The Resolution of the TESLA Energy Spectrometer

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The layout of the TESLA energy spectrometer is shown on Fig.7.3.1 of the TESLA Technical Design Report. It consists of 3 dipole magnets which displace the beam by about 5 mm, and finally return the beam to the nominal trajectory. This set up might be located in the final focus beam line in the last 300 m before the IP.

The beam energy is measured via the deflection angle of the trajectory before and after the spectrometer magnet. The beam energy is obtained by:

$$E = c\, Bl/\Theta \tag{1}$$

where E, Bl, Θ, c are the beam energy [GeV], the field integral [Tm], the deflection angle [mrad], and the speed of light c = 299.792. Formula (1) is valid for all types of magnets if Bl is the correct field integral.

The error of the energy measurement depends on the errors of the field integral and the deflection angle measurements. The total energy error results by error propagation from (1) to the quadratic sum:

$$(\sigma_E/E)^2 = (\sigma_{Bl}/Bl)^2 + (\sigma_\Theta/\Theta)^2 \tag{2}$$

The angle itself is no direct observable, instead the path of the beam trajectory is measured at different positions by so-called Beam Position Monitors (BPM). The number of BPMs, their distances and spatial resolution determine the angular error. At least 2 BPMs on each side of the spectrometer magnet are needed to measure the deflection angle. For this set-up the error of the deflection angle is given by:

$$\sigma_\Theta/\Theta = (2\, E/c\, Bl)\, (\sigma_X/D) \tag{3}$$

, where σ_X and D are the spatial resolution [μm] of the BPMs and their distances [m].

Up to now it was assumed that two BPMs are used at each side of the spectrometric magnet. The precision of the slope reconstruction can be improved if more BPMs are used. The slope error is given by the following formula:

$$\sigma_\Theta{}^2 = f(N)\, \sigma_X^2/D^2 \tag{4}$$

where $f(N)$ is a function which depends on the number N of BPMs. It has the form:

$$f(N) = N\, (N-1)^2/(\sum i^2 - (\sum i)^2) \tag{5}$$

For $N = 2, 3, 4...10$ one obtaines $f(N) = 2., 2., 1.8, 1.6, 1.43, 1.29, 1.17, 1.07, 0.98$. Combining formulas (2), (3) and (4) one obtains:

$$\sigma_E/E = \sqrt{(\sigma_{Bl}/Bl)^2 + 2\, f(N)\, (E\, \sigma_X/c\, D\, Bl)^2} \tag{6}$$

In the following the influence of the magnetic field error σ_{Bl}/Bl, the spatial resolution σ_X, the distance D of the outermost BPMs in each spectrometer arm and their number N is briefly illustrated by some plots. The energy resolution is calculated for a beam energy of 250 GeV. If not explicitely quoted the the following default values are used: $\sigma_{Bl}/Bl = 2\,10^{-5}$ with $Bl = 0.75Tm$, $\sigma_X = 200nm$, $D = 10m$, and $N = 3$.

