New aspects of the method "Electron Beam Energy Measurement Using Resonant Absorption of Laser Light by Electrons in a Magnetic Field”

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1. CONDITION OF RESONANT ABSORPTION

Electrons and photons are injected in a magnetic field B under small angles φ and θ to the z-axis, accordingly (Fig.1).

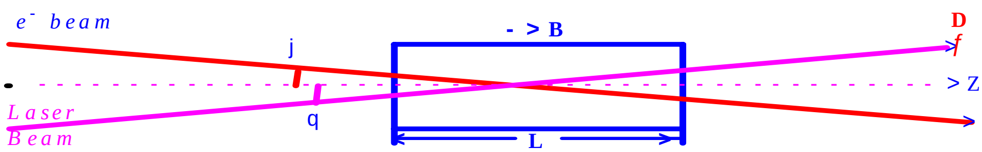

Fig.1

In a magnetic field the electrons have a discrete spectrum of energy [2]:

$$\varepsilon_{n,\zeta} = [m^2 + P_z^2 + eB(2n+1+\zeta)]^{1/2} \qquad (1)$$

Electrons in a magnetic field can be on equidistant energy levels (Fig.2).

Photons of frequency ω can be resonantly absorbed at transitions of electrons beween levels of energy.

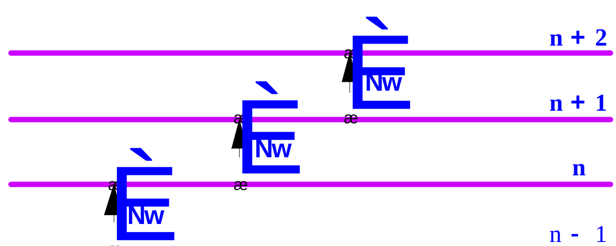

Fig. 2

Using the law of energy-momentum conservation for photon absorption:

$$\varepsilon_{n,\zeta} + \omega = \varepsilon_{n',\zeta}, \qquad P_{z,0} + \omega\cos\theta = P_z \qquad (2)$$

we find the condition for resonant absorption:

$$\omega\left[\gamma_0(1-V_{z,0}\cos\theta)+\frac{\omega}{m}\frac{(\sin\theta)^2}{2}\right]=\omega_c(n'-n), \qquad (3) \qquad n' - n = 1,$$

From (3) in approximation of $\hbar\omega/\varepsilon << 1$ the resonant condition of absorption can be written as:

$$\gamma_0-\cos\varphi\cos\theta\sqrt{\gamma_0^2-1}=\frac{\omega_c}{\omega} \qquad (4)$$

For electrons with angles $\varphi=\theta=0$ from (4):

$$\gamma_0=\frac{1}{2}\left(\frac{\omega}{\omega_c}+\frac{\omega_c}{\omega}\right) \qquad (5)$$

Dependence of electron energy from B - on Fig.3, Fig4

CO_2

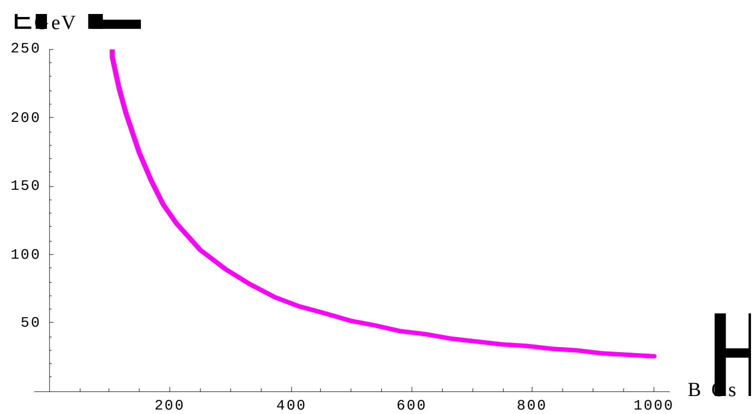

Fig.3

Nd:YAG

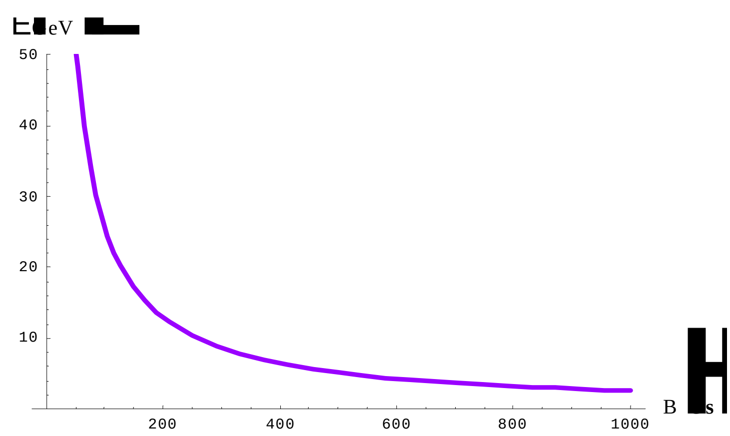

Fig.4

In a case of $\varphi\neq 0$ and $\theta\neq 0$:

$$\gamma_0=\frac{\Omega\pm\cos\varphi\cos\theta\sqrt{\Omega^2-1+(\cos\varphi\cos\theta)^2}}{1-(\cos\varphi\cos\theta)^2} \qquad (6)$$

$\Omega=\omega_c/\omega$

Dependence of the electron energy from B for some angles φ and θ are shown on Fig.5, Fig.6.

