ENERGY MEASUREMENT OF RELATIVISTIC ELECTRON BEAMS USING RESONANCE ABSORPTION OF LASER LIGHT BY ELECTRONS IN A MAGNETIC FIELD

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THE CONDITION OF RESONANT ABSORPTION

For measurement of electron beam energy we suppose to use:

-static and homogeneous magnetic field $\vec{B}$ directed along z–axis (Fig.1)

-laser beam

-detectot (D), for measuring of ratio of the number of absorbed photons to the total number of photons

-Electrons and photons are injected in a magnetic field under small angles φ and θ to the z-axis

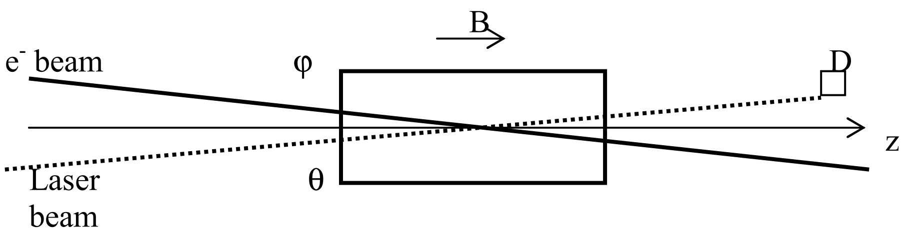

Fig.1

In a magnetic field the electrons have a discrete spectrum of energy:

$$\varepsilon_{n,\zeta} = [m^2 + P_z^2 + eB(2n+1+\zeta)]^{1/2} \qquad (1)$$

From (1) follows, that the electrons in a magnetic field can be on equidistant energy levels (Fig.2) and photons of frequency ω can be resonantly absorbed at transitions between electron energy levels. Using the spectrum of electron energy (1) and the law of energy-momentum conservation for photon absorption:

$$\varepsilon_{n,\zeta} + \omega = \varepsilon_{n',\zeta}, \quad P_{z,0} + \omega = P_z, \qquad (2)$$

we can find the condition for resonant electron transition:

$$\omega\left[\gamma_0(1 - V_{z,0}\cos\theta) + \frac{\omega}{m}\frac{(\sin\theta)^2}{2}\right] = \omega_c(n' - n), \qquad (3)$$

where: $\omega_c = eB/m$, $V_{z,0} = V_0\cos\varphi$, $n' - n = 1$, $\zeta = \zeta'$

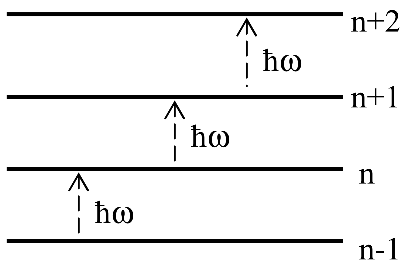

Fig. 2

For $\hbar\omega << \varepsilon$, the resonance condition can be written as:

$$\gamma_0 - \cos\varphi\cos\theta\sqrt{\gamma_0^2 - 1} = \frac{\omega_c}{\omega}, \qquad (4)$$

$\gamma_0 = \varepsilon_0/m$ is the electron relativistic factor

Solution of (4): $\gamma_0 = \dfrac{\Omega \pm \cos\varphi\cos\theta\sqrt{\Omega^2 - 1 + (\cos\varphi\cos\theta)^2}}{1-(\cos\varphi\cos\theta)^2}$, (5) $\Omega = \omega_c/\omega$

The dependence of the γ-factor from Ω for some fixed angles φ and θ is represented schematically on Fig.3.

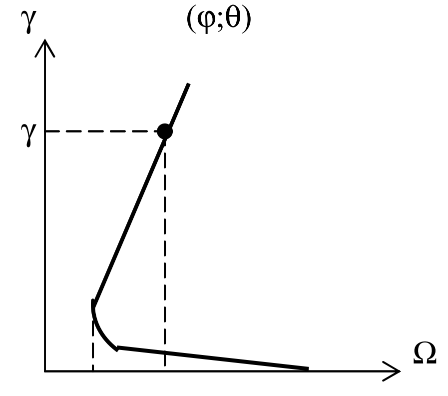

Fig. 3

-In case of one electron, if the angles φ and θ are known, then having measured a resonant frequency Ω, due to a unique dependence between γ and Ω, we can calculate γ-factor of electron according to the formula (5).

