$

3. The parameters of the microbunch

The number of the electrons in the microbunch is $n_e = 2 \cdot 10^{10}$. The length of the microbunch is $t=10^{-12}$ sec. The geometrical sizes of the microbunch are $\sigma_x = 2$ μm, $\sigma_y = 20$ μm, $\sigma_z = 300$ μm. The electron energy is 250 GeV. The relativistic factor for such electrons is $\gamma=4.892 \cdot 10^5$. The space charge density in the microbunch is $\rho=0.26$ C/cm^3.

Using the given parameters and the equation (1) let determine the position of the charge $q=n_e \cdot e$ relative to a point z_0 when the field in the point A is equal to zero. The given requirement is satisfied if the charge is in the range $z_0 \pm 1.5$ μm. If we take the effective field as $E \geq 10^2$ V/cm, then this condition fulfilment at the position of the charge in the pointes

$z_0 \pm 0.0001$ cm.

Hence if the centre of the bunch with the volume $12 \cdot 10^{-9}$ cm^3 is in the point z_0, then effective electric field in the point A will be produced by the electrons in the layer $z_0 \pm 0.0001$ cm thick.

If the electrons within the microbunch were described by the Gauss distribution, then the layer $z \leq 1$ μm thick contains the number of electrons $n_1 \approx 2 \bullet 10^8$. At the position of this layer near in the neighbourhood point z_0 the components of the field have the next values:

$$E_{\perp} = \frac{n_1 e}{4\pi\varepsilon_0 a^2} \cdot \gamma = 1.4 \cdot 10^7 V / cm \qquad (5)$$

$$E_{//} = \frac{n_e e}{4\pi\varepsilon_0 a^2} \cdot \frac{1}{\gamma^2} = 1.2 \cdot 10^{-10} V / cm \qquad (6)$$

Let the electron energy have decreased from 250 GeV to 230 GeV. In this case $\gamma = 4.5 \cdot 10^5$ and the value of the field strength along the movement direction will be equal to $E_{//} = 1.4 \cdot 10^{-10}$ V/cm. Comparing the field values for $E_{//}$ it is possible without a risk to take it equal to $1.3 \cdot 10^{-10}$ V/cm and to write the equation (4) in the form:

$$E = 1.3 \cdot 10^{-10} \cdot \gamma^3 \quad (7)$$

Differentiating the equation (7) we get

$$dE = 1.3 \cdot 10^{-10} \cdot 3\gamma^2 d\gamma \qquad (8)$$

From here we have the connection between the measurement accuracy of the bunch electric field and the required accuracy of the determination of γ

$$\frac{1}{3}\frac{dE}{E} = \frac{d\gamma}{\gamma} \qquad (9)$$

Because $d\gamma/\gamma = 10^{-5}$, then according (9), the value of the microbunch field can be measured with the accuracy

in 3 times less.

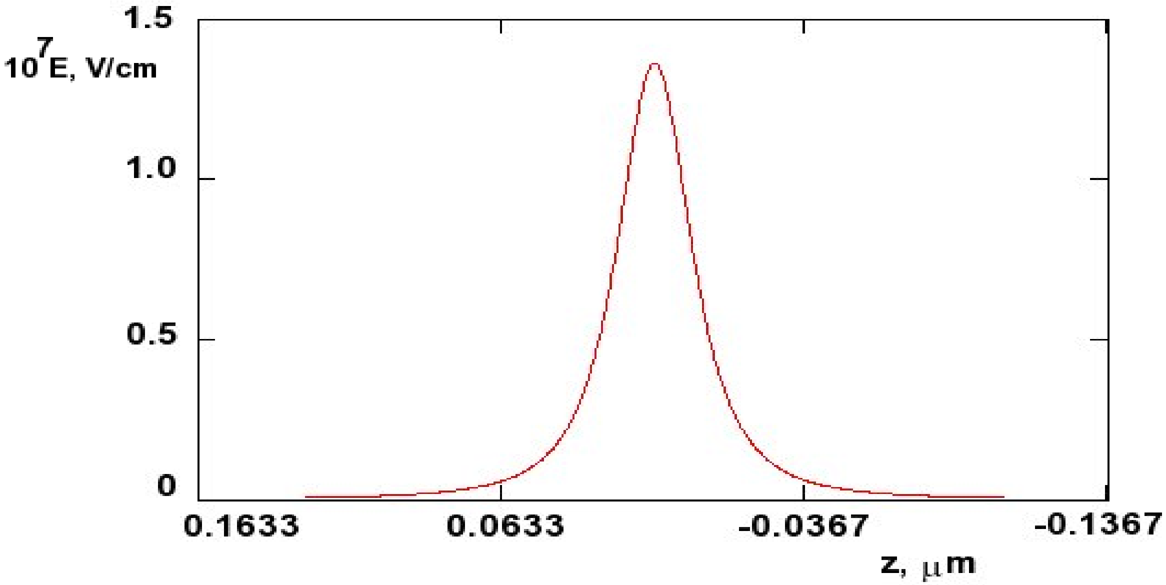


Fig.2. The electrical field of the electron layer of the moving microbunch.
$\gamma=4.892\cdot10^5$,
$N_e=2\cdot10^8$.

