THE POSSIBILITY OF PRECISE MEASUREMENT OF ABSOLUTE ENERGY OF THE ELECTRON BEAM BY MEANS OF "RESONANCE ABSORPTION" METHOD

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Abstract

In this report we consider a new conception of the "Resonant Absorption" method for measurement of the absolute energy of the electron beam for a wide (up to few hundred GeV) range of energies, with accuracy 10^{-4} . Detectors and lasers, necessary for the measurement of the energy of electrons by this method, are not unique and can be chosen from existing industrial samples. The parameters of the electron beam will not worsen during the measurements of the energy and this feature of the method allows carrying out the continuous monitoring of the electron beam energy.

1. Determination of electrons energy on the basis of condition of resonant absorption

As before[1], we suppose that electrons and photons are injected in a magnetic field $\vec{B}$, under small angles φ and θ to the z-axis, as shown on Fig.1.

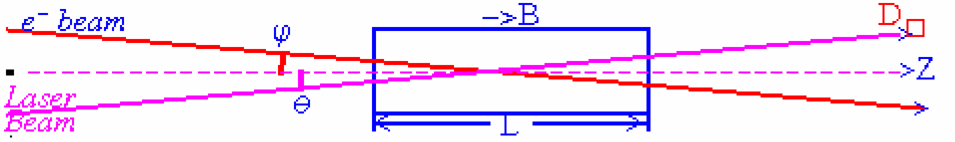

Fig.1

The energy of electrons can be determined using the condition of the resonant absorption of laser photons by electrons at transitions between quantum levels of energy in a magnetic field. For absorption of photons of optical or lower frequency ω **in approximation of** $\hbar\omega/\varepsilon << 1$ **the resonant condition can be written as:**

$$\gamma - \cos\varphi\cos\theta\sqrt{\gamma^2 - 1} = \Omega \,. \qquad \textbf{(1)}$$

where $\gamma = \varepsilon/mc^2$ **is the relativistic factor of electron,** $\Omega = \omega_c/\omega$ **,** $\omega_c = eB/mc$.

Equation (1) in relation to γ **has the solution:**

$$\gamma = \frac{\Omega \pm \cos\varphi\cos\theta\sqrt{\Omega^2 - 1 + (\cos\varphi\cos\theta)^2}}{1 - (\cos\varphi\cos\theta)^2} \qquad \textbf{(2)}$$

For simplicity, we first consider the dependence of γ **on** φ **in the case of** $\theta = 0$ **for fixed values of** Ω **(Fig.2). Then from (2) we have**

$$\gamma = \frac{\Omega \pm \cos\varphi\sqrt{\Omega^2 - \sin\varphi^2}}{\sin\varphi^2} \qquad \textbf{(3)}$$

The dependence ε **on** φ **for various values of** Ω **at** $(\varepsilon)_{\varphi=0} = 100$ **GeV is shown on Fig.2.**

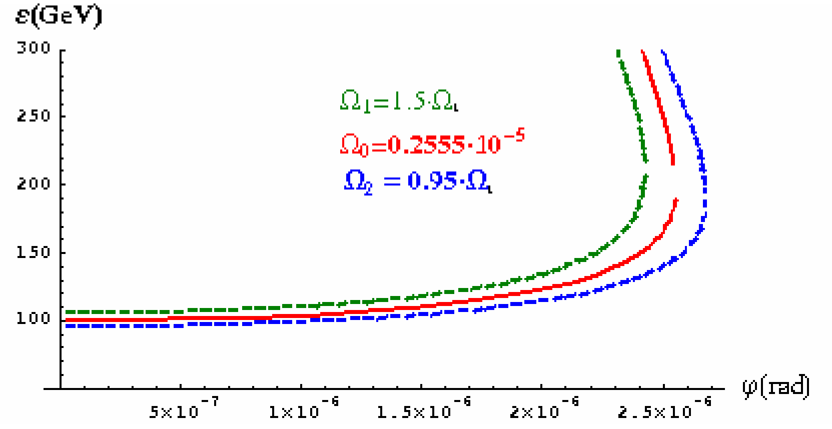

Fig.2

From (3) it follows, that:

1. γ **has a physical meaning if the following condition is satisfied :**

$$\sin\varphi \le \Omega \,. \qquad (4)$$

2. At $\varphi = 0$ **the** γ **factor of electron has the minimal value:**

$$\gamma_{\min} = \frac{1}{2\Omega} + \frac{\Omega}{2} \qquad (5)$$

3. At $\varphi = 0$ **the derivative of** γ **with respect to** φ **is equal to zero:**

$$\left(\frac{d\gamma}{d\varphi}\right)_{\varphi=0} = 0 \,. \qquad (6)$$

From Fig.2 and (6) it is clear that the value of ε **is less sensitive to changes of angle** φ **near to** $\varphi = 0$ **. This behavior of dependence** $\varepsilon = \varepsilon(\varphi)$ **we shall use for exact measuring of absolute energy of electron.**

From (3) it follows, that the lower branch of curve $\gamma(\varphi)$ **at** $\varphi << \Omega$ **can approximately be described as:**

$$\gamma \approx \frac{1}{2\Omega} + \frac{\Omega}{2} + \left(\frac{1}{8\Omega^3} + \frac{\Omega}{8} - \frac{1}{4\Omega}\right)\varphi^2 + 0[\varphi]^4 \,. \qquad (7)$$

2. Precise measurement of electron beam absolute energy, in case of electrons spread over angles φ in the interval $0 \le \varphi \le \varphi_{\max}$.