-Electrons of beam always have some spread on angles in limits $\varphi_i \le \varphi \le \varphi_f$ and laser light always diverges in limits of diffraction angles, therefore the precise measurement of absolute value of γ_0 immediately according to (5) is difficult.

Below we consider the possibility of determination of electron bunch energy using relation (5), taking in account the spread in angles of electrons and influence of light diffraction.

DETERMNATION OF ELECTRON BEAM ENERGY TAKING IN ACCOUNT THE SPREAD IN ANGLES OF ELECTRONS AND LIGHT DIFFRACTION

-The light beam of diameter D, because of diffraction diverges in limits of angle $\theta_d \cong \lambda/D$ around of direction of wave vector $\vec{k}$ of incident wave.

-Distribution of light intensity depending on angle of diffraction θ is determined by expression:

$$I(\theta) = I_0\left[\frac{2J_1(\alpha)}{\alpha}\right]^2, \quad (6) \qquad \alpha = Dk\theta/2., \qquad 0 \le |\theta| \le \lambda/D$$

and has the shape schematically represented on Fig.4.

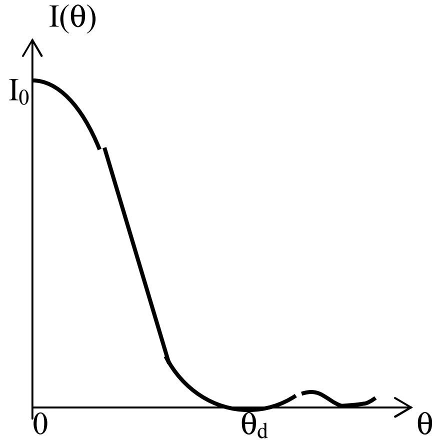

Fig. 4

From (5) follows that in a case of $\gamma >> 1$ and if $\Omega^2 >> 1-(\cos\varphi\cos\theta)^2$ the electron energy can be approximately determined by formula:

$$\gamma \cong \frac{2\Omega}{\varphi^2+\theta^2}, \qquad (7) \qquad \varphi^2+\theta^2 = \frac{2\Omega}{\gamma} = const \qquad (8)$$

On the graph with axes (φ, θ) the relation (8) represents the equation of a circle with radius $\sqrt{2\Omega/\gamma}$ (Fig.5).

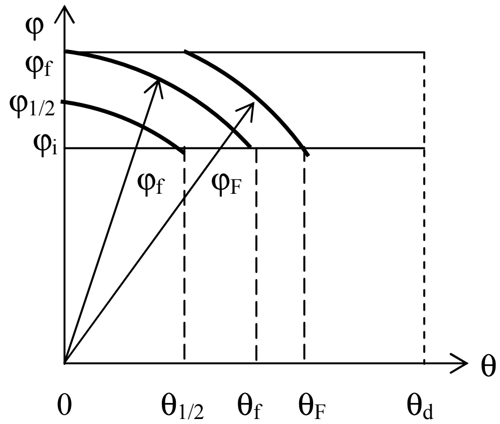

Fig. 5

-From (8) (and Fig.5) follows, that the intensity of a resonant absorption I_{abs} is minimum for electrons with angles $\varphi=\varphi_i$ and $\theta = 0$ at $\Omega_{\min} = \Omega_i = \gamma\varphi_i^2/2$ (Fig.6, Fig.7).

