ENERGY MEASUREMENT OF RELATIVISTIC ELECTRON BEAMS USING RESONANCE ABSORPTION OF LASER LIGHT BY ELECTRONS IN A MAGNETIC FIELD

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ABSTRACT

The possibility of a precise measurement of electron beam energy using a resonant absorption of laser light by electrons in a static homogeneous magnetic field is considered. The method permits to measure the average electron beam energy for wide range up to few hundred GeV energies. The relative accuracy of the electron beam energy measurement can reach 10^{-4}. The influence of a laser light diffraction and spread in angles of electrons on a measurement accuracy of electron beam energy is considered. The power of the required laser is estimated.

1. THE CONDITION OF RESONANT ABSORPTION

For a precise measurement of electron beam energy we suppose to use a resonant absorption of laser photons by electrons in a static and homogeneous magnetic field. Let's assume, that the magnetic field $\vec{B}$ directed along z–axis (Fig.1). It is supposed, that the electrons and photons are injected in a magnetic field under small angles φ and θ to the z-axis, accordingly. In a magnetic field the electrons have a discrete spectrum of energy [4]:

$$\varepsilon_{n,\zeta} = [m^2 + P_z^2 + eB(2n+1+\zeta)]^{1/2}, \qquad (1)$$

where n=0,1,2,… labels the electron energy levels, P_z is the z-component of the electron momentum, $\zeta = \pm 1$ is the projection of electron spin on a direction of $\vec{B}$. We use units for which $\hbar$=c=1. From (1) follows, that the electrons in a magnetic field can be on equidistant energy levels (Fig.2) and photons of frequency ω can be resonantly absorbed at transitions $\varepsilon_{n,\zeta} \rightarrow \varepsilon_{n',\zeta}$ between electron energy levels. Using the spectrum of electron energy (1) and the law of energy-momentum conservation for photon absorption:

$$\varepsilon_{n,\zeta} + \omega = \varepsilon_{n',\zeta}, \quad P_{z,0} + \omega = P_z, \qquad (2)$$

we can find the condition for resonant electron transition (see also [5], [6]):

$$\omega\left[\gamma_0(1 - V_{z,0}\cos\theta) + \frac{\omega}{m}\frac{(\sin\theta)^2}{2}\right] = \omega_c(n' - n), \qquad (3)$$

where $\varepsilon_{n,\zeta} = \varepsilon_0$ and $P_{z,0}$ are the energy and z-component of electron momentum before of photon absorption, ω_c = eB/m is the cyclotron frequency of electron, $\gamma_0 = \varepsilon_0/m$ is the electron relativistic factor, $V_{z,0} = V_0\cos\varphi$ is the z-component of electron velocity before absorption. We consider transitions without of change of electrons spin direction, because the probability of spin-flip transitions is negligible. Besides we consider only transitions on the main harmonic n' - n = 1, because the probability of transitions for higher harmonic is negligible.

For absorption of photons of optical and more lower frequencies of interest to us in approximation of $\omega/\varepsilon_0 << 1$ the resonance condition can be written as (see also [5,6]):

$$\gamma_0 - \cos\varphi\cos\theta\sqrt{\gamma_0^2 - 1} = \frac{\omega_c}{\omega}. \qquad (4)$$

In relation to γ_0 the relation (4) has a solution:

$$\gamma_0 = \frac{\Omega \pm \cos\varphi\cos\theta\sqrt{\Omega^2 - 1 + (\cos\varphi\cos\theta)^2}}{1 - (\cos\varphi\cos\theta)^2}, \qquad (5)$$

where $\Omega = \omega_c/\omega$. The dependence of the γ-factor from Ω for some fixed angles φ and θ is represented schematically on Fig.3. In case of one electron, if the angles φ and θ are known, then having measured a resonant frequency Ω, due to a unique dependence between γ and Ω, we can calculate γ-factor of electron according to the formula (5). Because in reality the electrons of beam always have some spread on angles in limits $\varphi_i \le \varphi \le \varphi_f$ and laser light always diverges in limits of diffraction angles, therefore the precise measurement of absolute value of γ_0 immediately according to (5) is difficult.

Below we consider the possibility of determination of electron bunch energy using relation (5), taking in account the spread in angles of electrons and influence of light diffraction.