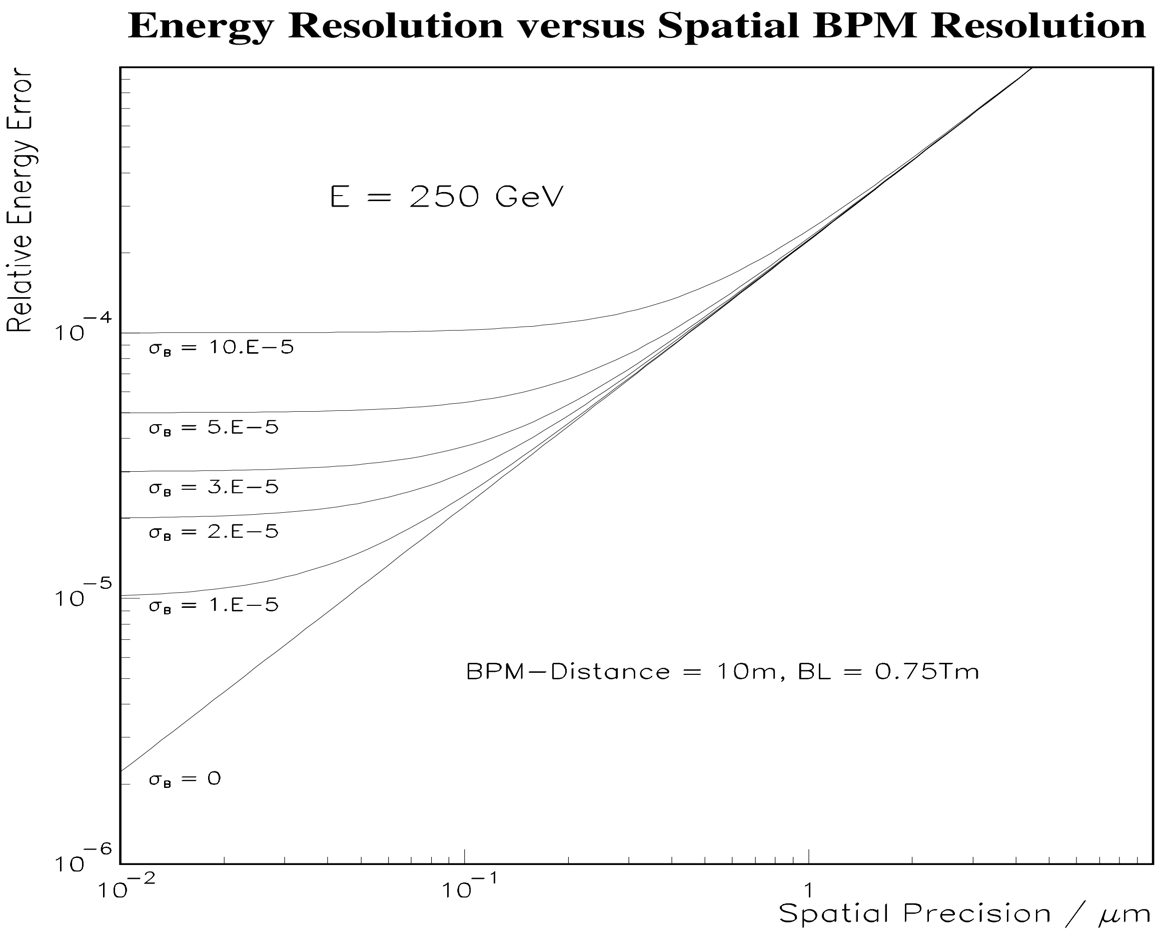

Figure 1: The energy resolution in dependence on the BPM spatial resolution. The lines describe different field precision in the range of a few 10^{-5}.