-From Fig.5 and Fig.6 it is obvious, that if $\Omega < \Omega_i$ then $I_{abs} = 0$

-Intensity reaches up to maximum at $\varphi = \varphi_f$ and $\Omega = \Omega_f = \gamma\varphi_f^2/2$ (Fig.5, Fig.7)

-At $\Omega > \Omega_f$ the intensity I_{abs} decreases because the interval of angles $\Delta\theta$ and therefore the intensity of light interacting with electrons decrease (Fig.5, Fig.7)

-It is clear from Fig.7, that on axis Ω the point $\Omega = \Omega_f$ is singled out and can be used for determination of electrons energy.

-If the electron beam interacts with light (see Fig.5) in limits of angles $0 < \theta < \theta_f$, then the diffraction divergence of this part of light will be smaller by factor of $\theta_f/\theta_d << 1$.

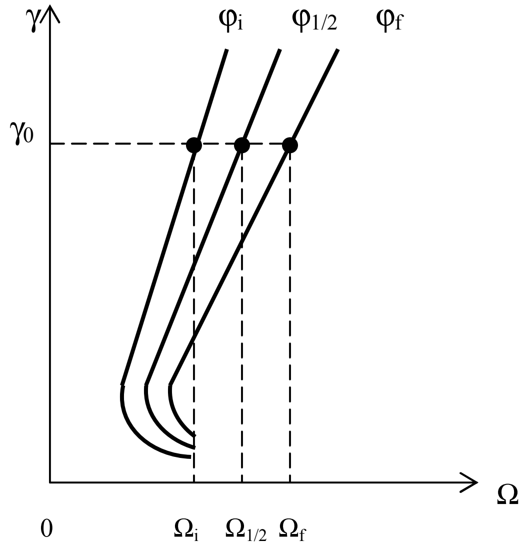

Fig.6

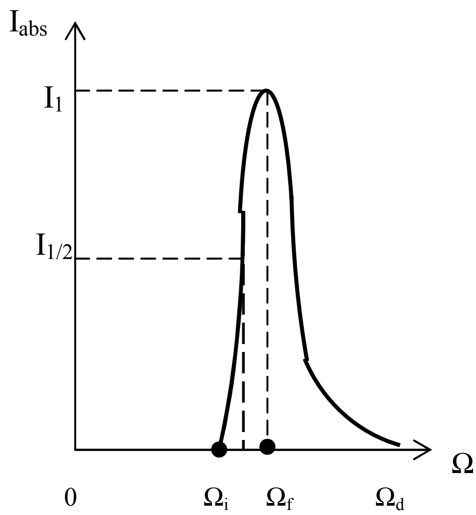

Fig.7

The first method: Determination of a relative energy of electrons.

-Assume that the γ-factor of electron beam γ_1 is known,

-The resonant frequency $\Omega_{f,1}$, which corresponds to the maximum of resonance absorption intensity $I_{abs}(\Omega_{f,1})$ for this γ-factor of electron beam (Fig.8), is measured

-we can calculate according to the formula (7) the angle: $\varphi_f = \sqrt{2\Omega_{f,1}/\gamma_1}$.

-Further we suppose, that for an unknown γ_2 the angular distribution of beam electrons does not vary and the maximum of resonant absorption intensity $I_{abs}(\Omega_{f,2})$ is fall on the same angle $\varphi_f = \sqrt{2\Omega_{f,2}/\gamma_2}$.

-the frequency $\Omega_{f,2}$ (see Fig.8) is measured

-we can calculate the value of unknown γ_2:

$$\gamma_2 = \frac{\Omega_{f,2}}{\Omega_{f,1}}\gamma_1 \tag{9}$$

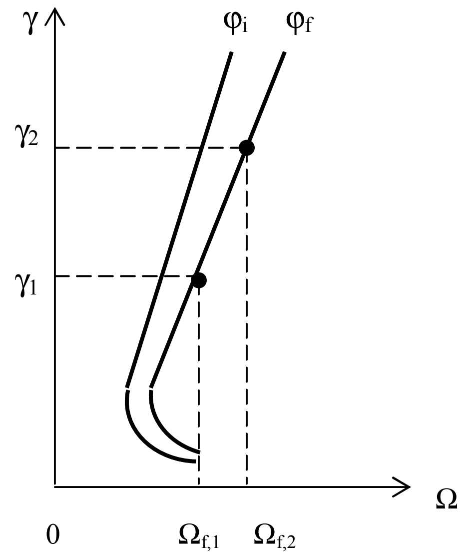

Fig. 8

The second method: Determination of a absolute energy of electrons.

a)-Let's assume, that we can turn the electron beam on some small angle $\delta\varphi = \varphi_F - \varphi_f$, known for us (Fig.9).