2. DETERMNATION OF ELECTRON BEAM ENERGY TAKING IN ACCOUNT THE SPREAD IN ANGLES OF ELECTRONS AND LIGHT DIFFRACTION

It is known, that the light beam of diameter D, because of diffraction diverges in limits of angle $\theta_d \cong \lambda/D$ around of direction of wave vector $\vec{k}$ of incident wave. Here $|\vec{k}| = \omega/c$, λ and ω are the length and frequency of wave. At diffraction of light the wave vector changes only direction, so that $|\vec{k}| = |\vec{k}'|$, where $|\vec{k}'|$ is wave vector after diffraction.

The distribution of light intensity depending on angle of diffraction θ is determined by expression (see for example [7], [8]):

$$I(\theta) = I_0\left[\frac{2J_1(\alpha)}{\alpha}\right]^2, \qquad (6)$$

and has the shape schematically represented on Fig.4. Here I_0 is the intensity of incident wave under angle $\theta = 0$, $J_1(\alpha)$ is the cylindrical function of the first order and $\alpha = Dk\theta/2$. From (6) follows, that the main part of radiation intensity is concentrated in interval of angles $0 \le |\theta| \le \lambda/D$.

From (5) follows that in a case of $\gamma >> 1$ and if $\Omega^2 >> 1 - (\cos\varphi\cos\theta)^2$ the electron energy can be approximately determined by formula:

$$\gamma \cong \frac{2\Omega}{\varphi^2 + \theta^2}. \qquad (7)$$

From (7) it is clear, that for fixed values of γ and Ω we have

$$\varphi^2 + \theta^2 = \frac{2\Omega}{\gamma} = const. \qquad (8)$$

On the graph with axes (φ, θ) the relation (8) represents the equation of a circle with radius $\sqrt{2\Omega/\gamma}$ (Fig.5). From (8) follows, that the intensity of a resonant absorption I_{abs} is minimum for electrons with angles $\varphi = \varphi_i$ and $\theta = 0$ at $\Omega_{\min} = \Omega_i = \gamma\varphi_i^2/2$, because the number of electrons absorbing photons resonantly, is minimum. From Fig.6 it is obvious, that if $\Omega < \Omega_i$ then $I_{abs} = 0$. With growth of Ω the intensity of absorption I_{abs} is increase, because in process of absorption will be included more electrons with angles in limits $\varphi_i \le \varphi \le \varphi_f$ and $0 \le \theta \le \theta_f$. Intensity reaches up to maximum at $\varphi = \varphi_f$ and $\Omega = \Omega_f = \gamma\varphi_f^2/2$. At the further growth of Ω the intensity of absorption I_{abs} decreases because the interval of angles $\Delta\theta$ and therefore the intensity of light interacting with electrons decrease. Thus the intensity of absorption I_{abs} as a function from Ω has a shape, which is schematically shown on Fig.7.

Note, that if the electron beam interacts with light (see Fig.5) in limits of angles $0 < \theta < \theta_f$, then the diffraction divergence of this part of light will be smaller by factor of $\theta_f/\theta_d << 1$.

It is clear from Fig.7, that on axis Ω the point $\Omega = \Omega_f$ is singled out and can be used for determination of electrons energy.

The first method: Determination of a relative energy of electrons.

If the energy of electron bunch γ_1 is known, then having measured the resonant frequency $\Omega_{f,1}$, which corresponds to the maximum of resonance absorption intensity $I_{abs}(\Omega_{f,1})$ for this energy (Fig.8), we can calculate according to the formula (7) the angle: $\varphi_f = \sqrt{2\Omega_{f,1}/\gamma_1}$. Further we suppose, that for an unknown energy γ_2 of electrons the angular distribution of electrons of beam does not vary and the maximum of resonant absorption intensity $I_{abs}(\Omega_{f,2})$ is fall on the same angle $\varphi_f = \sqrt{2\Omega_{f,2}/\gamma_2}$. Then having measured the frequency $\Omega_{f,2}$ (see Fig.6), we can calculate the value of unknown γ_2:

$$\gamma_2 = \frac{\Omega_{f,2}}{\Omega_{f,2}}\gamma_1.$$

(9)

It is obvious, that the accuracy of γ_2 will be determined by accuracy of γ_1, $\Omega_{f,1}$ and $\Omega_{f,2}$.

The second method: Determination of an absolute energy of electrons.