CO_2

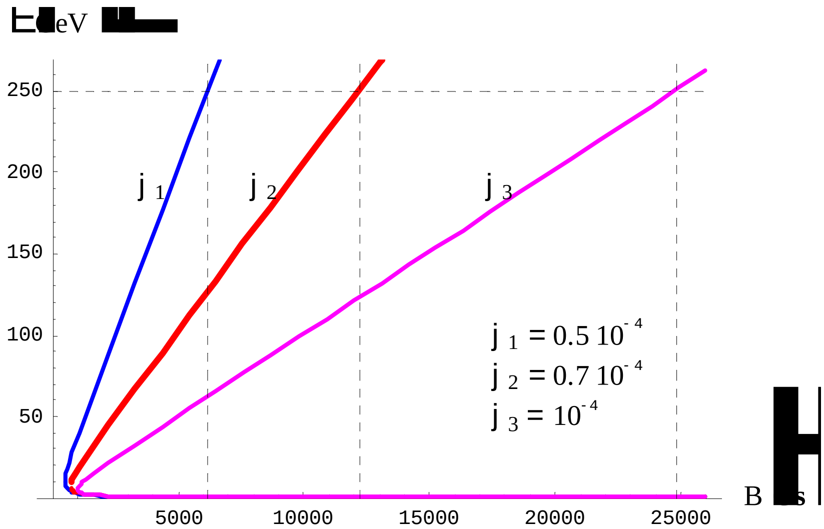

Fig.5

Nd:YAG

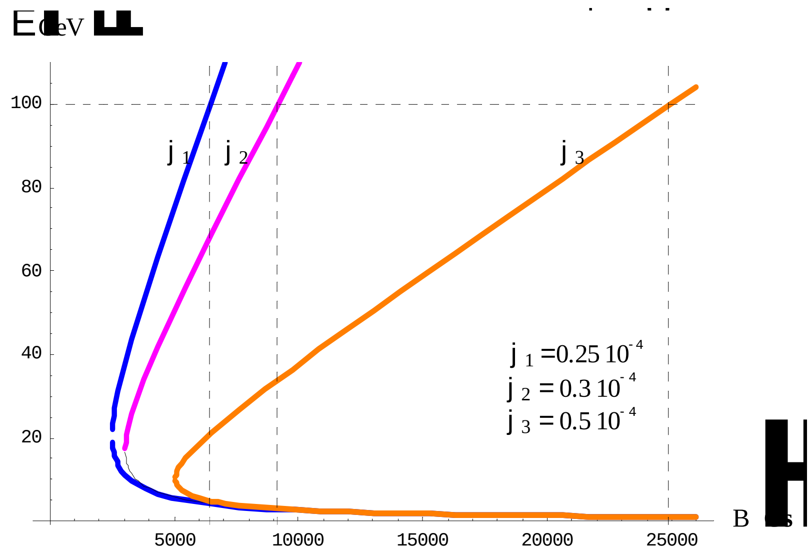

Fig.6

If in (6): $1-(Cos\varphi Cos\theta)^2 << \Omega^2$, then for lower branch on Fig.5, Fig.6 - approximately:

$$\gamma \approx \frac{\Omega}{2} + \frac{1}{2\Omega}\left[1 + \frac{1-(Cos\varphi Cos\theta)^2}{4\Omega^2} + \frac{(1-Cos\varphi^2 Cos\theta^2)^2}{8\Omega^4} + \cdots\right] \quad (8)$$

If in (8) :

$$\frac{1-(Cos\varphi Cos\theta)^2}{4\Omega^2} < 10^{-4}, \quad (9)$$

c
a

then γ can be determined with accuracy 10^{-4} without of dependence from φ and θ

$$\gamma \approx \frac{1}{2\Omega} + \frac{\Omega}{2} \quad (10)$$

This method is suitable for measurement of energy up to few hundreds MeV if to use the long-wavelength far infrared laser.