We assume that the electron beam has spread over energies $\varepsilon \pm \Delta\varepsilon$ **and over angles** φ **in the interval** $0 \le \varphi \le \varphi_{\max}$ **. Then, according to (7), the dependence** $\varepsilon(\varphi)$ **has the shape shown on Fig.3.**

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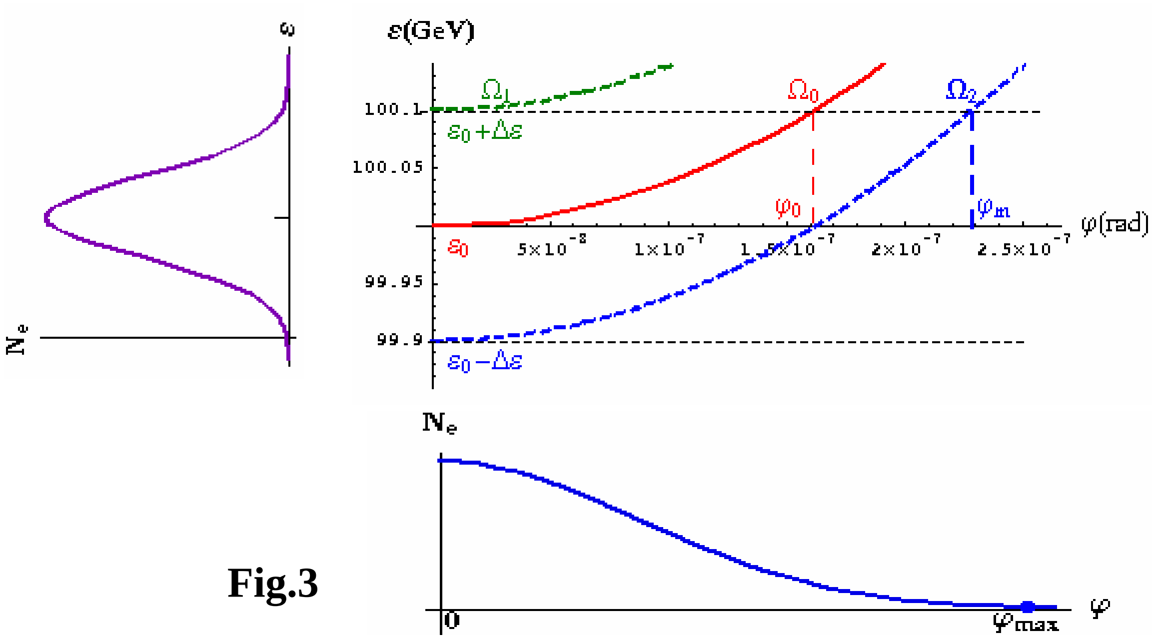


Fig.3

From the Fig.3 it is clear, if $\Omega \le \Omega_1$ **then electrons with energies** $\varepsilon \ge \varepsilon_0 + \Delta\varepsilon$ **cannot give the contribution to absorption intensity of photons (** $I_{abs} = 0$ **). It is obvious that with growth of** Ω **the absorption intensity gradually increases. When** $\Omega = \Omega_0$ **, then into intensity of absorption** I_{abs} **the contribution will gives electrons with energies in the interval** $\varepsilon_0 \le \varepsilon \le \varepsilon_0 + \Delta\varepsilon$ **and in the interval of angles** $0 \le \varphi \le \varphi_0$ **. Finally, if** $\Omega = \Omega_2$ **then into intensity of absorption will gives the contribution all electrons with energies in the interval** $\varepsilon_0 - \Delta\varepsilon \le \varepsilon \le \varepsilon_0 + \Delta\varepsilon$ **and in the interval of angles** $0 \le \varphi \le \varphi_m$ **. Then the absorption intensity will be maximal.**

Dependence of absorption intensity I_{abs} on Ω has the shape, shown on Fig.4.

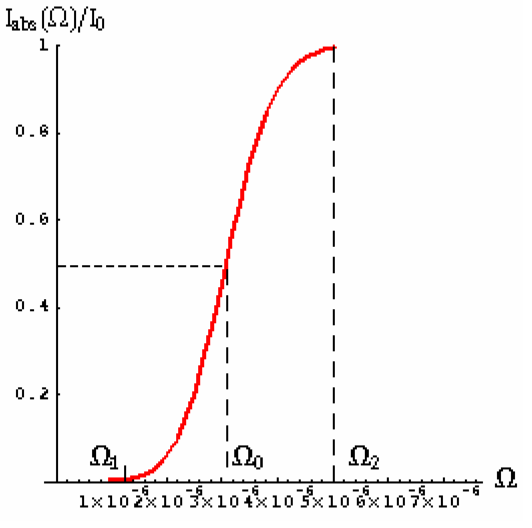


Fig.4

Using the relations (7) and (5) we can estimate quantity φ_0 and φ_m (Fig.3) approximately as:

$$\varphi_0 \approx 2\Omega_2 \cdot \sqrt{\frac{\Delta\gamma}{\gamma}} \approx \frac{1}{\gamma} \cdot \sqrt{\frac{\Delta\gamma}{\gamma}}, \qquad \varphi_m \approx \sqrt{2} \cdot \varphi_0. \tag{8}$$