-resonant frequencies Ω_F and Ω_f, which correspond to the maximum of resonance absorptions intensity at angles $\varphi_F = \sqrt{2\Omega_F/\gamma}$ and $\varphi_f = \sqrt{2\Omega_f/\gamma}$, are measured (Fig.10)

-we can calculate the absolute value of γ factor of electrons:

$$\gamma = \frac{1}{2}\left(\frac{\sqrt{\Omega_F} - \sqrt{\Omega_f}}{\delta\varphi}\right)^2 \qquad (10)$$

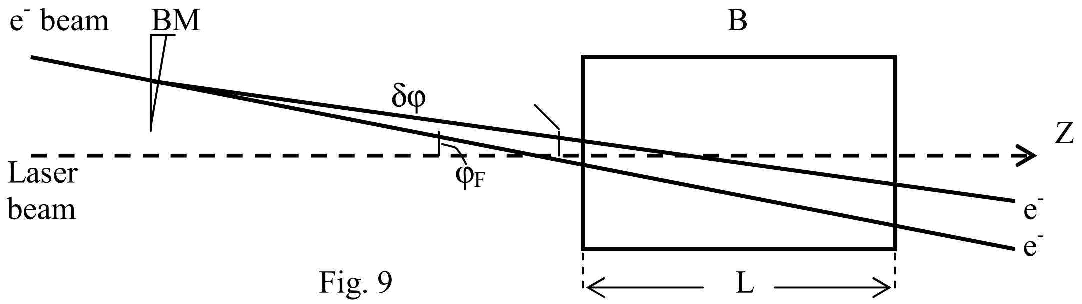

Fig. 9

b)-In particular it is possible to use a bending magnet (BM) for turn of electron beam on some small angle $\delta\varphi = \varphi_F - \varphi_f$ (Fig.9), which is determined by the formula:

$$\delta\varphi = \frac{eH}{mc\gamma}\frac{l}{c}, \qquad (11)$$

-resonant frequencies Ω_F and Ω_f, which correspond to a maximum of resonance absorptions intensity at angles $\varphi_F = \sqrt{2\Omega_F/\gamma}$ and $\varphi_f = \sqrt{2\Omega_f/\gamma}$ (Fig.10, Fig.11, Fig12), we have the relation:

$$\delta\varphi = \varphi_F - \varphi_f = \sqrt{\frac{2}{\gamma}}\left(\sqrt{\Omega_F} - \sqrt{\Omega_f}\right) = \frac{eH}{mc\gamma}\frac{l}{c} \qquad (12)$$

-From (12) we can calculate the absolute value of γ factor of electron:

$$\gamma = \frac{1}{2}\left(\frac{eH}{mc}\frac{l}{c}\frac{1}{(\sqrt{\Omega_F} - \sqrt{\Omega_f})}\right)^2. \qquad (13)$$

-At both methods the precise measurement of a resonant value Ω_f is difficult because of some width of resonance of absorption intensity $I_{abs}(\Omega)$ (Fig.7). However, prove to be, that by tuning of the laser intensity parameter $\xi = eE/m\omega c$ (or length of a magnet L) it is possible to increase the accuracy of Ω_f determination