2. INFLUENCE OF SPREAD IN ANGLES OF ELECTRONS AND LIGHT DIFFRACTION ON DETERMNATION OF ELECTRON BEAM ENERGY

Light beam of diameter D diverges in limits of angle: $\theta_d \cong \lambda / D$

λ - length of wave

Distribution of light intensity (Fig.7):

$$I(\theta) = I_0 \left[\frac{2J_1(\alpha)}{\alpha} \right]^2 \qquad (12)$$

$\alpha = Dk\theta/2$

$J_1(\alpha)$ - is the cylindrical function of the first order

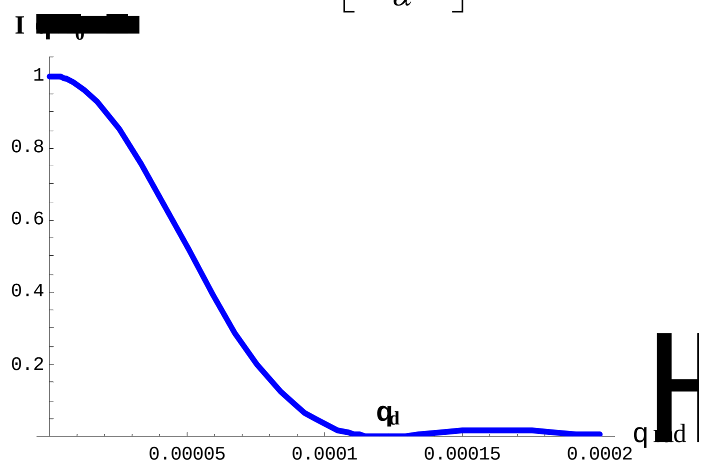

Fig.7

From (6) in a case of $\gamma >> 1$ and $\Omega^2 >> 1 - (\cos\varphi \cos\theta)^2$ for upper branch on Fig.5 and Fig.6 electron energy is approximately:

$$\gamma \approx \frac{2\Omega}{\varphi^2 + \theta^2} \qquad (13)$$

$$\varphi^2 + \theta^2 \approx \frac{2\Omega}{\gamma} = const \qquad (14)$$

(14) is eq. of a circle with radius $\sqrt{2\Omega/\gamma}$ (Fig.8).

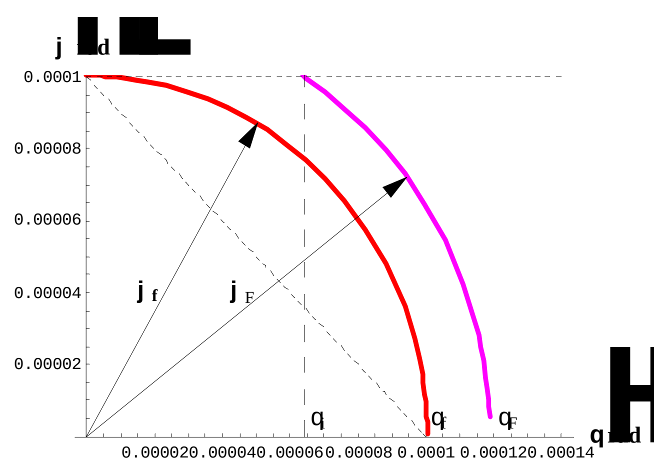

Fig.8

We have assumed that function of distribution of electrons on angles φ is Gaussian. Then, dependence of number of electrons N_e from angles φ has the shape shown on Fig.10.

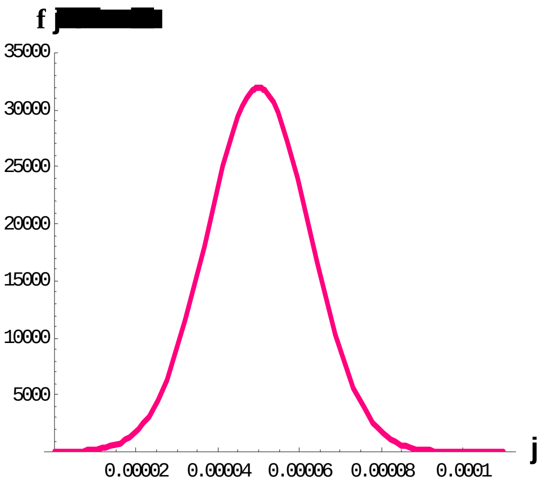

Fig.9

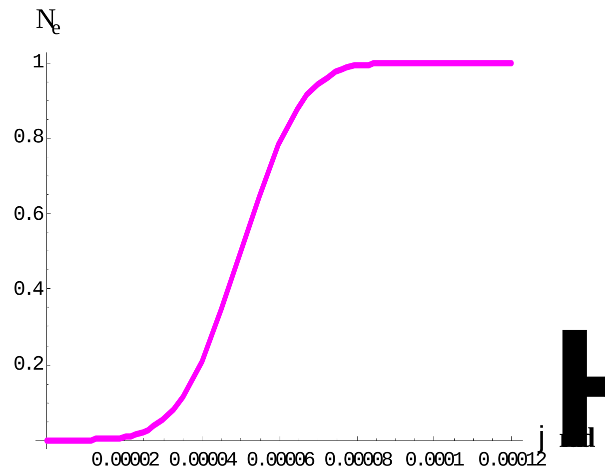

Fig.10

To consider behavior of I_{abs} we notice: $I_{abs} \sim n_e \cdot n_\gamma$ (15)

From Fig.8, Fig.10 and (15) follows: $I_{abs} \sim \left(\varphi^2 + \theta^2\right)^{1/2}$