If the spread over energies of electrons is equal to $\Delta\varepsilon/\varepsilon = 10^{-3}$ then from (8) follows that:

$$\varphi_m \approx \sqrt{2} \cdot 2\sqrt{10^{-3}} \cdot \Omega_2 << \Omega_2. \tag{9}$$

From the Fig.3 it is clear, that if the electron beam has angular distribution in the interval $0 \geq \varphi > \varphi_m$ then in case of Ω=Ω_2 into absorption intensity I_{abs} will gives contribution electrons with the angles $\varphi < \varphi_m$. In other words, the pseudo-collimation of the electron beam occurs.

According to (5) and Fig.3 we have:

$$(\gamma)_{\varphi=0} = \gamma_{\min} = \frac{1}{2\Omega}\cdot\left(1+\Omega^2\right) \qquad \textbf{(10)}$$

From (10) follows if $\Omega_0^2 < 10^{-4}$ **then we can determine** $\gamma_{\min}$ **with accuracy** $\delta\gamma/\gamma < 10^{-4}$ **according to the formula:**

$$\gamma_{\min} = \frac{1}{2\Omega}. \qquad \textbf{(11)}$$

At the same time the condition $\Omega_0^2 < 10^{-4}$ **imposes a restriction on** γ **. Really, from (11) we have** $\Omega = \frac{1}{2\gamma} \le 10^{-2}$ **, whence follows:** $\gamma \ge 50$ **or** $\varepsilon \ge 25.55 \quad MeV$ **.**

Now let's find the width of $\Omega_2 - \Omega_1$ **(Fig. 4). Taking into account that according to (11)**

$$\gamma_0 + \Delta\gamma = \frac{1}{2\Omega_1}, \quad \gamma_0 - \Delta\gamma = \frac{1}{2\Omega_2} \quad \textbf{and} \quad \gamma_0 = \frac{1}{2\Omega_0}, \qquad \textbf{(12)}$$

we receive approximately:

$$\Omega_2 - \Omega_1 \approx 2\Omega_0 \cdot \frac{\Delta\gamma}{\gamma} = \frac{1}{\gamma_0}\cdot\frac{\Delta\gamma}{\gamma}. \qquad \textbf{(13)}$$

For example if $\gamma_0 = 2\cdot 10^5$ **and** $\Delta\gamma/\gamma = 10^{-3}$ **then** $\Omega_2 - \Omega_1 = 0.5\cdot 10^{-8}$ **.**

From (12) follows the relation:

$$\frac{\delta\gamma}{\gamma} = \frac{\delta\Omega}{\Omega} \qquad \textbf{(14)}$$

According to (14) to find energy γ_0 **with accuracy** $\delta\gamma/\gamma_0 < 10^{-4}$ **it is necessary to find** Ω_0 **with accuracy** $\delta\Omega/\Omega_0 < 10^{-4}$ **.**

The quantity of Ω_0 with accuracy $\delta\Omega/\Omega_0 < 10^{-4}$ can be found using behavior of dependence I_{abs} from Ω (Fig.4).

It is obvious from the (Fig.3), that the greatest contribution to the intensity of absorption will be given by the electrons of distribution centre, with energy γ_0 or when $\Omega = \Omega_0$. From Fig.4 it is clear, that for points Ω_1 and Ω_2 the partial derivative $\frac{\partial I_{abs}}{\partial \Omega} = 0$, while for point Ω_0 the private derivative $\frac{\partial I_{abs}}{\partial \Omega}$ is maximal. Using these properties of function $I_{abs}(\Omega)$ we can find the behavior of dependence $\frac{\partial I_{abs}}{\partial \Omega}$ on Ω (Fig.5).

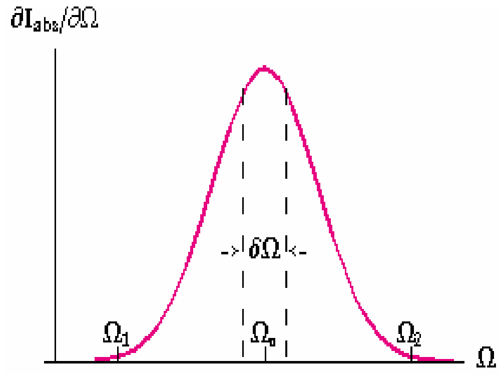


Fig.5

Using (13) and (14) we can find the ratio of $\delta\Omega$ to $\Omega_2 - \Omega_1$, and its value in a case when $\Delta\gamma/\gamma < 10^{-3}$, $\delta\gamma/\gamma_0 < 10^{-4}$ then:

$$\frac{\delta\Omega}{\Omega_2 - \Omega_1} = \left(\frac{\delta\gamma}{\gamma}\right) \cdot \frac{1}{2\left(\frac{\Delta\gamma}{\gamma}\right)} = \frac{1}{20} \qquad \textbf{(15)}$$