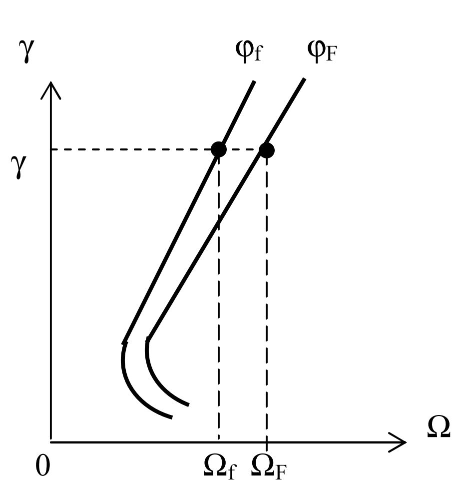

Fig. 10

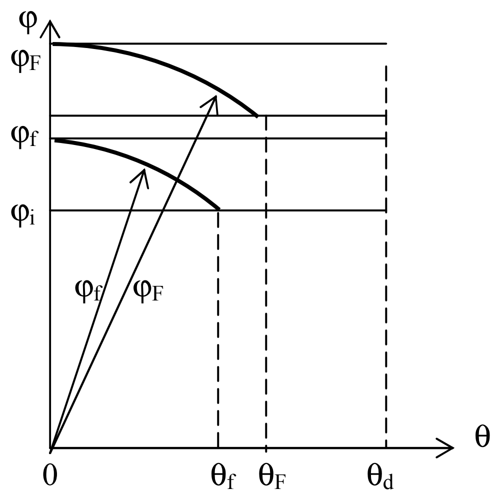

Fig. 11

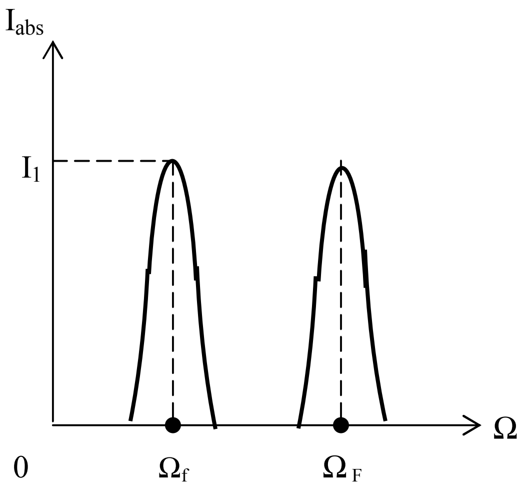

Fig. 12

ESTIMATION OF INTENSITY OF A RESONANT ABSORPTION

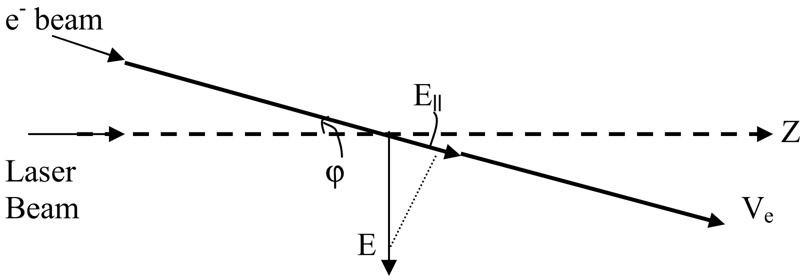

Fig. 13

-If the laser beam intersects the electron beam under angle φ, then electrons can accelerated in a direction of velocity $\overrightarrow{V_e}$ under action of a component $E_{\text{II}} = \sin\varphi E \cong \varphi E$ and growth of electron energy will be $eEL\varphi$.

-At acceleration the electrons can absorb only integer photons ν, therefore:

$$\nu\hbar\omega = eE\varphi L = mc^2\xi\varphi L\frac{\omega}{c}, \qquad \text{or} \qquad \nu = \frac{mc}{\hbar}\xi\varphi L \cong \frac{L[cm]10^{12}}{38{,}61}\xi\varphi \ .$$

If electron absorb 1 photon, then we have relation:

$$\frac{mc}{\hbar}\xi\varphi L \geq 1$$

If $\xi < \xi_{1/2}$ then:

-electrons with angles in interval $\varphi_i < \varphi < \varphi_{1/2}$ will not absorb photons (Fig.15),

-electrons with angles $\varphi > \varphi_{1/2}$, will absorb (Fig.15).

