Some Remarks, Problems, Questions to the Method of Resonance Absorption for the Beam Energy Determination

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In this talk I would like to discuss some problems of the RA method, indicate some possible useful parameters and point out questions respectively problems which should be evaluated in the future

Basic assumption: ***the RA method is reliable***

In the following, only **CO_2** (with λ = 10.6 μm)
and **Nd:YAG** lasers (with λ = 1.06 (or 0.53, 0.27) μm) are considered

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Check of the main condition for RA:

$$\gamma_0 = \frac{\Omega \pm \cos\varphi \cos\theta \sqrt{\Omega^2 - (1 - \cos\varphi^2 \cos\theta^2)}}{1 - \cos\varphi^2 \cos\theta^2}$$

with $\Omega = \frac{\omega_c}{\omega} = \frac{eB}{m\omega}$

with the magnetic field B and the laser frequency ω

$$\Omega^2 \geq 1 - (\cos\varphi \cos\theta)^2$$ **?**

some examples:

Laser	B [T]	Ω^2
CO_2	0.5	2.5 10-7
	2.0	3.9 10-6
	6.0	3.6 10-5
Nd:YAG Λ=1.064 µm	0.5	2.4 10-9
	2.0	3.8 10-8
	6.0	3.4 10-7
Nd:YAG Λ=0.266 µm	0.5	1.5 10-10
	2.0	2.5 10-9
	6.0	2.2 10-8

with φ = 0.

θ [rad]	$1 - (\cos\varphi\cos\theta)^2$
1 10-4	0.9 10-8
2 10-4	4.0 10-8
4 10-4	1.6 10-7
5 10-4	2.5 10-7
8 10-4	6.4 10-7
1 10-3	1.0 10-6
5 10-3	2.5 10-5

From comparison of the two tables, some lessons follow:

- if CO_2 is used
  - low B-fields of < 0.5 T require θ < $5 \cdot 10^{-4}$ rad;
  - larger B-fields (~6 T) allow to increase θ up to 5 mrad
- if Nd:YAG (λ=1.064 μm) is used
  - low B-fields of < 0.5 T require very small θ, significantly below 10^{-4} rad;
  - with increasing B-field, θ can be larger, e.g. B=6 T, θ=$5 \cdot 10^{-4}$ rad
- if Nd:YAG (λ=0.266 μm) is used
  - only large B-fields (> 5 T) in conjunction with very small θ (<10^{-4} rad) are allowed
- in many discussions Robert M. applied the condition

$$\Omega^2 >> 1-(\cos\varphi\cos\theta)^2$$

↑

which is only rarely fulfilled,
→ avoid its general application !
(or check it for a given set of parameters)

What about the constraints to estimate the intensity of the Resonance Absorption process ?

Two constraints were used: **$\xi \ll 1$** and **$\omega/2\omega_c \ll \gamma_0$** **?**

If absorption occurs within the B-field, the electron gets some acceleration.
If the number of photons absorbed by an electron is denoted by n_γ

The ***energy growth of the electrons*** near the resonance condition can be expressed by

$$E_b - E_b^0 \cong n_\gamma E_\gamma \approx mc^2 \xi \omega \sqrt{\frac{2\Omega}{\gamma_0}} \cdot t_i$$

where $\xi = e\varepsilon / mc\omega$ a non-dimensional parameter of the laser intensity, with **$\xi \ll 1$** !

ε = amplitude of energy vector of the laser [V/cm]

ω = frequency of the laser
$\omega_c = eB/m$
} with **$\omega/2\omega_c \ll \gamma_0$**

$\xi << 1$?

Using - CO_2 and Nd:YAG laser (infrared … ultraviolet)
- E_b = 45, 250 and 500 GeV
- B-fields = 0.5 …. 6 T
- magnet lengths = 50 … 100 cm

$\xi < 10^{-7}$

$\omega/2\omega_c << \gamma_0$?