Let's notice that real possibilities of measurement of B and ω allow us to find Ω with accuracy $\delta\Omega/\Omega < 10^{-4}$ because

$$\frac{\delta\Omega}{\Omega} = \pm\frac{\delta B}{B} \mp \frac{\delta\omega}{\omega} \qquad \textbf{(16)}$$

So, if spread over angles is $0 \le \varphi \le \varphi_{\max}$ then energy of electron beam can be determined with accuracy $\delta\Omega/\Omega_0 < 10^{-4}$ according to the formula:

$$\gamma_0 = \frac{1}{2\Omega_0}, \qquad \textbf{(17)}$$

where the quantity of Ω_0 we can find over maximum of value of $\frac{\partial I_{abs}}{\partial\Omega}$.
Dependence of absorption intensity I_{abs} from B for $\varepsilon = 100\ GeV$ and $\varepsilon = 2\ GeV$ in case of use Nd:YAG and CO_2 lasers has the shape, shown on Fig.6.

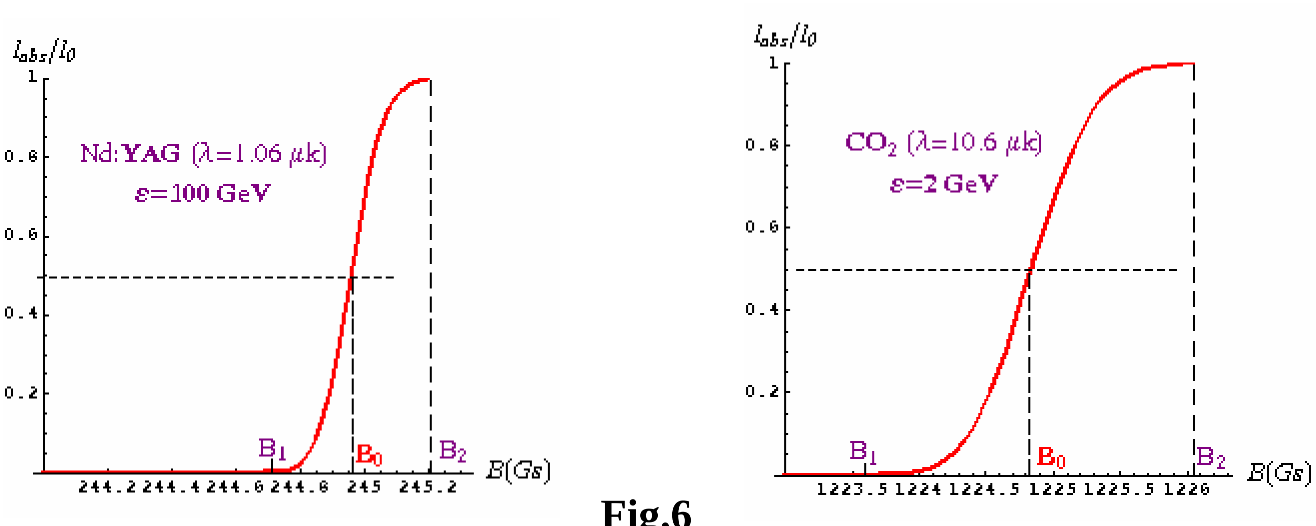


Fig.6

Dependence of electron beam energy ε from magnetic field B in case of use Nd:YAG and CO_2 lasers has the shape, shown on Fig.7.

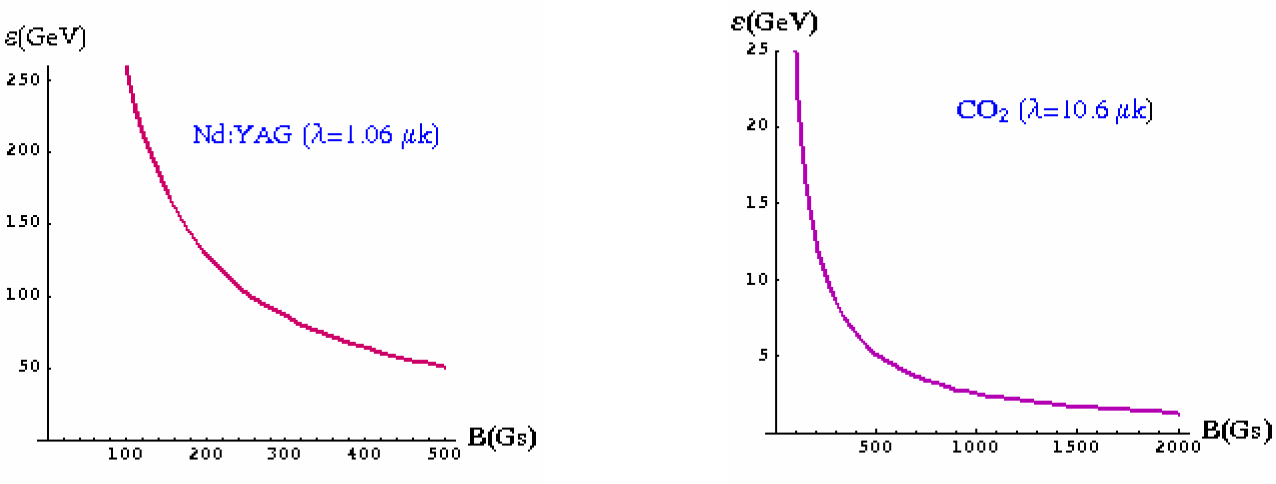


Fig.7

3. Measurement of absolute energy of electron beam in case of spread of electrons over spatial angles around of axis $\varphi = 0$ in interval $-\varphi_{\max} \le \varphi \le \varphi_{\max}$.

