An electron/positron energy monitor based on synchrotron radiation.

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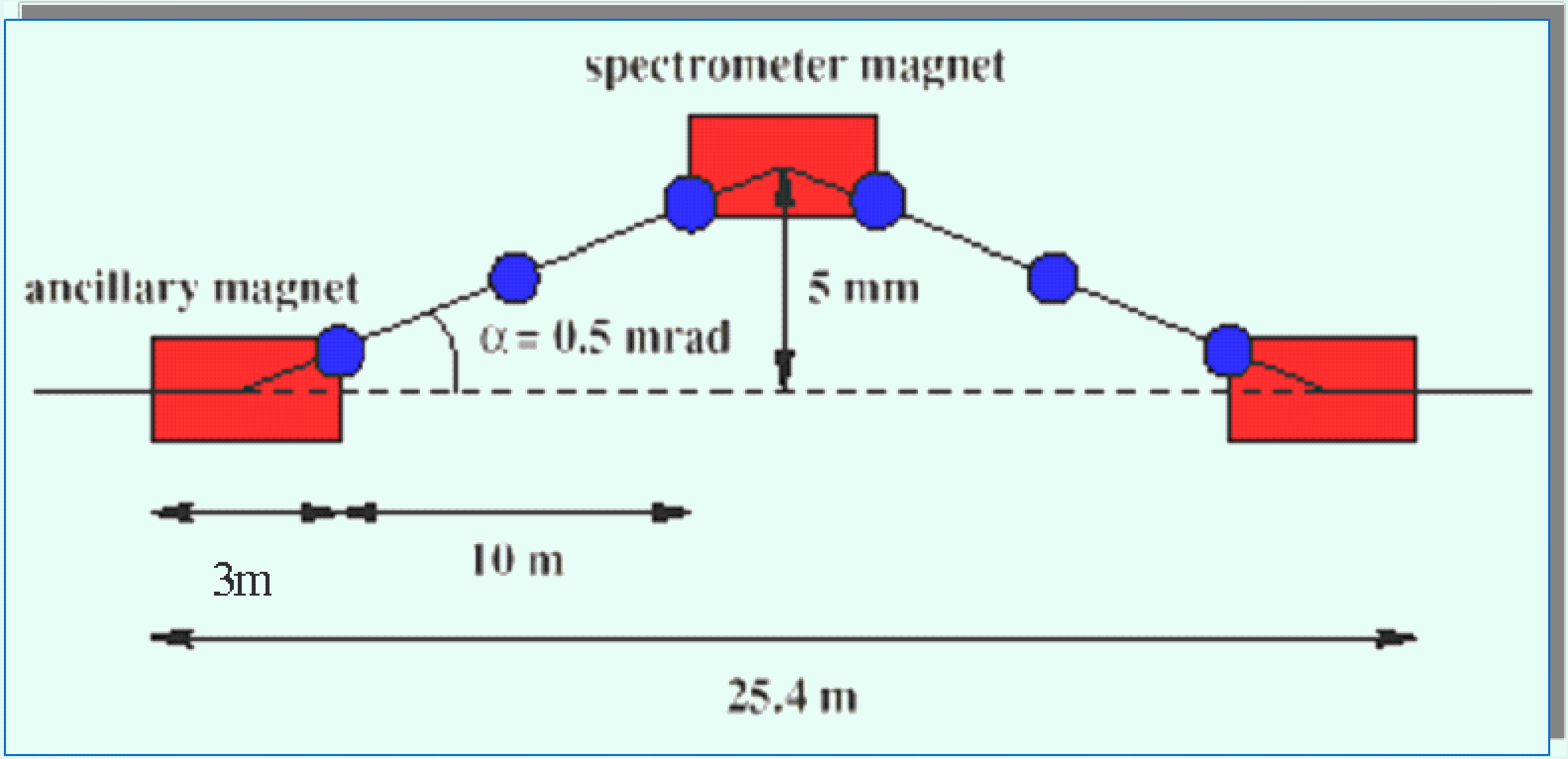