Laser	B [T]	$\omega/2\omega_c$	E_b / γ_0
CO_2	0.5	1000	45 / 8.8 10 4
	2.0	250	250 / 4.9 10 5
	6.0	83	500 / 9.8 10 5
Nd:YAG Λ=1.064 µm	0.5	1.0 10 4	45 / 8.8 10 4
	2.0	2.6 10 3	250 / 4.9 10 5
	6.0	857	500 / 9.8 10 5
Nd:YAG Λ=0.266 µm	0.5	4.0 10 5	45 / 8.8 10 4
	2.0	1.0 10 5	250 / 4.9 10 5
	6.0	3.4 10 3	500 / 9.8 10 5

and conclude:

- the ultraviolet Nd:YAG laser violates this constraint for low B-fields (< 2 T or so) at all beam energies. If one intends to use such a laser, the B > 5 T.

- if the infrared Nd:YAG or the CO_2 laser will be used, the constraint is appr. fulfilled.

Laser power needed to achieve RA ?

We require $n_\gamma = 1$
(in a classical approach)

$$\xi = \frac{\lambda_c}{L}\sqrt{\frac{\gamma_0}{2\Omega}} = \frac{\lambda_c}{L}\sqrt{\frac{\gamma_0 m\omega}{2eB}}$$ [· n_γ (=1)]

Remember, ξ is proportional to the laser energy vector and hence proportional to the square of the laser power

$$\Rightarrow \quad Power_{\min}\left[\frac{W}{cm^2}\right] \cong \left[\frac{\varepsilon[V/cm]}{19.4}\right]^2$$

i.e., the min laser power required scales with:

- $1/L^2$
- $1/B$
- E_b
- $1/\lambda^3$ (ω^3)

?
Θ,φ dependence

The number of incident laser photons related to the laser power is

$$N_{I,\gamma}\left[cm^{-2}\sec^{-1}\right] = \frac{Power[W/cm^2]}{E_\gamma[eV]} * \frac{1}{1.6022\cdot 10^{-19}}$$

Conversion factor to Joule

For a ***CO_2*** laser:

min. $N_{l,\gamma}$ [cm^{-2} sec^{-1}] ranges between $1.5\ 10^{18}$ … $7.2\ 10^{20}$
in dependence on γ_0, L, B-field:

- with increasing E_b, $N_{l,\gamma}$ increases
- shorter magnets needs more $N_{l,\gamma}$
- smaller B-field require more $N_{l,\gamma}$

Examples:

E_b = 50 GeV,	L = 100 cm,	B = 2 T	→	$N_{l,\gamma} = 4.5\ 10^{18}$
E_b = 250 GeV,	L = 100 cm,	B = 2 T	→	$N_{l,\gamma} = 2.3\ 10^{19}$
E_b = 250 GeV,	L = 100 cm,	B = 6 T	→	$N_{l,\gamma} = 7.5\ 10^{18}$

For a ***Nd:YAG*** (infrared) laser:

min. $N_{l,\gamma}$ [cm^{-2} sec^{-1}] ranges between $1.5\ 10^{20}$… $7.2\ 10^{22}$

Examples:

E_b = 50 GeV,	L = 100 cm,	B = 2 T	→	$N_{l,\gamma} = 4.5\ 10^{20}$
E_b = 250 GeV,	L = 100 cm,	B = 2 T	→	$N_{l,\gamma} = 2.3\ 10^{21}$
E_b = 250 GeV,	L = 100 cm,	B = 6 T	→	$N_{l,\gamma} = 7.4\ 10^{20}$

So far, **min. $N_{l,\gamma}$ is given in $cm^{-2} \cdot sec^{-1}$**, but in practice we are forced

- to reduce the laser spot size

- should account for a 'pulsed' laser

$$N_{l,\gamma} = N_{l,\gamma}[cm^{-2}\, sec^{-1}] \cdot S \cdot T$$

with **S**, the laser spot size in [cm^2]
and **T**, the laser pulse length, in [sec]

For a ***CO_2*** laser with **T = 1 msec** pulse length

→ $N_{l,\gamma}^{min} = (1.5\ 10^{15} \ldots 7.2\ 10^{17}) \cdot S$

For a ***Nd:YAG*** laser with the option **of much shorter pulse lengths** (100 … 50 … 10 psec)

→ $N_{l,\gamma}^{min} = (1.5\ 10^{9} \ldots 7.2\ 10^{11}) \cdot S$

(for a **10 psec** long laser pulse)

so that with **N_e/bunch = 2 10^{10}** at ILC → ***all laser photons might be absorbed for a perfect overlap of the laser with the electron beam*** ***(?)***

But what about the laser spot size S ?