I_{abs} is max at $\left(\varphi^2 + \theta^2\right)^{1/2} = \varphi_f$

If $\left(\varphi^2 + \theta^2\right)^{1/2} > \varphi_f$ then I_{abs} decreases, because interval of angles $\Delta\theta = \theta_F - \theta_i$ decrease. So, I_{abs} as a function from $\left(\varphi^2 + \theta^2\right)^{1/2}$ has a resonant form (Fig. 11)

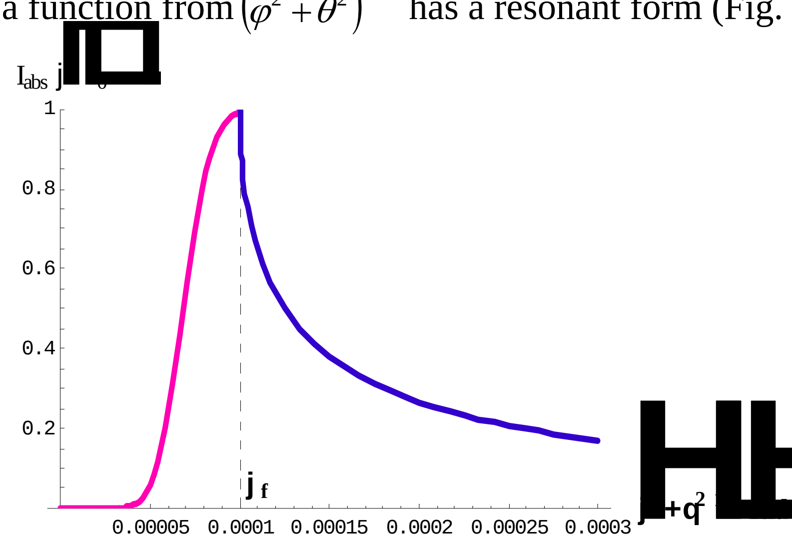

Fig.11

- Position of the point φ_f on Fig.11 is determined only by distribution of electron beam. Point φ_f is singled out and can be used for determination of energy of electrons.

Below will be shown that by change of light polarization can be carried out the “collimation” of electron beam. Here “collimation” means that on the colored area of Fig.9a electrons can not absorb photons. “Collimation” of electron beam allow us to get the sharp left edge of intensity of photon absorption (Fig.12).

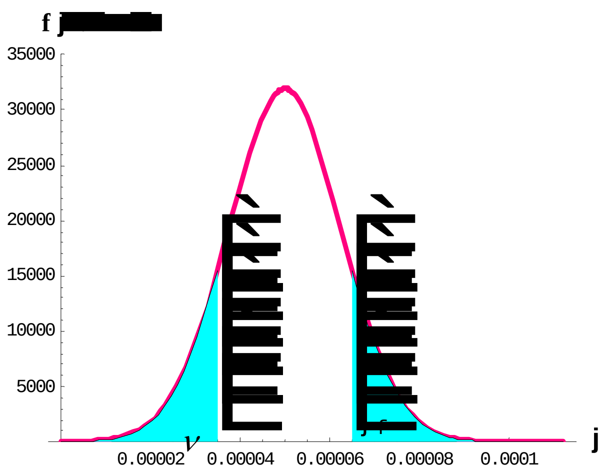

Fig.9a

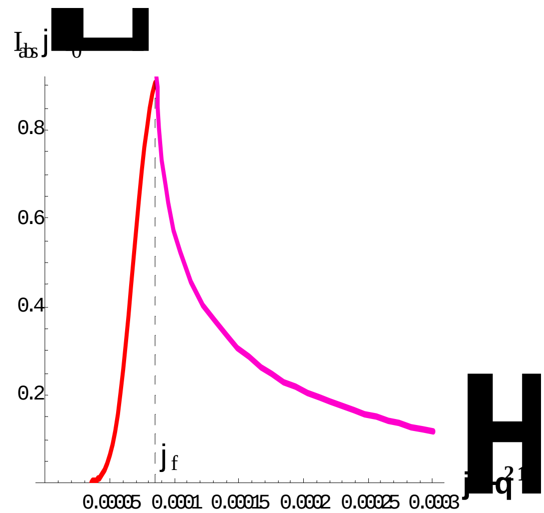

Fig.12

Below it will be shown, that for electrons with given γ and Ω resonant absorption of photons is possible only for laser of the certain minimal intensity.

3. ESTIMATION OF INTENSITY OF A RESONANT ABSORPTION

It is known in field of circularly polarized wave at the presence of magnetic field electrons can be accelerated owing to absorption of photons.
The number ν of the absorbed laser photons on length of interaction l_i we can estimate using the classical formula for energy growth of electron:

$$\nu \cdot \hbar\omega = \varepsilon - \varepsilon_0 \approx mc^2 \xi\omega \sqrt{\frac{2\Omega}{\gamma_0}} \cdot t_i \tag{21}$$