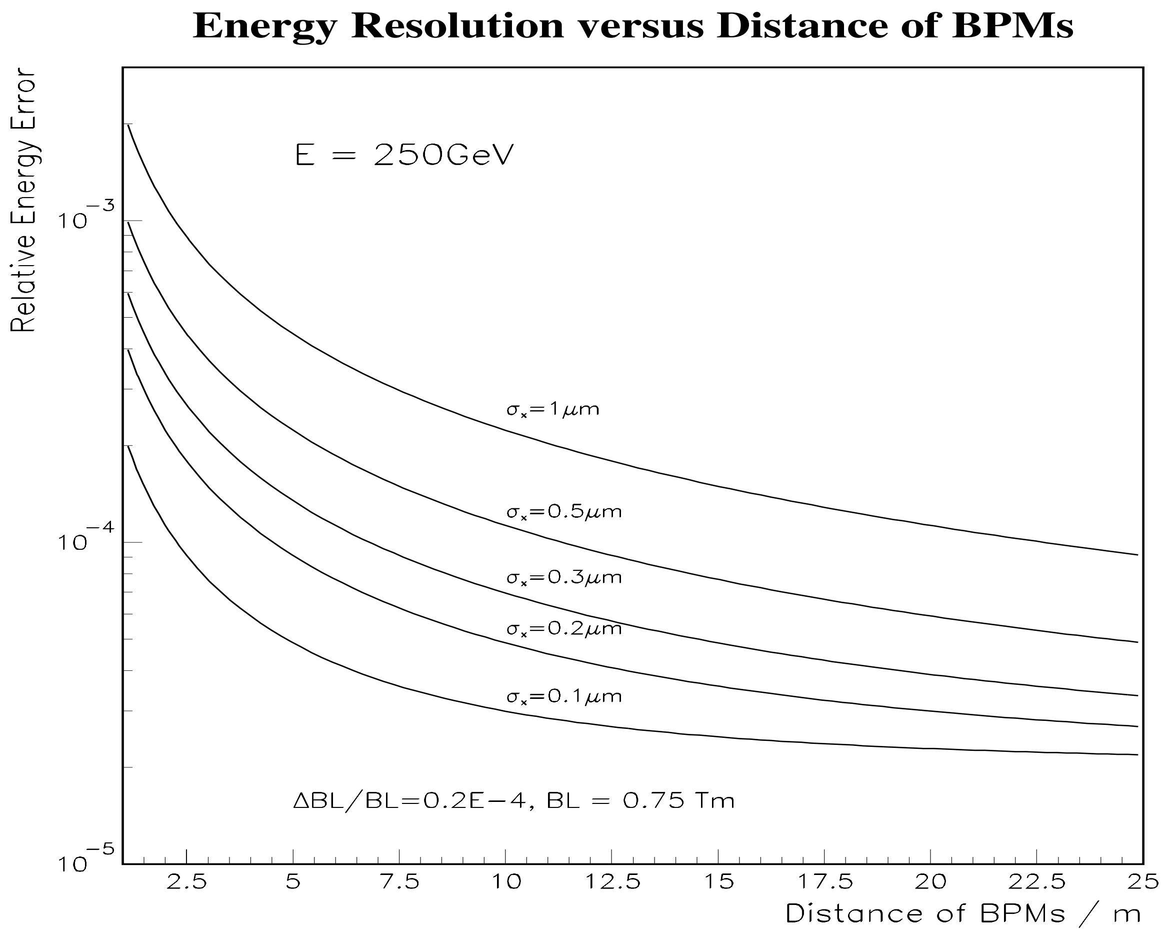

Figure 2: The energy resolution in dependence on the distance of the BPMs. The lines describe different BPM spatial resolution.