A distribution of an electrical field in the neighbourho – od of z_0 point when the numbers of the electrons in the layer are taken equal to $2\cdot10^8$ and $\gamma = 4.892\cdot10^5$ is shown in fig.2.

The coordinates [0.0633; 0.0367] on the z-axis corres – pond to the field equals $5 \cdot 10^5$ V/cm. It will be seen from the distribution that the field of the electrons is localized within the range of the width 0.1 μm.

If γ is equal to $4.89237 \cdot 10^5$, then the energy of the electrons is equal to $\varepsilon_1 = 250.000107 \cdot 10^9$ eV and when n_e is equal to $2 \cdot 10^8$ then the electrical field is equal to

$$E_1 = 1.1124525 \cdot 10^7 \text{ V/cm.}$$

When γ changes to quantity of $4.892382 \cdot 10^5$ we obtain for ε_2 and E_2 the next quantities:

$250.000720 \cdot 10^9$ eV and $1.1124866 \cdot 10^7$ V/cm.

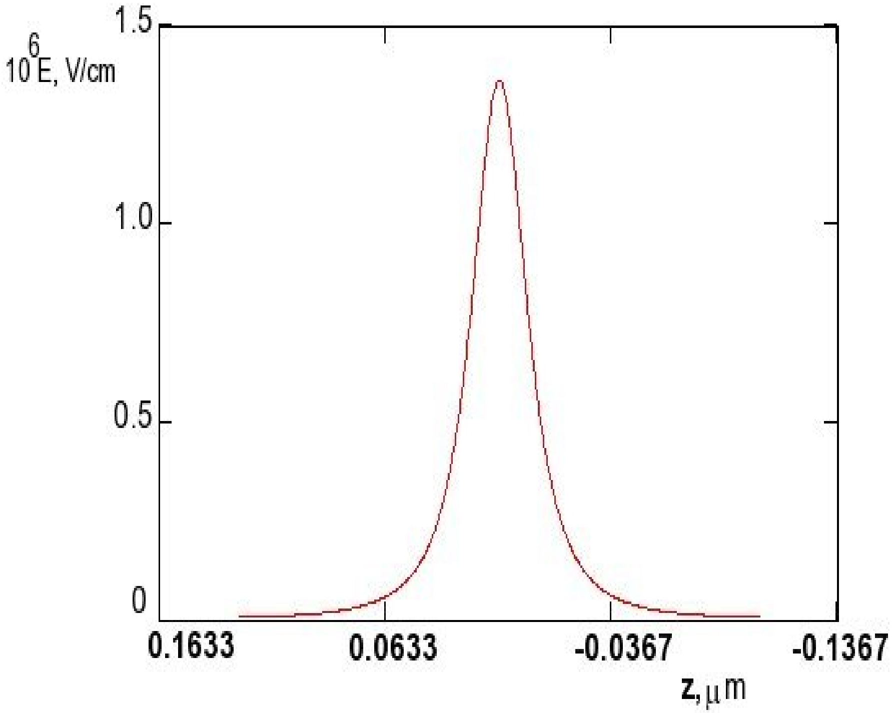
It follows from these quatities that

$$\frac{\Delta\gamma}{\gamma} = 1.02 \cdot 10^{-5}, \qquad \frac{\Delta E}{E} = 3 \cdot 10^{-5}$$

and the relationship (9) is carried out.

It should be noted the proposal method can by useful to determine the space position of the microbunch.

The dependence of the electrical field from the numbers of the electrons in the neighbourhood point z_0 in shown in Fig.3 and Fig.4.