CO_2 laser (λ=10.6 µm)

E_b [GeV]	γ_0	L [cm]	B [T]	ξ	ε [V/cm] needed	min. power [W/cm^2]	$N_{l,\gamma}$ [cm^{-2}s^{-1}]
50	1 10 5	100	2.0	1.93 10 -9	5.63	0.085	4.53 10 18
		100	0.5	3.86 10 -9	11.26	0.337	1.80 10 19
		100	6.0	1.12 10 -9	3.27	0.028	1.49 10 18
		50	2.0	3.86 10 -9	11.26	0.337	1.80 10 19
		50	0.5	7.72 10 -9	2.52	1.348	7.19 10 19
		50	6.0	2.24 10 -9	6.54	0.114	6.10 10 18
250	5 10 5	100	2.0	4.32 10 -9	12.61	0.422	2.25 10 19
		100	0.5	8.63 10 -9	25.18	1.685	8.98 10 19
		100	6.0	2.50 10 -9	7.30	0.141	7.52 10 18
		50	2.0	8.63 10 -9	25.18	1.685	8.98 10 19
		50	0.5	17.3 10 -9	50.36	6.740	35.9 10 19
		50	6.0	5.01 10 -9	14.62	0.568	3.03 10 19
500	1 10 6	100	2.0	6.10 10 -9	17.79	0.841	4.48 10 19
		100	0.5	12.2 10 -9	35.58	3.364	17.9 10 19
		100	6.0	3.54 10 -9	10.33	0.284	1.52 10 19
		50	2.0	8.63 10-9	25.18	3.370	18.0 10 19
		50	0.5	17.3 10 -9	50.36	13.48	71.9 10 19
		50	6.0	5.01 10 -9	14.62	1.140	6.08 10 19

Nd:YAG laser (infrared with λ=0.1064 μm)

E_b [GeV]	γ_0	L [cm]	B [T]	ξ	ε [V/cm] needed	min. power [W/cm^2]	$N_{l,\gamma}$ [cm^{-2}s^{-1}]
50	1 10 5	100	0.5	1.22 10 -8	354.2	333.35	17.85 10 20
		100	2.0	0.61 10 -8	177.1	83.34	4.47 10 20
		100	6.0	0.35 10 -9	101.6	27.44	1.47 10 20
		50	0.5	2.44 10-8	708.4	1334.0	71.5 10 20
		50	2.0.	1.22 10 -8	216.1	124.1	6.65 10 20
		50	6.0	0.71 10 -8	206.1	112.9	6.05 10 20
250	5 10 5	100	0.5				
		100	2.0.				
		100	6.0	*√5		*5	
		50	0.5				
		50	2.0				
		50	6.0				
500	1 10 6	100	0.5				
		100	2.0				
		100	6.0	*√10		*10	
		50	0.5				
		50	2.0				
		50	6.0				