Let's consider the version when the electron beam has spread over energies $\varepsilon \pm \Delta\varepsilon$, and directions of velocities of electrons are concentrated in narrow spatial angle around of axis $\varphi = 0$ in interval: $-\varphi_{\max} \le \varphi \le \varphi_{\max}$.

In this case of distribution over angles φ the dependence ε from φ will be described by the formula (3) and for $\varepsilon = 100$ GeV has the form, shown on Fig.8.

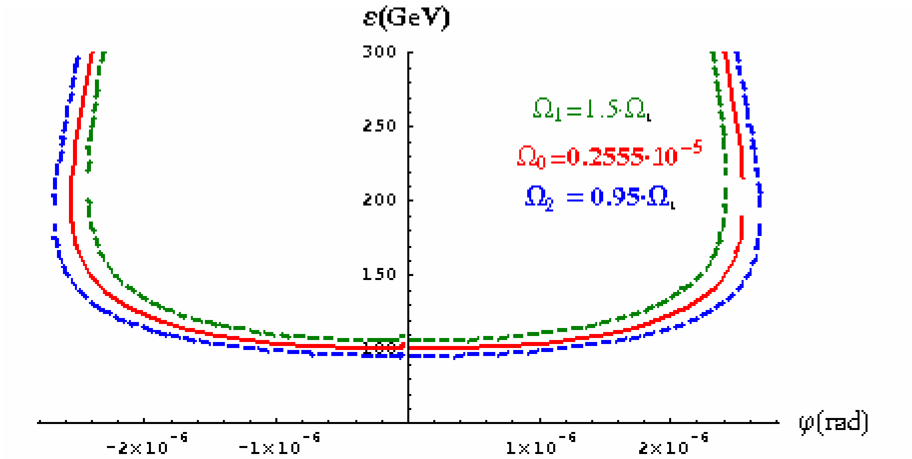


Fig.8

According to (3) (or (7)) for angles $\varphi << \Omega$ the dependence of $\varepsilon(\varphi)$ has the form, shown on (Fig.9). Therefore for angles $-\varphi_{\max} \le \varphi \le \varphi_{\max}$ the dependence of $I_{abs}(\Omega)$ will have the form similar shown on Fig.4. Energy of electron beam can be determined by means of the formula (17).

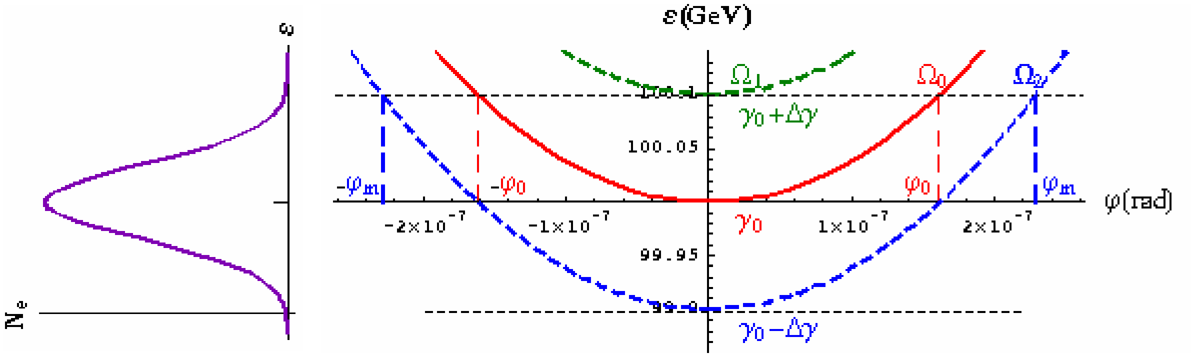


Fig.9

4. Measurement of electron beam energy in case of spread of electrons over spatial angles in intervals $-\varphi_{\max} \le \varphi \le \varphi_{\max}$ and $-\theta_{\max} \le \theta \le \theta_{\max}$.

Dependence ε from φ and θ for constant $\Omega = 0.25 \cdot 10^{-5}$ according to formula (2) has shape shown on Fig.10.