Because electron can absorb only an integer ν photons and using (21) we have the condition:

$$\nu = \xi \frac{l_i}{\lambdabar_c}\sqrt{\frac{2\Omega}{\gamma_0}} = \xi \frac{l_i}{\lambdabar_c}\left(\varphi^2 + \theta^2\right)^{1/2} \geq 1,2,... \qquad (22)$$

where $\lambdabar_c = \hbar/mc \approx 3.861 \cdot 10^{-11}\, cm$

$$I\left[w/cm^2\right] \cong \left(\frac{E\left[V/cm\right]}{19,4}\right)^2 \qquad (23)$$

Using (22), (23) we find the minimal intensity of the laser necessary for absorption of photons:

$$I_{\min} \geq \left(\frac{\lambdabar_c}{L}\frac{mc^2}{\lambda}\frac{2\pi}{19,4}\right)^2 \frac{\gamma}{2\Omega} = \left(\frac{\lambdabar_c}{L}\frac{mc^2}{\lambda}\frac{2\pi}{19,4}\right)^2 \frac{1}{\varphi^2 + \theta^2} \qquad (24)$$

For example, in case of parameters: $\left(\varphi^2 + \theta^2\right)^{1/2} = \sqrt{\frac{2\Omega_f}{\gamma}} = \varphi_f = 10^{-4}\, rad \quad \lambda = 10,6\mu k$

$L = 100 cm \qquad I_{\min} \approx 0.36 w/cm^2$

Using expression (12) and condition (24) we can find the area of angles φ and θ where absorption of photons is possible:

$$\left(\varphi^2 + \theta^2\right)^{1/2}_{\min} \geq \left(\frac{\lambdabar_c}{L}\frac{mc^2}{\lambda}\frac{2\pi}{19,4}\right)\frac{\alpha}{\sqrt{I_0[2J_1(\alpha)]}} \qquad \alpha = \pi d\theta/\lambda \qquad (25)$$

If $I(\theta_f) = I_{\min}$ then the relation (25) can be rewritten as:

$$\left(\varphi^2 + \theta^2\right)^{1/2}_{\min} \geq \theta \cdot \frac{J_1(\alpha_f)}{J_1(\alpha)} \qquad (28)$$

From (28) follows:

1. In region $\left(\varphi^2 + \theta^2\right)^{1/2} > \left(\varphi^2 + \theta^2\right)^{1/2}_{\min}$ all electrons can absorb laser photons (Fig.14).

2. In region $\left(\varphi^2 + \theta^2\right)^{1/2} < \left(\varphi^2 + \theta^2\right)^{1/2}_{\min}$ absorption of photons is impossible (colored area of angles on Fig.14 is fobidden zone of absorption of photons

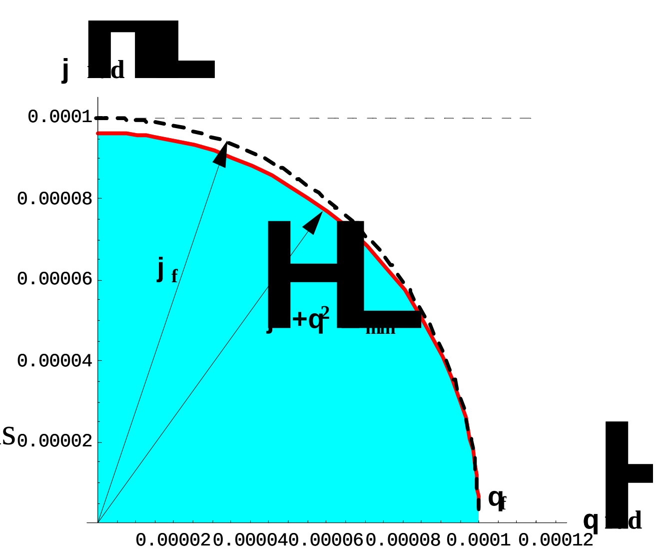

Fig.14

Owing to small difference between curves $\varphi = \left(\varphi_f^2 - \theta^2\right)^{1/2}$ (see Fig.14) and $\varphi = \theta \cdot \left(\left[\frac{J_1(\pi d\theta_f / \lambda)}{J_1(\pi d\theta / \lambda)}\right]^2 - 1\right)^{1/2}$ (found from the formula (28)) curve of absorption I_{abs} has very sharp left edge (Fig.16), what is important for accuracy of determination of energy of electrons.

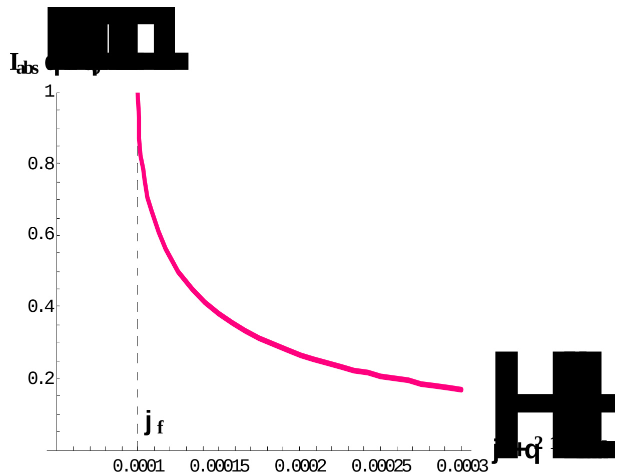

Fig.16

For parameters $\left(\varphi^2 + \theta^2\right)^{1/2} = \varphi_f = 10^{-4}\, rad$ $L = 100cm$ $d = 0.2cm$ $\lambda = 1{,}06\,\mu k$

according to (24) the required minimal intensity of laser is: $I_{\min} \approx 36{,}34 w / cm^2$

and dependence of intensity of absorption I_{abs} from $\left(\varphi^2 + \theta^2\right)^{1/2}$ has the form,. shown on Fig.17.