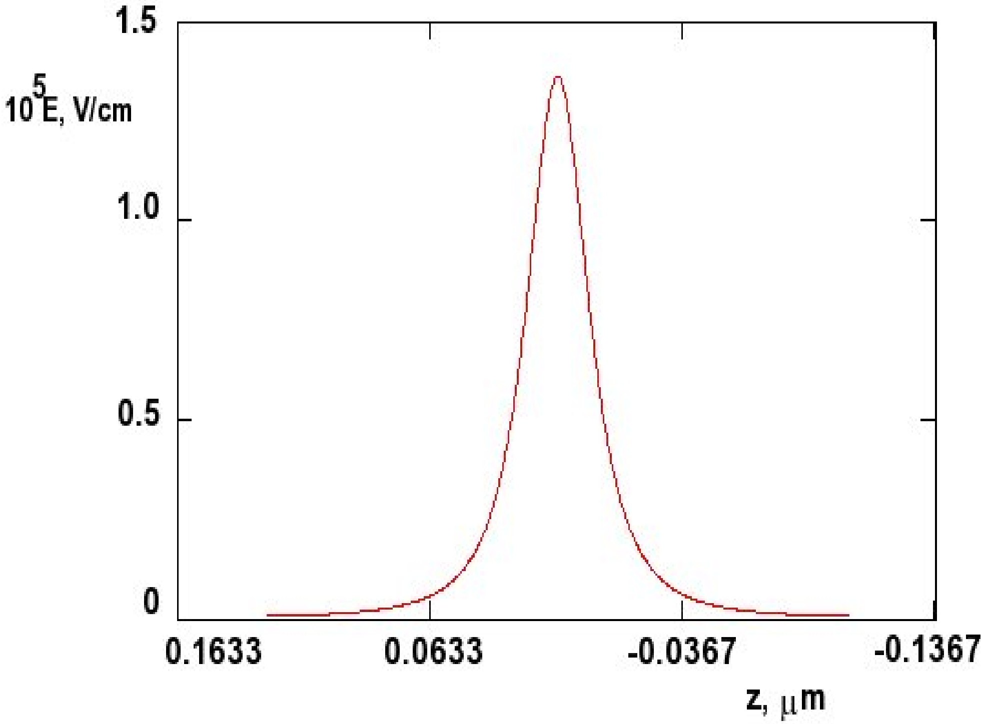


Fig.3. $\gamma=4.892 \cdot 10^5$, $n_e=2 \cdot 10^7$.

Fig.4. $\gamma=4.892 \cdot 10^5$, $n_e=2 \cdot 10^6$.

4. Version of the polarization detector for detecting the electric field of the microbunch.

- As is known, polarization occurs in a metal or a dielectric placed in a constant or variable. Out of all this variety of polarization processes the one important for us is electron polarization established within the time about **10^{-15} s**. After the voltage is switched off, electron polarization disappears within the time about **10^{-15} s**.

- Ion polarization or elastic-ion polarization is established within the time **10^{-12} – 10^{-13} s**.

Let the electric field point in the x direction. The equation of motion of the electron in the field E_x has the form

$$m^* \ddot{x} = eE_x$$

→

$$\dot{x} = V_{x0} + \frac{eE_x t}{m^*}$$

The average velocity gain V_x of the electron accelerated within the time between two collisions is equal to half its final velocity. The electron charge flux (considering $V_{x0}=0$) is expressed by the formula. Substituting the value V_x into it, we get an expression for current

$$V_x = \frac{eE_x \tau}{m^*}$$

→

$$J_x = NeV_x$$

$$J_x = \frac{Ne^2\tau}{m^*} E_x = \sigma E_x$$

N – is the total number of valence electrons in **1m^3**.
m* -- is the effective mass of the electron possessing inertia.
Let us find the number **N** of polarized electrons produced in copper ring of radius **R=1cm** and cross section radius equal to **150 μ**.

N_{Cu}=8.5 10^{28} m^{-3}; **m*=**1.2m; In a ring of volume 4.4 x $10^{-9}$$m^3$ there are 4 x 10^{20} valence electrons.

E = 10^9 V/m. **σ = 1/r** =5.8 x $10^7$$\Omega^{-1}$$m^{-1}$

$$\tau = \frac{1.2m\sigma}{Ne^2} = 2.9 \cdot 10^{-14}\ s.$$

Substituting the values obtained into equation and taking into account that E = 10^9V/m, we get

$$J = \frac{Ne^2\tau}{1.2m}E = 2.7\cdot 10^4\, A/cm = 2.7\cdot 10^4\, C/s\cdot cm^2 \qquad \textbf{(16)}$$

The charge of electrons producing current through the ring surface for a period of 10^{-12} s can be found from the result (16)

$$q = JT_{bunch}S_{ring} = 2.7\cdot 10^4\cdot 10^{-12}\cdot 0.3 = 8\cdot 10^{-9}\,C$$

$Q = 8 \times 10^{-9}$ C.

This charge is quite large and may be detected. However, the detector itself should be manufactured in compliance with all requirements to microwave frequency systems [RC ≈ (1-5) 10^{-12} s]

and have a high figure of merit.

5. References

[1] The Beam Energy Spectrometer at the International Linear Collider, DESY Report, LC-DET-2004-031

[2] E.M. Purcell, Electricity and magnetism (Berkeley physics course, Vol.2)