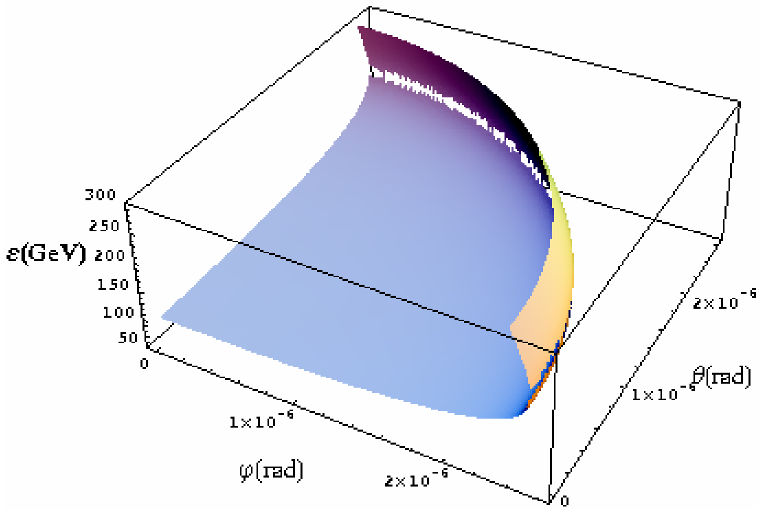


Fig.10

From (2) follows, that:

1. γ has physical sense if satisfies the condition:

$$\varphi^2 + \theta^2 \le \Omega^2 \ . \qquad \textbf{(18)}$$

2. In a point $\varphi = 0$ and $\theta = 0$, energy γ has the minimal value:

$$\gamma_{\min} = \frac{1}{2\Omega} + \frac{\Omega}{2} \qquad \textbf{(19)}$$

3. In a point $\varphi = 0$ and $\theta = 0$, the derivative of γ with respect φ and θ are equal to zero:

$$\left(\frac{d\gamma}{d\varphi}\right)_{\varphi=0} = 0 \ , \qquad \left(\frac{d\gamma}{d\varphi}\right)_{\theta=0} = 0 \ . \qquad \textbf{(20)}$$

From (2) follows, that the lower branch of curve $\gamma(\varphi,\theta)$ in case of $\varphi << \Omega$ and $\theta << \Omega$ can be described approximately as:

$$\gamma \approx \frac{1}{2\Omega} + \frac{\Omega}{2} + \frac{1}{8\Omega^3}\left(1+\Omega^4 - 2\Omega^2\right)\varphi^2 + \frac{1}{8\Omega^3}\left(1+\Omega^4 - 2\Omega^2\right)\theta^2 + 0[\varphi]^4 + 0[\theta]^4 \quad \textbf{(21)}$$

Using (21) and (19) we can estimate the quantity θ_0 and θ_m (similar to estimation of φ_0 and φ_m according to (8)) approximately as:

$$\theta_0 \approx 2\Omega_2 \cdot \sqrt{\frac{\Delta\gamma}{\gamma}} \approx \frac{1}{\gamma} \cdot \sqrt{\frac{\Delta\gamma}{\gamma}} \ , \quad \theta_m \approx \sqrt{2} \cdot \theta_0 \quad \textbf{(22)}$$

Energy of electron beam can be found in the form described above, by means of the formula (17).

5. Intensity of the laser necessary for absorption of photons by electrons

We have shown that for determination of electrons energy γ_0 (17), in case of constant laser frequency ω it is necessary to measure only the magnetic field B_0, which corresponds to absorption of photons by electrons of distribution centre over energy (Fig.3, Fig.6, Fig.9). In the formula (17) for determination of energy there is no explicit dependence on intensity of the laser I_{las}, magnet length L_M. However it is supposed that these parameters should satisfy to the certain conditions.

Intensity of the laser necessary for absorption of photons on the length ℓ_a we can find using the classical formula for a gain of energy of electrons in considered fields [1,3]:

$$\frac{d\gamma}{dt} = \xi\omega \cdot \sqrt{\frac{2\Omega}{\gamma}}, \qquad (23)$$

where $\xi = eE/m\omega c$ is parameter of laser intensity, E is amplitude of electric field of wave. Let's take into account that ℓ_a is the length of formation of photon absorption with energy $\Delta\varepsilon = \hbar\omega$ during time $\Delta t = \ell_a / V_e \cong \ell_a / c$, $\gamma = 1/2\Omega$ and $E\left[\frac{V}{cm}\right] = 19.4\sqrt{I_{las,a}\left[\frac{W}{cm^2}\right]}$, then the relation (23) can be written as:

$$I_{las,a} = \left(\frac{\hbar\omega}{19.4} \cdot \frac{\gamma}{\ell_a}\right)^2. \qquad (24)$$

According to (24) dependence of laser intensity $I_{las,a}$ from energy of electrons ε in case of Nd:YAG and CO_2 lasers , $L_M = \ell_a = 100cm$, has the shape, shown on Fig.11.

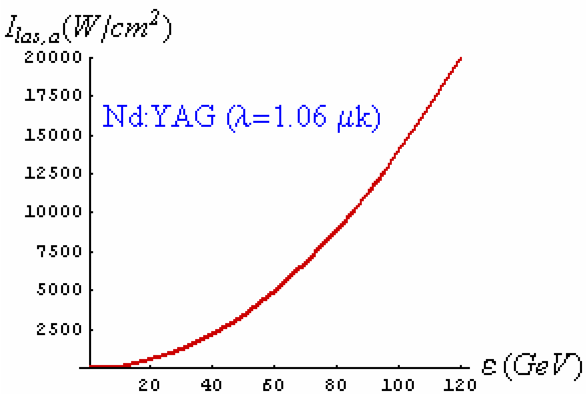
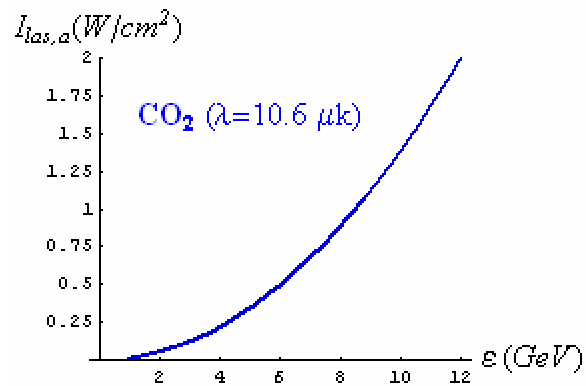