-for $\Omega < \Omega_{1/2}$ there is not resonant absorption of photons (see Fig.15, Fig.16).

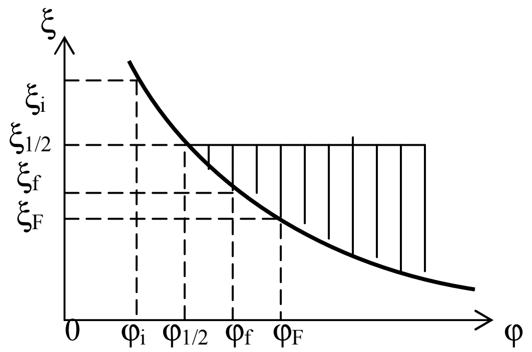

Fig. 14

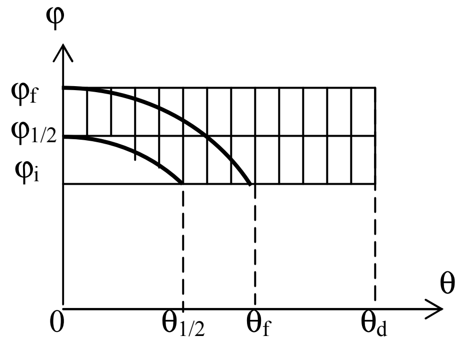

Fig.15. $\xi < \xi_{1/2}$

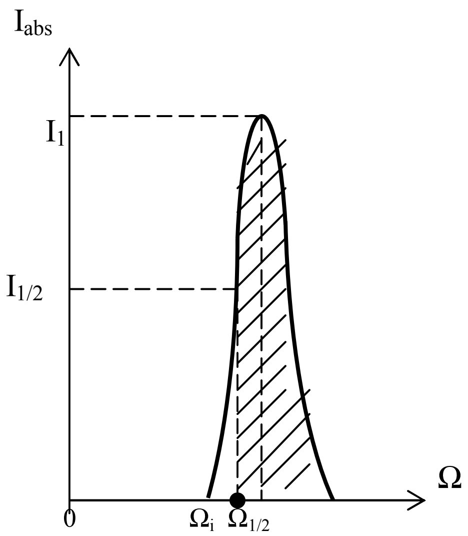

Fig.16. $\xi \leq \xi_{1/2}$

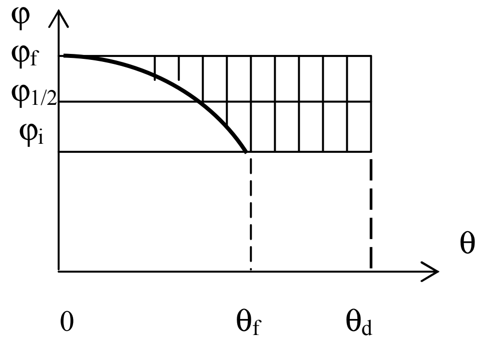

Fig.17. $\xi \leq \xi_f$

If $\xi \leq \xi_f$ and $\xi \leq \xi_F$ (Fig.14) then electrons will be absorb photons only at realization of requirements $\Omega > \Omega_f$ and $\Omega > \Omega_F$ (Fig.17, Fig.18 and Fig.19, Fig.20).

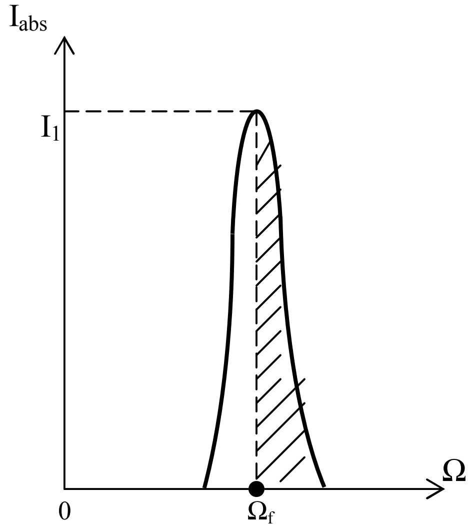

Fig.18. $\xi \le \xi_f$.