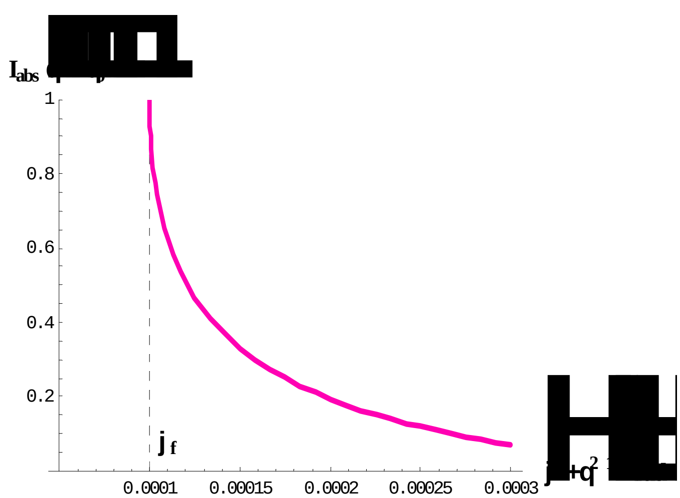

Fig.17

In case of linear polarization of wave: Number ν of the absorbed photons on interaction length L we can estimate using formula for growth of energy of electrons:

$$\nu \approx \frac{\xi L \psi}{\lambdabar_c} \geq 1 \qquad (29)$$

$\psi = \varphi + \theta$ - is angle between electron and laser beam. Condition (29) defines area of angles φ and θ where absorption of photons is possible (Fig.8):

$$\varphi \geq -\theta + \left(\frac{\lambdabar_c}{L} \frac{mc^2}{\lambda} \frac{2\pi}{19{,}4} \right) \frac{1}{\sqrt{I_{\min}}} \qquad (30)$$

Elliptic polarization of wave: area of angles φ and θ where absorption of photons is possible, obviously, will be outlined by an ellipse (Fig.18, Fig.19).