Fig.11



6. Accompanying radiation of electrons.

It is known that in fields considered above both processes are possible: acceleration (due to absorption of photons) and deceleration (and in consequence of that the radiation of photons) of electron in dependence of its initial phase[2,3].

Let's analyze conditions of radiation of electrons for the given parameters $\gamma,\ \ \omega,\ \ L_M,\ \ B,\ \ I_{las}$.

Earlier we have noted that frequency of radiation of electrons is determined by the classical formula [3]:

$$\omega_r = \frac{V}{\ell_r\left(1-\frac{V}{c}\cos\alpha\right)}, \qquad \textbf{(25)}$$

where α **is angle between electron velocity** $\vec{V}$ **and direction of radiation,** ℓ_r **is the length of electron trajectory, onto which the radiation of photon is formed.**

On the other hand, using the discrete spectrum of electron energy in a magnetic field and the law of preservation of energy-momentum during radiation, for frequency of radiation we have [3]:

$$\omega_r = \frac{(n-n')\omega_c}{\gamma\left(1-\frac{V_z}{c}\cos\psi\right)} \qquad \textbf{(26)}$$

where ψ **is angle between direction of magnetic field** $\vec{B}$ **and direction of radiation,**

$n-n'=1,\quad 2,\quad 3,\ \ldots$

In a case of $\gamma >> 1$, the main part of radiation intensity is concentrated in a narrow interval of angles $0 \le \alpha \le 1/\gamma$ or $0 \le \psi \le 1/\gamma$ and for simplicity we shall consider for angles $\alpha = \psi = 0$ the frequency ω_r , which according to (25) and (26) will be:

$$\omega_r \approx \frac{c \cdot 2\gamma^2}{\ell_r} \cong (n - n') \cdot 2\gamma\omega_c \approx (n - n') \cdot \omega \tag{27}$$

It is clear that radiation of a electron is possible if

$$\ell_r \approx \frac{c \cdot 2\gamma^2}{(n - n')\omega} \le L_M = \ell_a \ . \tag{28}$$

Intensity of the laser $I_{las,r}$, necessary for radiation of electrons with frequency $\omega_r = (n - n') \cdot \omega$ on the length ℓ_r we can find by means of similar to (24) formula:

$$I_{las,r} = \left(\frac{\hbar\omega_r}{19.4} \cdot \frac{\gamma}{\ell_r} \right)^2 \tag{29}$$

As radiation of electron is possible if $\ell_r \le L_M$ then using relation (29) we receive a condition for radiation of electron:

$$\ell_r = \frac{(n - n') \cdot \hbar\omega \cdot \gamma}{19.4 \cdot \sqrt{I_{las,r}}} \le L_M \tag{30}$$

Taking into account (24), the condition (30) can be written as:

$$(n - n') \cdot \sqrt{I_{las,a}} \le \sqrt{I_{las,r}} \tag{31}$$

If for absorption of electrons with energy γ **the necessary parameters** $\omega,\ L_M,\ I_{las,a}$ **are already chosen, then radiation of electrons for the same parameters is possible if** $I_{las,r} \le I_{las,a}$ **or if to use** (31)

$$(n-n')\cdot\sqrt{I_{las,a}} \le \sqrt{I_{las,r}} \le \sqrt{I_{las,a}}\ .$$

This condition can be carried out only if $n-n'=1$ **. In case of** $n-n'=1$ **, using (28) we shall obtain a restriction on energy** γ **of electron:**

$$\gamma \le \sqrt{\frac{\omega\cdot L_M}{2c}} \qquad \textbf{(32)}$$

Thus radiation of electrons is possible, if their energy is limited by condition (32), while electrons with energy $\gamma \ge \sqrt{\frac{\omega\cdot L_M}{2c}}$ **can absorb photons.**

For example if $L_M = 100cm$ **and we use Nd:YAG and** CO_2 **lasers then radiation of electron is possible if are satisfied accordingly the conditions:** $\varepsilon \le 879.71$ **MeV and** $\varepsilon \le 278.19$ **MeV.**

7. Influence of process of measurement on parameters of electron beam

During the measurement of the electron beam energy his parameters will not worsen.

a) As energy of the absorbed photons is much less than energy of electrons $\hbar\omega << \varepsilon$ **then the change of energy of electron beam because of absorption will be negligible.**

b) Let's estimate change of direction of velocity $\vec{V}_e$ **of electron in magnetic field. For that we resolve** $\vec{B}$ **onto components parallel (**$B_\parallel$**) and perpendicular (**$B_\perp$**) to direction** $\vec{V}_e$ **(Fig.1).**