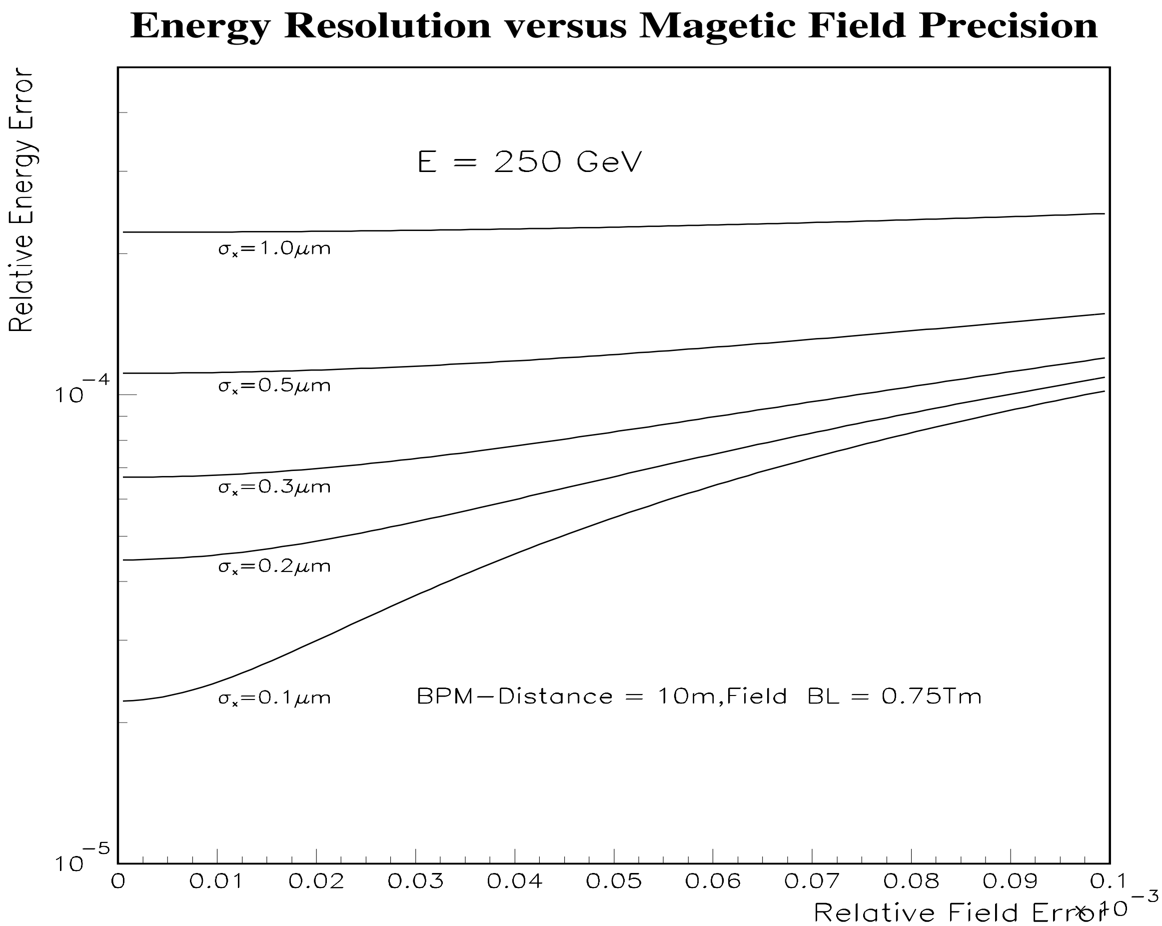

Figure 3: The energy resolution in dependence on the relative field precision. The lines describe different BPM spatial resolution.