Let's assume, that we can turn the electron beam on some small angle $\delta\varphi = \varphi_F - \varphi_f$, known for us (Fig.9). Then having measured resonant frequencies Ω_F and Ω_f (Fig.10), which correspond to a maximum of resonance absorptions intensity at angles

$\varphi_F = \sqrt{2\Omega_F/\gamma}$ and $\varphi_f = \sqrt{2\Omega_f/\gamma}$, we can calculate the absolute value of γ factor of electrons:

$$\gamma = \frac{1}{2}\left(\frac{\sqrt{\Omega_F} - \sqrt{\Omega_f}}{\delta\varphi}\right)^2 \qquad (10)$$

In particular it is possible to use a bending magnet (BM) for turn of electron beam on some small angle $\delta\varphi = \varphi_F - \varphi_f$ (Fig.9), which is determined by the formula:

$$\delta\varphi = \frac{eH}{mc\gamma}\frac{l}{c}, \qquad (11)$$

where H is the value of the magnetic field, and l is the length of bending magnet. Having measured resonant frequencies Ω_F and Ω_f, which correspond to a maximum of resonance absorptions intensity at angles $\varphi_F = \sqrt{2\Omega_F/\gamma}$ and $\varphi_f = \sqrt{2\Omega_f/\gamma}$ (Fig.10, Fig.11, Fig12), we have the relation:

$$\delta\varphi = \varphi_F - \varphi_f = \sqrt{\frac{2}{\gamma}}\left(\sqrt{\Omega_F} - \sqrt{\Omega_f}\right) = \frac{eH}{mc\gamma}\frac{l}{c}. \qquad (12)$$

From here we can calculate the absolute value of γ factor of electron:

$$\gamma = \frac{1}{2}\left(\frac{eH}{mc}\frac{l}{c}\frac{1}{(\sqrt{\Omega_F} - \sqrt{\Omega_f})}\right)^2.$$

(13)

In this case the accuracy of γ will be determined by accuracy of parameters Ω_F, Ω_f, H and l.

At both methods the precise measurement of a resonant value Ω_f is difficult because of some width of resonance of absorption intensity $I_{abs}(\Omega)$ (Fig.7). However, prove to be, that by tuning of the laser intensity parameter $\xi = eE/m\omega c$ (or length of a magnet L) it is possible to increase the accuracy of Ω_f determination and to increase a spectral resolving power of measuring method.

3. ESTIMATION OF INTENSITY OF A RESONANT ABSORPTION

Let's estimate the parameter of the laser intensity ξ at absorption by electron of ν laser photons on length of interaction L and for angle of incidence φ of electrons (Fig.13). It is clear from Fig.13, that under action of a component $E_{II} = \sin\varphi E \cong \varphi E$ of electric vector of light, the electron will be accelerated in a direction of velocity $\overrightarrow{V_e}$ of electron and the growth of electron energy will be $eEL\varphi$. If at acceleration the electron absorbs ν photons, then we have the relation:

$$\nu\hbar\omega = eE\varphi L = mc^2\xi\varphi L\frac{\omega}{c}, \qquad (14)$$

whence we have

$$\nu = \frac{mc}{\hbar}\xi\varphi L \cong \frac{L[cm]10^{12}}{38{,}61}\xi\varphi. \qquad (15)$$

In order to the electron would absorb even 1 photon, it is necessary to satisfy the requirement:

$$\frac{mc}{\hbar}\xi\varphi L \geq 1. \qquad (16)$$

For fixed length of a magnet L, the dependence ξ from φ have the shape, schematically shown on Fig.14. It is clear from Fig.14, that for example in a case of $\xi < \xi_{1/2}$ the electrons with angles in an interval $\varphi_i < \varphi < \varphi_{1/2}$ will not absorb photons, while the electrons with angles $\varphi > \varphi_{1/2}$, will absorb. It means, that for $\Omega < \Omega_{1/2}$ there is not resonant absorption of photons (see Fig.14, Fig.15). Analogously, in cases of $\xi < \xi_f$ and $\xi < \xi_F$ (Fig.14) the electrons will be absorb photons only at realization of requirements $\Omega > \Omega_f$ and $\Omega > \Omega_F$, accordingly (Fig.17, Fig.18 and Fig.19, Fig.20). Thus it is obvious (Fig.18 and Fig.20), that at $\xi < \xi_f$ and $\xi < \xi_F$ the intensity of absorption have sharp left-hand edge and this property of dependence of $I_{abs}(\Omega)$ allows increasing the accuracy of determination of Ω. It is clear, that in case of determination of energy of electrons according to the first and second method described above, we suppose that $\xi > \xi_i$.

Let's remark, that the absorption of photons does not increase noticeably the spread of electron beam energy $\Delta\varepsilon_b$ because of relation: $\hbar\omega << \Delta\varepsilon_b$.