It is clearly that the trajectory of a electron can be bent only under influence of perpendicular component $B_{\perp} \cong \varphi \cdot B$. Therefore angle of bend α on length L_M of electron trajectory, we can estimate according to the formula:

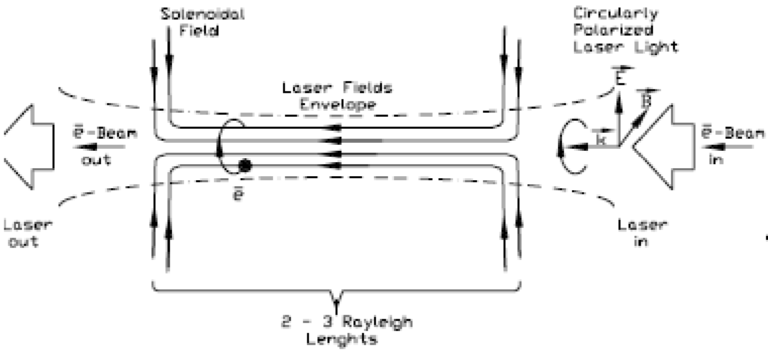
$$\alpha \approx \frac{L_M \omega_c}{c\gamma}\varphi = \frac{L_M \omega}{2c\gamma^2}\varphi \quad . \qquad \textbf{(33)}$$

For example, if $\gamma = 2 \cdot 10^5$ **,** $L_M = 100cm$ **,**$\lambda = 1.06\mu k$ **then** $\alpha \cong 10^{-4}\varphi$ **. It means, that the change of direction of electron velocity** $\vec{V}_e$ **is negligible.**

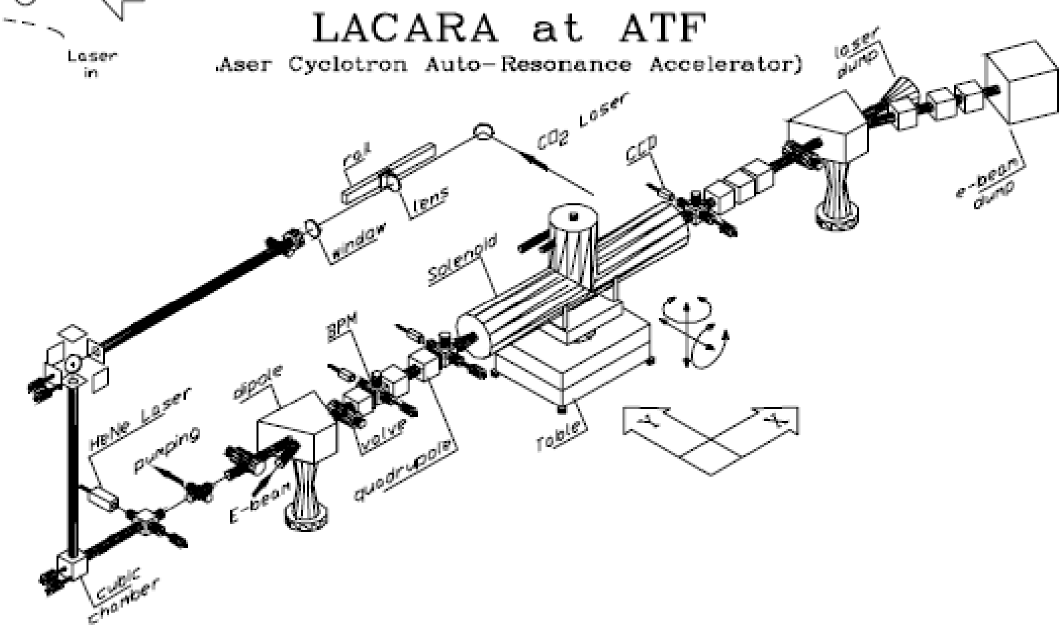
8. Experimental confirmations

Acceleration of electrons in the electromagnetic wave and the magnetic fields, due to absorption of photons by electrons, has been experimentally confirmed [4,5].
The obtained results are in the good agreement with the theoretical predictions regarding validity of the resonant condition (1) and the electrons energy gain (23).
It is known also about planned experiment in BNL: Laser Cyclotron Auto-Resonance Acceleration (LACARA) , with use of the CO_2 **laser [6]. It seems this set-up is suitable for testing the considered conception.**

Laser Cyclotron Auto-Resonance Accelerator, (LACARA), AE25
Jay Hirshfield and Tom Marshall



40 to 50 MeV
Expected energy gain



9. Summary

1. The considered conception of "Resonant Absorption" allows one to measure the absolute energy of the electron beam, for a wide range (up to few hundred GeV) energies, with accuracy 10^{-4} .
2. The detectors and lasers required for measurement of the electron beam energy are not unique, and can be chosen from existing industrial samples.
3. In this conception, for measurement of high energy of electron beam, the required magnetic fields have relatively small value, equal a few hundred Gs.
4. The parameters of the electron beam will not worsen during the measurement of the energy . This feature of the method allows carrying out the continuous monitoring of the electron beam energy.

10. References

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