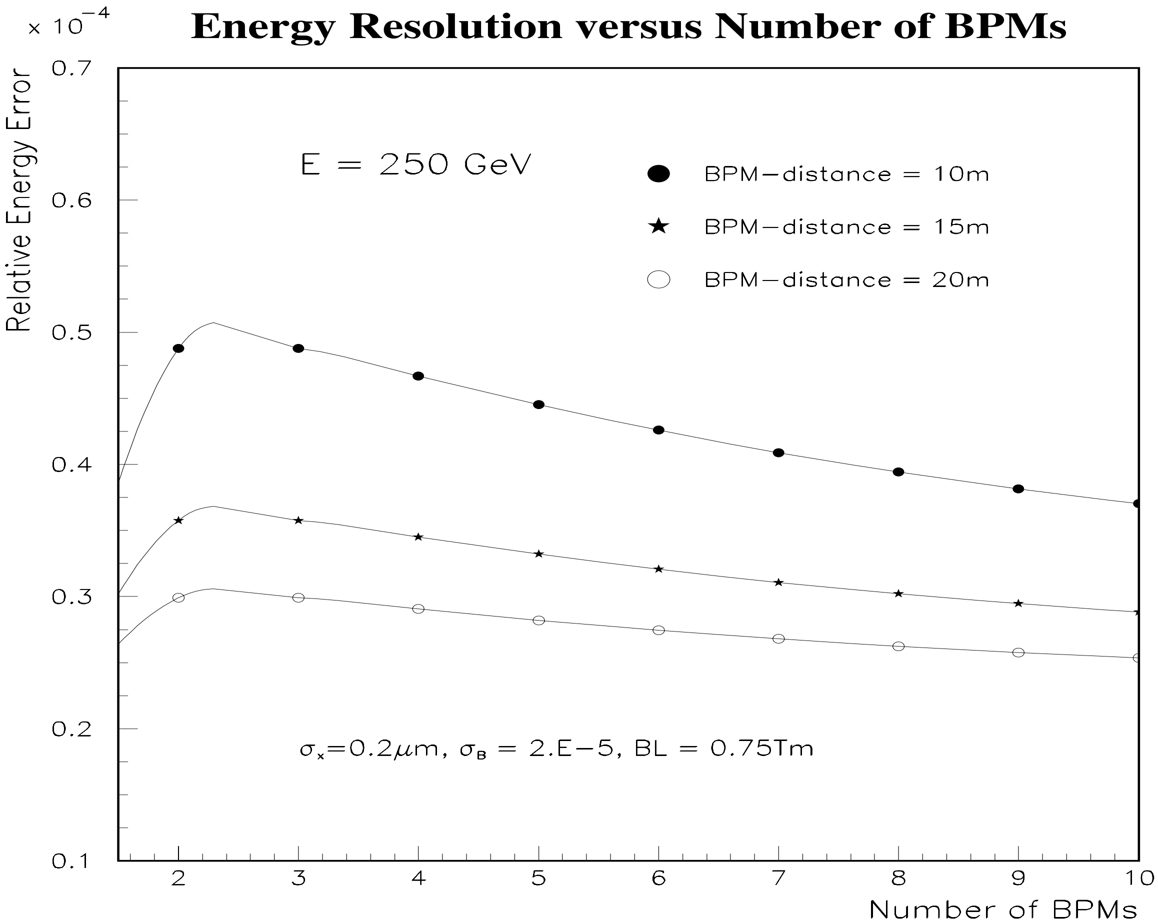

Figure 4: The energy resolution in dependence on the numbers of BPMs in each spectrometer arm. The lines describe different distances of the outermost BPMs.