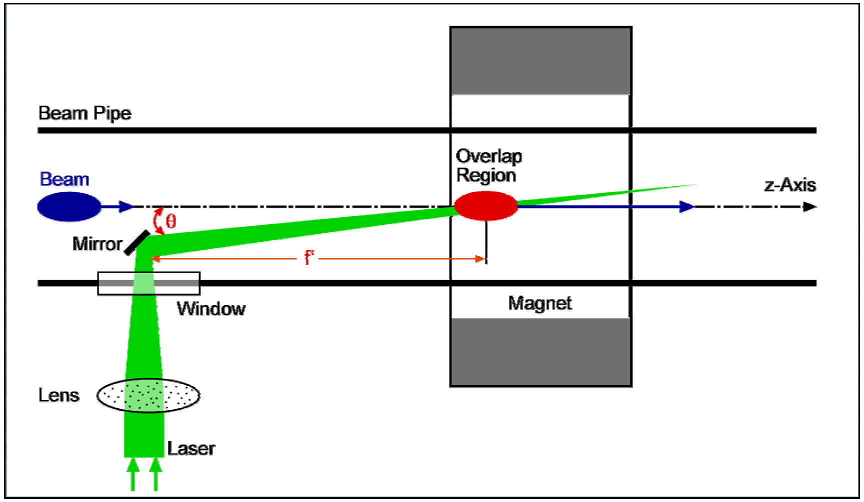
Laser spot size / laser emittance

Spot size of the laser within the magnet: $\boldsymbol{\sigma_{x\gamma}, \sigma_{y\gamma}}$

$\sigma_{x\gamma,y\gamma}$ can be expressed in terms of emittance of the laser $\varepsilon_{x.y}$ and its angular aperture $\theta_{x,y}$

$$\sigma_{x\gamma,y\gamma} = \frac{\varepsilon_{x,y}}{\theta_{x,y}} = \frac{f}{a_{x,y}} \cdot \varepsilon_{x,y}$$

(at small $\theta_{x,y}$)



$\boldsymbol{a_{x,y}}$ – ***effective half aperture*** of laser beam
f – ***focal length*** (~ f' in the fig.)
$\boldsymbol{\Theta_{x.y}}$ – ***angular half aperture of laser; it is not the angle between e$^-$ beam and laser***, (***a*** being in the order of few mm)

Now, the ***best possible emittance of a perfect laser is limited by laws of optics:***

$$\varepsilon_{x.y}^{\min} = \frac{\lambda}{4\pi}$$

i.e. it is determined by the laser wavelength, so that for a given λ → ***minimum of laser spot size***

$\sigma_{x,y}^{min}$ ***is proportional to the focal length f and the min. emittance*** ε^{min}
but inverse proportional to the effective half aperture a

Examples:

wavelength [μm]	***f [m]***	***a [mm]***	σ^{min} ***[μm]***
CO_2 λ = 10.6	3	3	844 ***!***
	10	6	1407 ***!***
	10	3	2814 ***!***
Nd:YAG λ = 0.1064	3	3	85
	10	6	141
	10	3	285
λ = 0.532	3	3	42
	10	6	71
	10	3	142
λ = 0.266	3	3	21
	10	6	35
	10	3	70

In practice, however, the spot size is typically two times larger

If CO_2: to reach a spot size of σ = 500 μm or less,
→ f < 3 m and a ≤ 3 mm;
but θ is then large
(few $\cdot 10^{-3}$ rad ***!***)

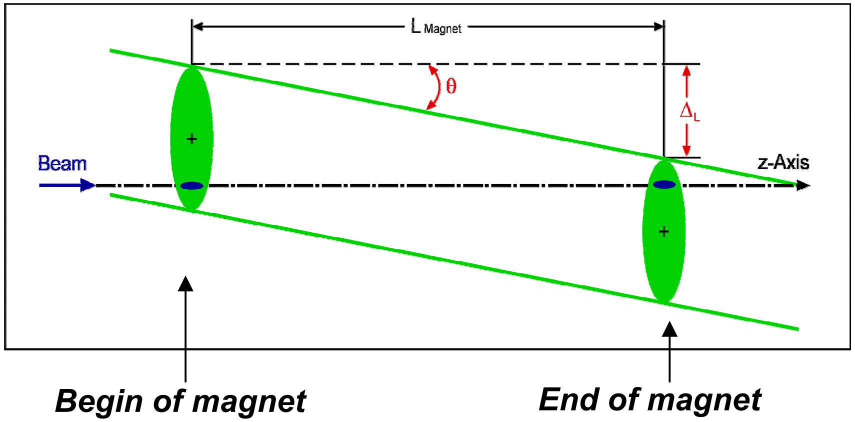
Basic resonance condition

$$\Omega^2 \geq 1-(\cos\varphi\cos\theta)^2$$

ok ?