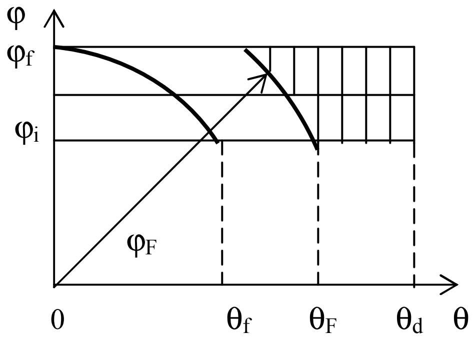

Fig.19. $\xi \le \xi_F$

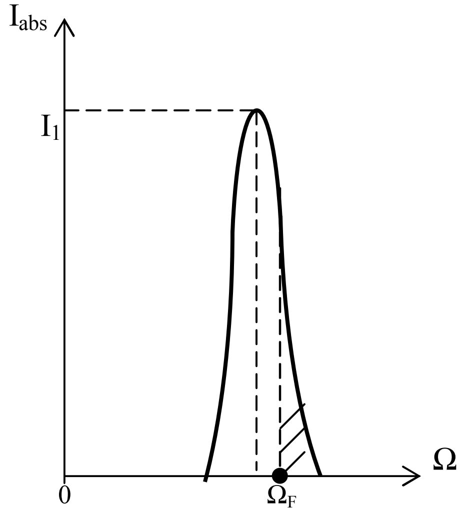

Fig.20. $\xi \le \xi_F$

-So, from Fig.18 and Fig.20 we see, that for $\xi < \xi_f$ and $\xi < \xi_F$ the intensity of absorption have sharp left-hand edge and this property of dependence of $I_{abs}(\Omega)$ allows increasing the accuracy of determination of Ω and therefore the accuracy of determination of γ.

-We can estimate number of photons N_{abs} absorbed by electron beam of length l_b taking into account that $l_b >> L$ and, that each electron transits the region of interaction L only once.

To observe the resonant absorption we suppose, that in (15) the parameters ξ, φ and L are chosen so that $\nu \ge 1$.

-If the number of electrons in the beam is N_e, then $N_{abs} = N_e$.

-If the parameters ξ and ω are known, then we have $eE = \xi m\omega c$ and the minimum power of the laser can be found by a relation:

$$W\left[W/cm^2\right] \cong \left(\frac{E\left[V/cm\right]}{19{,}4}\right)^2$$

-Now, using the known value of W we can find the number $N_{l,ph}$ of laser photons, incident on a surface S during time $T_b = l_b / c$:

$$N_{l,ph} = \frac{W}{\hbar\omega} S T_b$$

-Resonance absorption can be observed by measuring the ratio of the absorbed photons N_{abs} to total number of photons, incident on area of interaction $N_{l,ph}$ or (I_{abs}/I_{tot}).

-Resonance absorption can be observed by measuring the ratio $N_{abs} / N_{l,ph}$ or (I_{abs}/I_{tot}).

For $\gamma = 10^5$, $\lambda = 10{,}6$ micrometer (CO_2 laser) and $\varphi = 2 \cdot 10^{-4}$, $\theta = 0$: $B = 20.2$ kGs.

For $L = 50$ cm and $T_b = 0.64$ msec (TESLA): $\xi \cong 3.8 \cdot 10^{-9}$ and $W \cong 0.36 W/cm^2$.

If $N_e = 4 \cdot 10^{13}$ e/pulse and $S = 0.25 cm^2$, then: $N_{l,ph} \cong 3 \cdot 10^{15}$ photon/pulse and $N_{abs}/N_{l,ph} \cong 1.3 \cdot 10^{-2}$.

-So, near to a resonance, the ratio $N_{abs}/N_{l,ph}$ has a reasonable value for measuring.