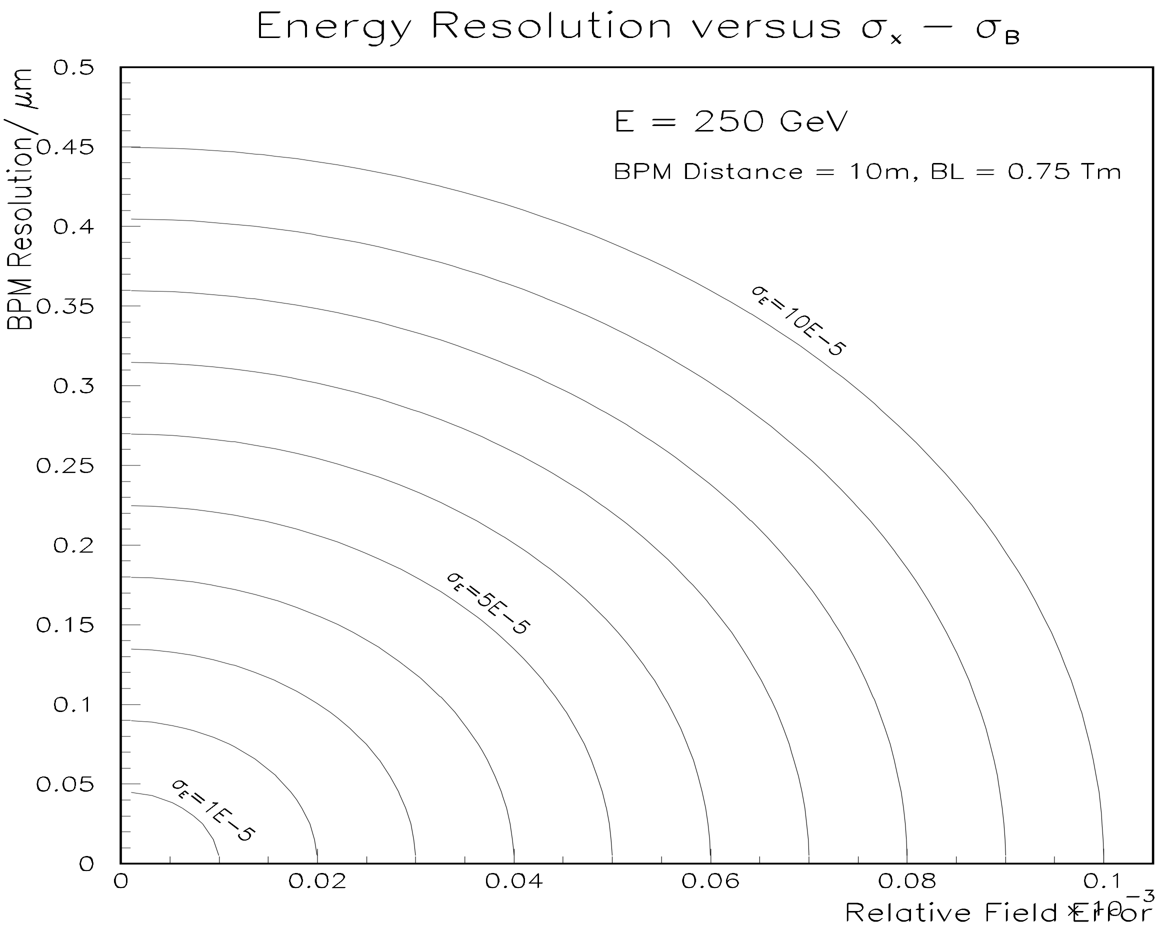

Figure 5: The lines of constant energy resolution in dependence on the relative field precision and the BPM spatial resolution.

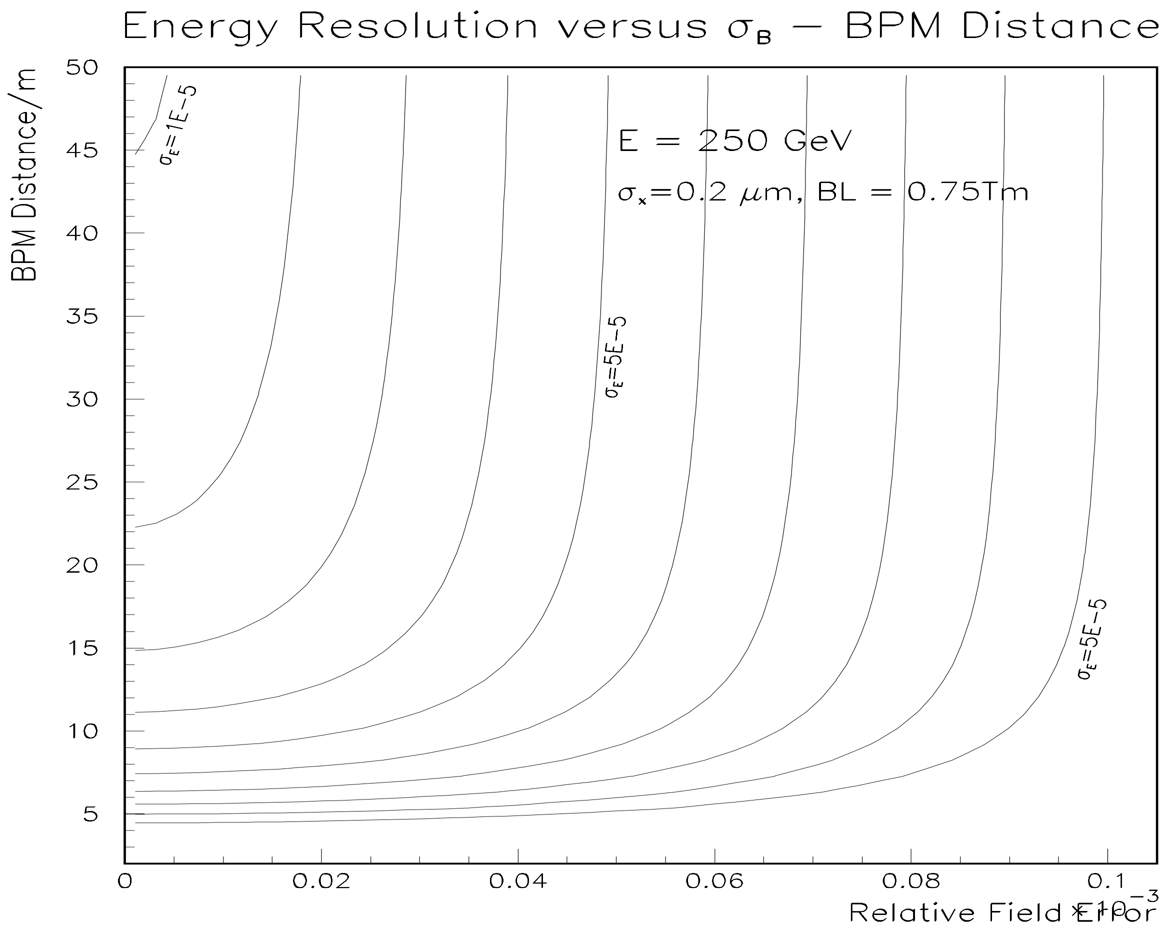

Figure 6: The lines of constant energy resolution in dependence on the relative field precision and the BPM distance.