Large vs. small θ, as seen from an other point of view:

The laser should cross the electron beam with θ over the ***total length L of the magnet*** in some optimized way



Aim: ensure that the electron bunch remains in overlap with the laser spot.
Due to some Gaussian intensity distribution of the laser photons, ***keep Δ_L small*** (to about ¼ … ½ of the laser spot size)
→ ***good absorption rate***

With $\Delta_L = L \cdot \tan\theta$ → ***some values for ΔL [μm]***

Θ [rad]	*L = 1 m*	*L = 0.5 m*	*L = 0.25 m*
$1\ 10^{-3}$	1000	500	250
$5\ 10^{-4}$	500	250	125
$1\ 10^{-4}$	250	50	25

If one needs a small laser spot size
→ ***avoid θ ~ 10^{-3} rad, even for very short magnets.***

If however θ should large (~1 mrad), → ***account for a large laser spot size (~1 mm) or short magnet (~25 cm)***

Vertical vs. horizontal beam crossing ?

- the size of the laser focus dominates over the electron beam size

- the electron beam is flat, i.e. $\sigma_x >> \sigma_y$, and one should probably locate the set-up at large beam sizes (resp. at large β-functions)

- less radiation (SR) is expected above and below the electron beam line

⇒ ***vertical beam crossing is preferred***

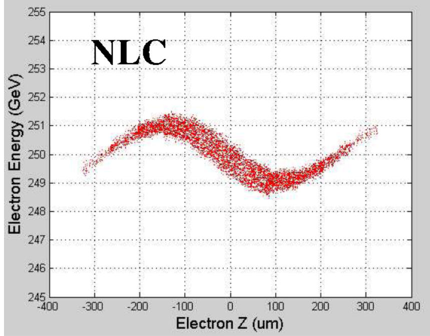
Significance of the RA method

The number of absorbed photons, $\boldsymbol{n_\gamma}$, is considered as an ***ideal number***

$$E_b - E_b^0 \cong n_\gamma E_\gamma \approx mc^2 \xi\omega \sqrt{\frac{2\Omega}{\gamma_0}} \cdot t_i$$

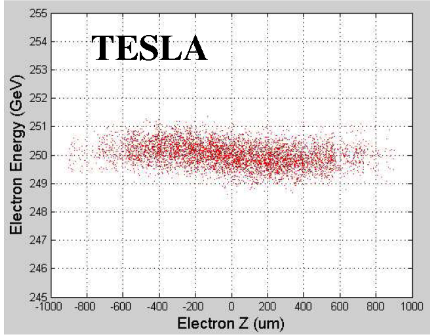
classical approach !

$\boldsymbol{n_\gamma}$ is a ***function*** of - the beam energy, $\boldsymbol{\gamma_0}$,
- the ***B-field***
- the laser frequency $\boldsymbol{\omega}$
- (possibly) on $\boldsymbol{\theta}$ ***and*** $\boldsymbol{\varphi}$



Now, **during the time of measurement** (e.g. within 1 msec)

→ the beam energy is not a fixed number
→ the laser frequency has a tiny (Gaussian ?) spread
→ θ and φ vary over some (Gaussian ?) ranges
→ the B-field may slightly vary within the time of measurement



Thus, over the period of measurement no fixed values exist for all these quantities which enter the RA condition

Strictly speaking, the ***RA condition is 'never' fulfilled***.
It is only a mathematical relation between some quantities,
with the answer YES or NO for a given set of input, and ***NO*** is the usual answer:

$$\gamma_0 = \frac{\Omega \pm \cos\varphi \cos\theta \sqrt{\Omega^2 - (1-\cos\varphi^2 \cos\theta^2)}}{1-\cos\varphi^2\cos\theta^2}$$