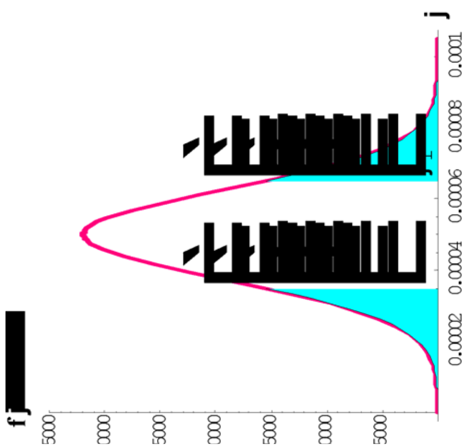

Fig.9a

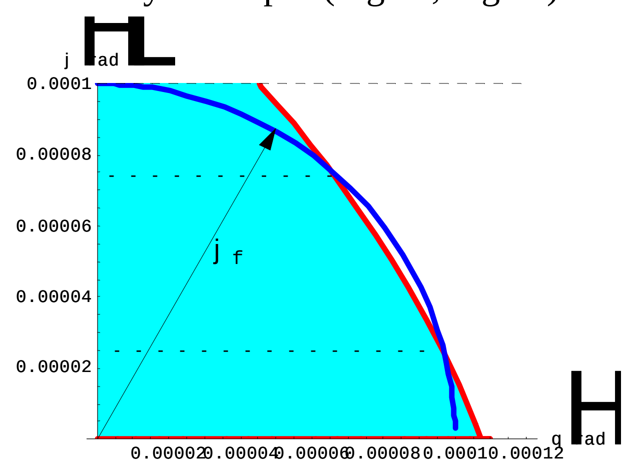

Fig.18

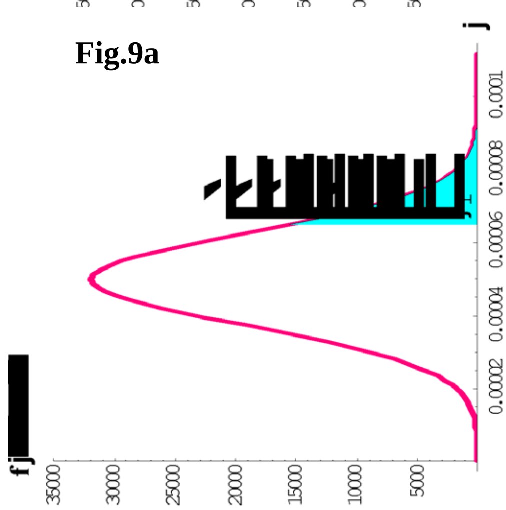

Fig.9b

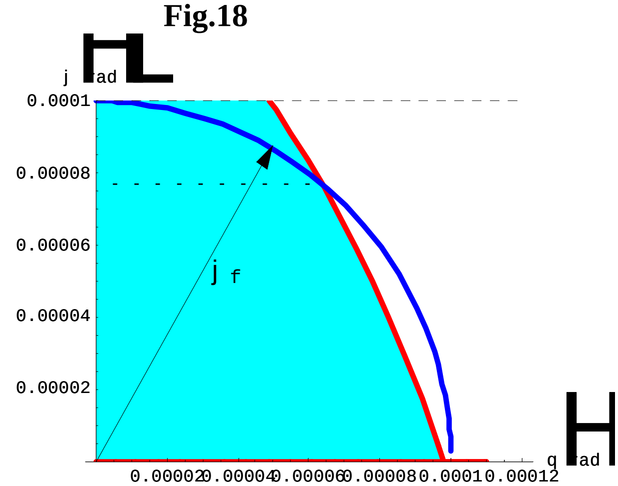

Fig.19

From Fig.18 and Fig.19 it is clear that by changing of wave polarization we can change parameters of the forbidden zone or in other words to **“collimate”** electron beam, which allow us to have sharp left edge of I_{abs}.

4. DETERMINATION OF ENERGY OF THE ELECTRON BEAM

For determination of electron energy we can use:1) the fact that I_{abs} is maximal for angle φ_f and 2) the threshold character of absorption intensity.
We consider three possible variants of measurement of electron beam energy.

4.1. Determination of the relative energy of electron beam.

At first, for a known energy ε_1 of electron beam and for the given parameters ω, L we measure the quantity of $B_{f,1}$ (Fig.20), which corresponds to the max of I_{abs}. Then we can find angle:

$$\varphi_f = \sqrt{\frac{2\Omega_{f,1}}{\gamma_1}} = \sqrt{\frac{2eB_{f,1}}{mc\omega}\frac{mc^2}{\varepsilon_1}} \qquad (31)$$

We assume: electron beam is “collimated” and distribution of electrons on angles φ for energy ε_1 and for unknown energy ε_2 are identical, and maximum of I_{abs} falls on the same angle φ_f. Then having measured $B_{f,2}$ which corresponds to the maximum of intensity I_{abs} we find angle:

$$\varphi_f = \sqrt{\frac{2\Omega_{f,2}}{\gamma_2}} = \sqrt{\frac{2eB_{f,2}}{mc\omega}\frac{mc^2}{\varepsilon_2}} \qquad (32)$$

From (31) and (32) we find the unknown energy ε_2 :

$$\varepsilon_2 = \varepsilon_1 \frac{B_{f,2}}{B_{f,1}} \qquad (33)$$

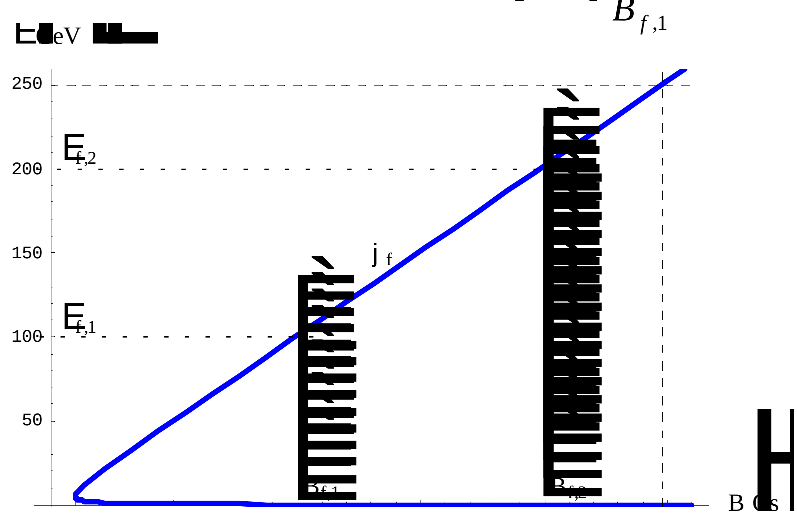

Fig.20

4.2. Determination of absolute energy of electrons using two magnets.

Electron beam passes through two magnets B_1 and B_2 of identical length L (Fig.21) B_1 is directed along axis Z. Axis Z' of magnetic B_2 is inclined to axis Z on some angle $\delta\varphi = \varphi_{f,2} - \varphi_{f,1}$. First, on maximum of I_{abs} in magnet B_1 (while magnet B_2 is switched off) we find $\Omega_{f,1}$ for the given γ and $\varphi_{f,1}$. Then we can find angle:

$$\varphi_{f,1} = \sqrt{\frac{2\Omega_{f,1}}{\gamma}} \qquad (34)$$

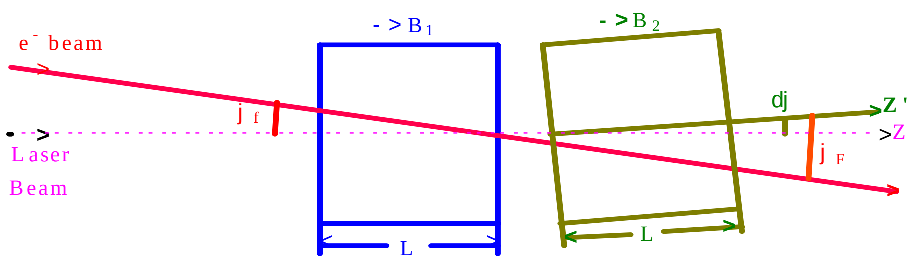

Fig.21

Similarly, on max of I_{abs} in magnet B_2(while the magnet B_1 is switched off) we find $\Omega_{f,2}$ for the given γ and $\varphi_{f,2}$. Then we find angle:

$$\varphi_{f,2} = \sqrt{\frac{2\Omega_{f,2}}{\gamma}} \qquad (35)$$

From (34) and (35):

$$\delta\varphi = \sqrt{\frac{2}{\gamma}}\left(\sqrt{\Omega_{f,2}} - \sqrt{\Omega_{f,1}}\right) \qquad (36)$$

$\delta\varphi$ depends only from disposition of magnets and is constant for all energies of electrons. Then we can find $\delta\varphi$ if to measure $\Omega_{f,1,0}$ and $\Omega_{f,2,0}$ for known energy γ_0 So, from (36) we can find the **absolute energy** of electrons:

$$\varepsilon = 2mc^2 \cdot \left(\frac{\sqrt{\Omega_{f,2}} - \sqrt{\Omega_{f,1}}}{\delta\varphi}\right)^2 = 4\mu_B \frac{mc^2}{\hbar\omega} \cdot \left(\frac{\sqrt{B_{f,2}} - \sqrt{B_{f,1}}}{\delta\varphi}\right)^2 \qquad (37)$$

where $\mu_B = \frac{e\hbar}{2mc} \approx 9.274 \cdot 10^{-21}$ erg/Gs – is Bohr magneton.

- **4.3. Determination of absolute energy of electrons by change of "collimation" of electron beam**

We asume by change of "collimation" of electron beam we can change distribution of electrons on some angle $\delta\varphi$ known for us. Having measured resonant frequencies $\Omega_{f,1}$ and $\Omega_{f,2}$ which correspond to maximum of resonance absorptions intensity for angles $\varphi_{f,1}$ and $\varphi_{f,2}$ we have:

$$\delta\varphi = \varphi_{f,2} - \varphi_{f,1} = \sqrt{\frac{2}{\gamma}}\left(\sqrt{\Omega_{f,2}} - \sqrt{\Omega_{f,1}}\right) \qquad (38)$$

From here we find the absolute energy of electrons:

$$\varepsilon = 2mc^2 \cdot \left(\frac{\sqrt{\Omega_{f,2}} - \sqrt{\Omega_{f,1}}}{\delta\varphi}\right)^2 = 4\mu_B \frac{mc^2}{\hbar\omega} \cdot \left(\frac{\sqrt{B_{f,2}} - \sqrt{B_{f,1}}}{\delta\varphi}\right)^2 \qquad (39)$$

5. Determination of the ratio of absorption intensity to the laser intensity

Absorption of photons can be observed by measuring the ratio of the number of absorbed photons $N_{a,ph}$ to total number $N_{l,ph}$ of laser photons, incident on area of interaction (or I_{abs}/I_{las}). The number of photons $N_{a.ph}$ absorbed by electron beam of length l_b can be estimate taking into account that each electron passes through region of interaction L only once. Therefore if the number of electrons in beam is N_e, then $N_{a,ph} = N_e$. Intensity of the laser I_0 and the number of photons incident on surface S during of time $\tau_b = l_b/c$:

$$N_{l,ph} = \frac{I_0}{\hbar\omega} S\tau_b \qquad (40)$$

Estimation of the ratio $N_{a,ph}/N_{l,ph}$ for design of the TESLA in a case of $\varepsilon = 100GeV$, $L = 100cm$, $B = 10^4 Gs$, $d = 0.2cm$, $\lambda = 10.6\mu k$. In TESLA-500 the electron bunches (2820) are stored in a train (frequency - 5 Hz) [7] and pulse train length is $\tau_b = 0.95m\sec$. Each bunch contains $2\cdot 10^{10} e$. So, $N_{a,ph}$ by one electron train (pulse) will be: $N_{a,ph} = 2820\cdot 2\cdot 10^{10} = 5.64\cdot 10^{13}$ photon/pulse. For synchronization of laser pulse τ_{las} with pulse of electron beam we take $\tau_{las} \approx 3\cdot\tau_b$. If $I_0 = 0.36 w/cm^2$ then using (40) for TESLA we have: $N_{a,ph}/N_{l,ph} \approx 6\cdot 10^{-2}$.

It means that $N_{a.ph}$ is enough for observation of photon absorption process.