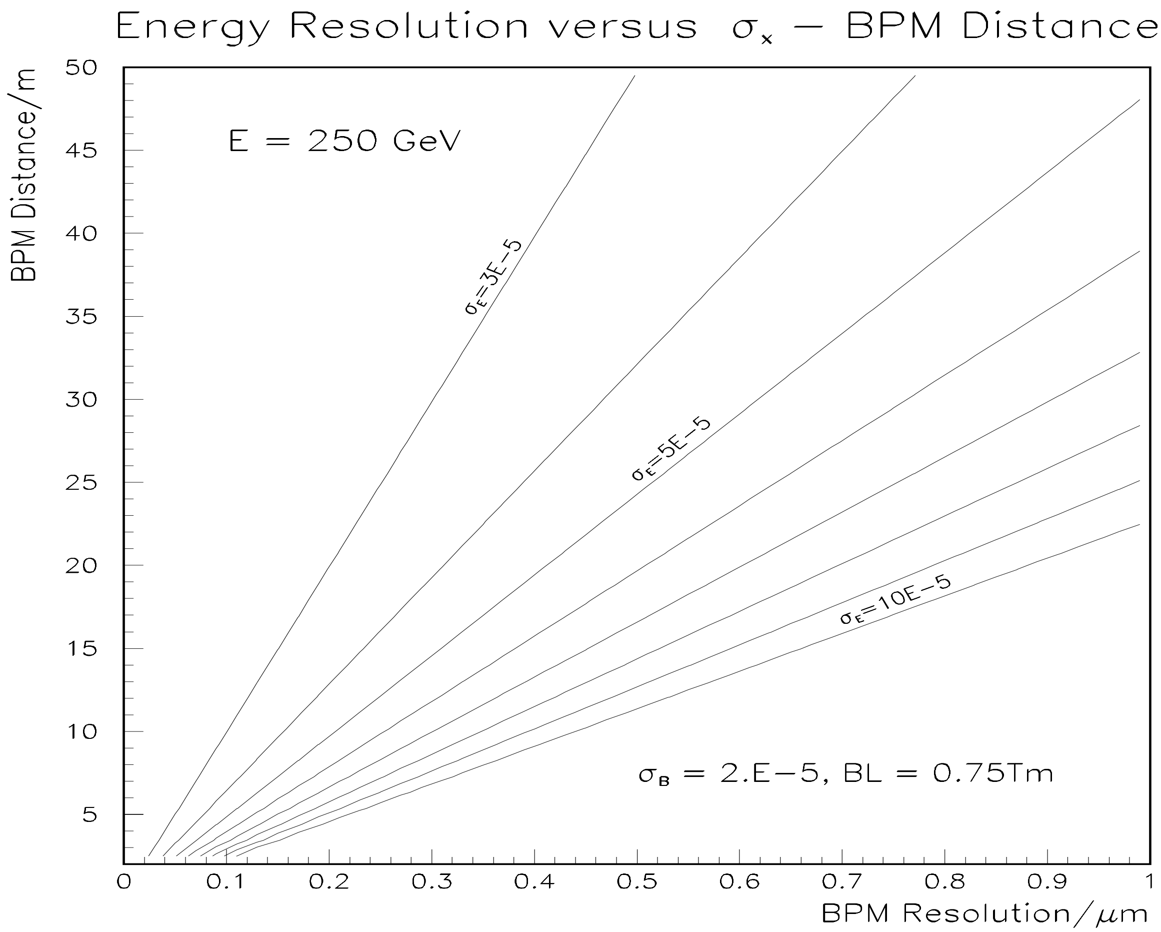

Figure 7: The lines of constant energy resolution in dependence on the BPM spatial resolution and the BPM distance.