→ ***We need some convolution of the resonance absorption condition with the possible (Gaussian ?) spreads of all involved quantities, E_b, θ, φ, ω and B*** **!**

The ***same argument*** holds for the ***estimation of n_γ***, the number of absorbed laser photons

→ ***realistic n_γ^{real} is obtained after convolution with exp. spread expected for E_b, θ, φ, ω and B***

→ $n_\gamma^{real} < n_\gamma$ but
- **how much smaller ?**
- **is it below the threshold of a possible laser ray detector ?**

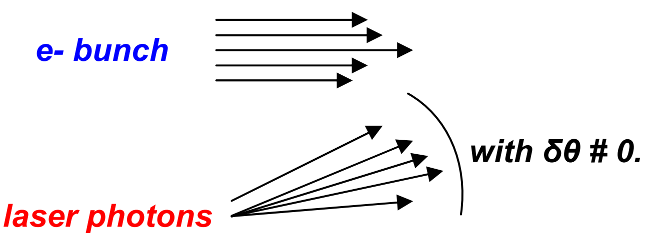
Needs to be evaluated

Divergence of the laser and electron beams

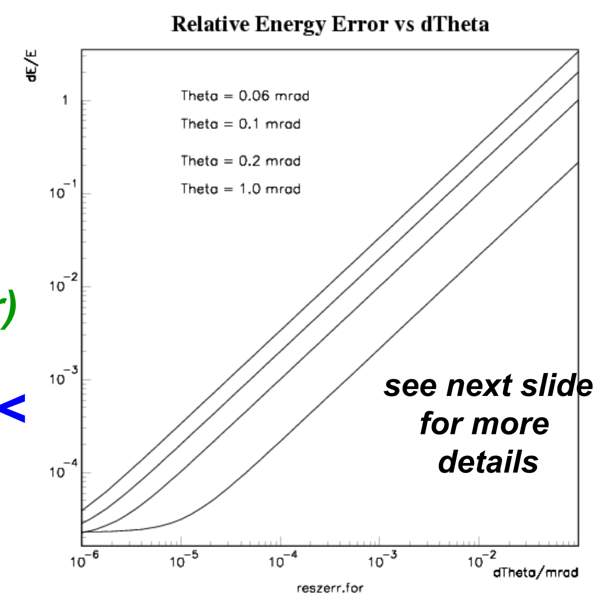
Assume - a **perfect aligned electron beam** with no divergence;
→ φ_i for each electron are all identical

- the **laser has some divergence** governed by optical laws;
→ θ_i for each photon vary within a certain range

The **RA condition** $$\gamma_0 = \frac{\Omega \pm \cos\varphi \cos\theta \sqrt{\Omega^2 - (1 - \cos\varphi^2 \cos\theta^2)}}{1 - \cos\varphi^2 \cos\theta^2}$$



very, very sensitive to the angles θ (and φ); small variations of one of them → **tremendous effect on E_b!**



see next slide for more details

Very important question:

Is it possible to have a laser with δθ ~10^{-9} rad, i.e. a laser with practical no divergence ?

(needed to achieve a beam energy precision of 10^{-4} or better)