Let's estimate number of photons N_{abs} absorbed by electron beam of length l_b, if $l_b >> L$ and taking into account, that each electron transits the region of interaction L only once. . To observe the resonant absorption we suppose, that in (15) the parameters ξ, φ and L are chosen so that $\nu \geq 1$. If the number of electrons in the beam is N_e, then $N_{abs} = N_e$.

If the parameters of the laser ξ and ω are known, then we have $eE = \xi m\omega c$ and the minimum power of the laser can be found by a relation:

$$W\left[W/cm^2\right] \cong \left(\frac{E[V/cm]}{19,4}\right)^2. \qquad (17)$$

Actually, because only the part of laser light interacts with electron beam, the necessary minimum of a laser power (intensity) should be more (see Fig.5).

Now, using the known value of W we can find the number $N_{l,ph}$ of laser photons, incident on a surface S during time $T_b = l_b/c$:

$$N_{l,ph} = \frac{W}{\hbar\omega}ST_b. \qquad (18)$$

Resonance absorption can be observed by measuring the ratio of the absorbed photons N_{abs} to total number of photons, incident on area of interaction $N_{l,ph}$ or (I_{abs}/I_{tot}).

Let's consider a numerical example for $\gamma = 10^5$, $\lambda = 10,6$ micrometer (CO_2 laser) and $\varphi = 2\cdot 10^{-4}$, $\theta = 0$. Then according to (7) the field strength is $B = 20.2$ kGs. For $L = 50$ cm and $T_b = 0.64$ msec (TESLA) using (15) and (17) we have: $\xi \cong 3.8\cdot 10^{-9}$ and $W \cong 0.36 W/cm^2$. If $N_e = 4\cdot 10^{13}$ e/pulse and $S = 0.25 cm^2$, then using (18) we find $N_{l,ph} \cong 3\cdot 10^{15}$ photon/pulse and $N_{abs}/N_{l,ph} \cong 1.3\cdot 10^{-2}$.

Thus, the numerical estimations show, that near to a resonance, the ratio $N_{abs}/N_{l,ph}$ has a reasonable value for measuring.

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Figure Captions

Fig. 1. Conceptual sketch of the set-up.
e^- - electron beam, L – laser beam, D – detector, M – magnet.

Fig. 2. Equidistant energy levels of electrons in magnetic field.

Fig. 3. Depedence of electron γ-factor from Ω for fixed angles φ and θ.

Fig. 4. Distribution of laser light intensity $I_{(\theta)}$ from angle of diffraction θ.

Fig. 5. The dependence of angle φ from angle θ.

Fig. 6. Dependence of γ-factor of electrons from Ω for different angles φ_I, $\varphi_{1/2}$, φ_f. The intensity is minimum for $\varphi=\varphi_i$ and $\Omega=\Omega_i$. I_{abs} reaches to maximum at $\varphi=\varphi_f$ and $\Omega=\Omega_f$. At $\Omega>\Omega_f$ the intensity I_{abs} decreases.

Fig. 7. Intensity I_{abs} of photon absorption by electrons as a function from Ω.

Fig. 8. Determination of the unknown γ_2-factor of electrons relative to the known γ_1-factor.

Fig. 9. Sketch of the arrangement for determination of the absolute energy of electrons. BM – bending magnet, L – the length of the main magnet.

Fig. 10, Fig. 11, Fig.12. Measuring of frequencies Ω_f and Ω_F relevant to the maximum of resonant absorption at angles φ_f and φ_F.

Fig. 13. Laser beam intersects the electron beam under angle φ. Electrons can accelerated in direction of electron velocity V_e under action of a component $E_{\|} = E\cdot\sin\varphi$.

Fig. 14. The dependence laser intensity parameter of ξ from φ for fixed length of magnet L.

Fig. 15. In case of $\xi< \xi_{1/2}$ electrons with angles in interval $\varphi<\varphi<\varphi_{1/2}$ absorb photons, while electrons with angles $\varphi>\varphi_{1/2}$ will absorb.

Fig. 16. In case of $\xi< \xi_{1/2}$ for values of $\Omega<\Omega_{1/2}$ there is not resonant absorption of photons.

Fig. 17, Fig. 18. In case of $\xi< \xi_f$ electrons will be absorbs photons only at $\Omega\geq\Omega_f$. Intensity of absorption have sharp left-hand edge at $\Omega=\Omega_f$.

Fig. 19, Fig 20. In case of $\xi\leq \xi_F$ electrons can absorb photons only at $\Omega\geq\Omega_F$. Intensity of absorption have sharp left-hand edge at $\Omega=\Omega_F$.