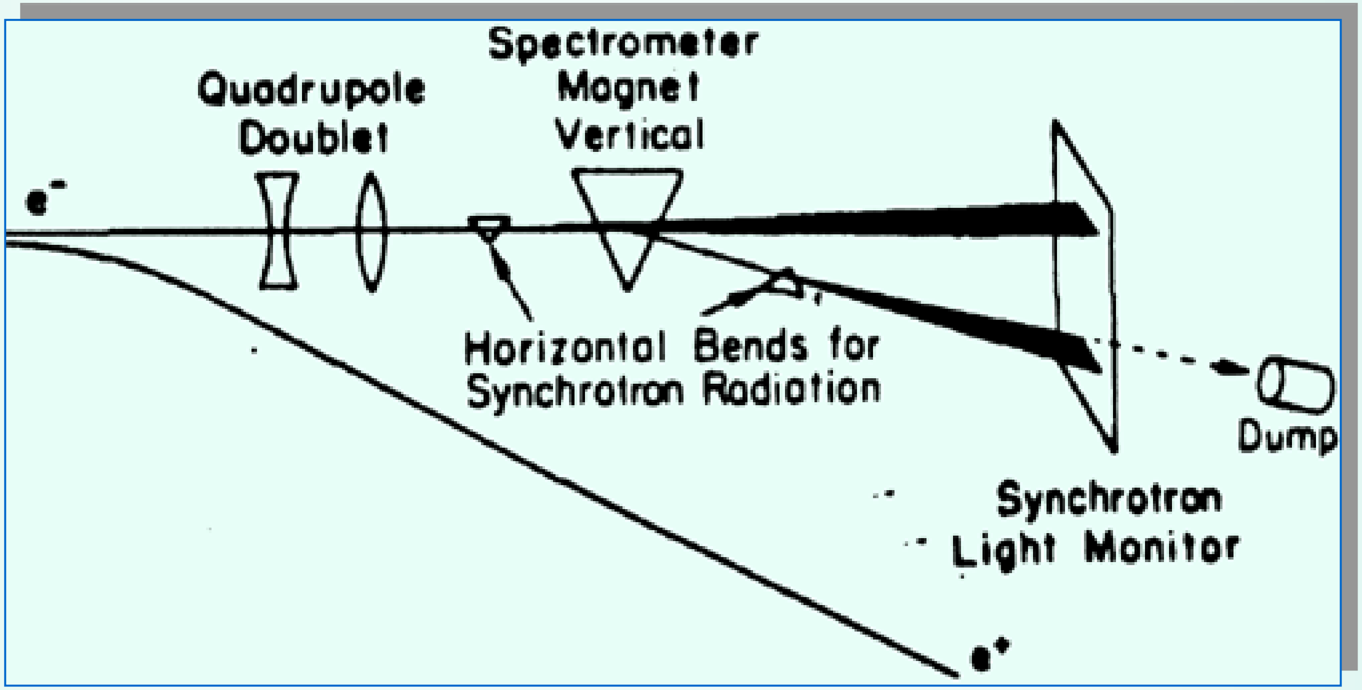
Magnetic spectrometer with an energy resolution of a few 10^{-4} is proposed for TESLA energy calibration experiment