>> IS THAT THE END OF THE RA STUDIES ? <<

→ ***Laser experts are needed !***

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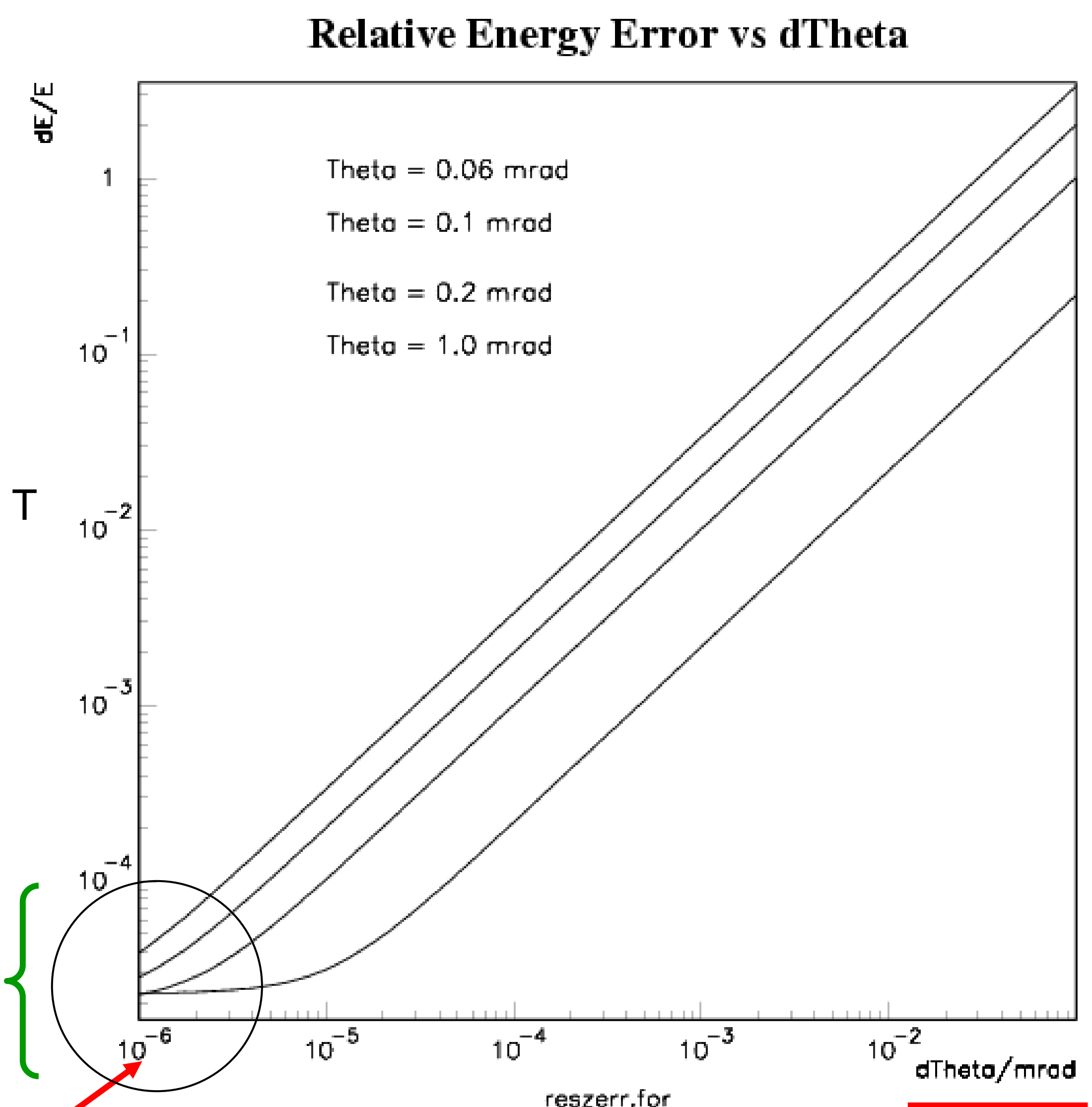
from Karlheinz



For $\Delta B/B = 2\ 10^{-5}$,
and B is in the range 1 … 5 T
to fulfill the RA condition
for a CO_2 laser

(an analogous plot exists for dφ)

Region of interest

10^{-9} rad !

First (personal) summary

- **Some problems related to the Resonance Absorption method were indicated; many questions were not at all discussed (e.g. stability issues, laser light detector, the 'best' position within the BDS, data taking strategies, …)**
- **Some of them might be solved in a reasonable manner, but require both theoretical and experimental effort**
- **The divergence of the laser beam (as well as that of the electron beam) is the most serious problem in order to achieve a beam energy precision of 10^{-4} or better**
- **To solve this basic problem laser experts have to be consulted immediately**
- **Any volunteers ?**