Scheme of magnetic spectrometer.

The energy spectrometer based on synchrotron radiation was used in SLAC for precision measurements of the SLC beam energy.

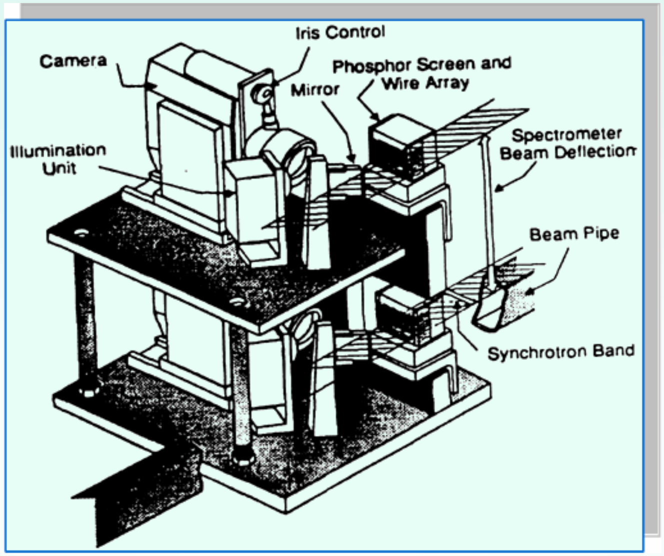


Conceptual design of the extraction-line SLC spectrometers

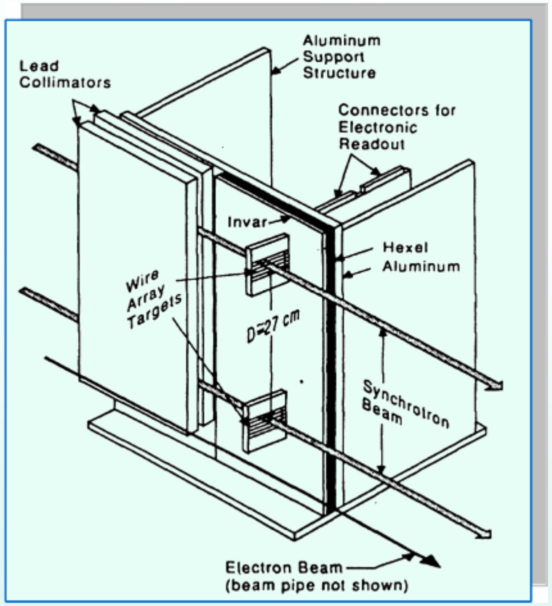
Basic spectrometers magnet parameters

	SLC	LEP	CEBAF	TESLA (Proposal)
Energy E (GeV)	42 – 50	40 – 100	0.5 – 7	45 – 400
Absolute accuracy of energy measurement ΔE/E	5×10^{-4}	1×10^{-4}	1×10^{-4}	1×10^{-4} - 1×10^{-5}
Bending angle (mrad)	18.286	3.75		1
Magnetic field range (T)	0.88 – 1.1	0.086 – 0.216	0.04 – 0.6	0.05 – 0.44
Magnetic field integral (T•m)	2.56 – 3.05	0.5 – 1.242	0.12 – 1.8	0.3 – 1.33
Magnetic measurement error of the field integral (relative)	7×10^{-5}	3×10^{-5}	1×10^{-5}	3×10^{-5}
Magnet iron length (m)	2.5	5.75	3	3
Effective magnet length (m)				3.045
Gap height (mm)	31.7	100	25.4	35
Magnet type	H	C	C	C
Laboratory ∫B•dl measurement technique	Mowing wire, mowing probe (NMR, Hall)	Mowing probe (NMR, Hall), search coil	NMR probe, 2 search coils	Should be estimated
Operational ∫B•dl measurement technique	Flip coil, fixed probes (NMR)	Fixed probes (NMR)		Should be estimated
Energy loss due to synchrotron radiation (max) (MeV)		3.55		120

SLC Precision Synchrotron radiation detectors



The SLC phosphorescent screen monitor.



The SLC wire imaging synchrotron radiation detector

Magnetic spectrometer parameters

$\Delta = \frac{R\alpha^2}{2}$ $\approx$ **0.9 mm - the electron displacement in the dipole magnet**

$\alpha = \frac{\ell}{R}$ $\approx$ **0.6 mrad - the electron deflection angle,**

R=5 km is curvature radius of electron orbit,

l **=3 m - the magnet length.**

B = 0.3 T - the magnetic field,

$\gamma = 8 \cdot 10^5$ **- relativistic factor**

$$R_{[cm]} = \frac{1{,}7 \cdot 10^3 \gamma}{\mathrm{B}(G)}$$

The total electron beam displacement

$\Delta_{tot} \cong R\alpha^2 + L\alpha \cong$ **8 mm,**

$L \cong 10$ **m is the distance between ancillary and spectrometer magnets.**

Synchrotron Radiation parameters

$$\theta_{SR} \cong \frac{1}{\gamma} \cong 1$$ **μrad - the synchrotron radiation divergence angle;**

L_{SR}=R/γ ≈ 5 mm is the length of the shining electron trajectory in the spectrometer magnet.

The SR from other part of electron trajectory is collimated;

$$\delta \cong L_{S-d}\theta_{SR} \cong \frac{L_{S-d}}{\gamma}$$ **- the SR spot diameter in the detector,**

L_{S-d} - spectrometer–detector distance.

$$\lambda_{cr} = \frac{2\pi R}{\gamma^3} \cong 4 \cdot 10^{-4}\,\text{Å}$$ **- the critical SR wave length;**

$$\varepsilon_{cr\,[keV]} = \frac{2.2\varepsilon^3[GeV]}{R[m]} \approx$$ **28 MeV – the critical SR photon energy;**

$$\Delta\varepsilon_{SR[MeV]} = 1{,}26\,10^{-3} l_{mag[m]} \cdot \mathrm{B}_{[T]}^{2} \cdot \varepsilon_{[GeV]}^{2} \approx 44\,MeV$$

- the electron energy loses ;

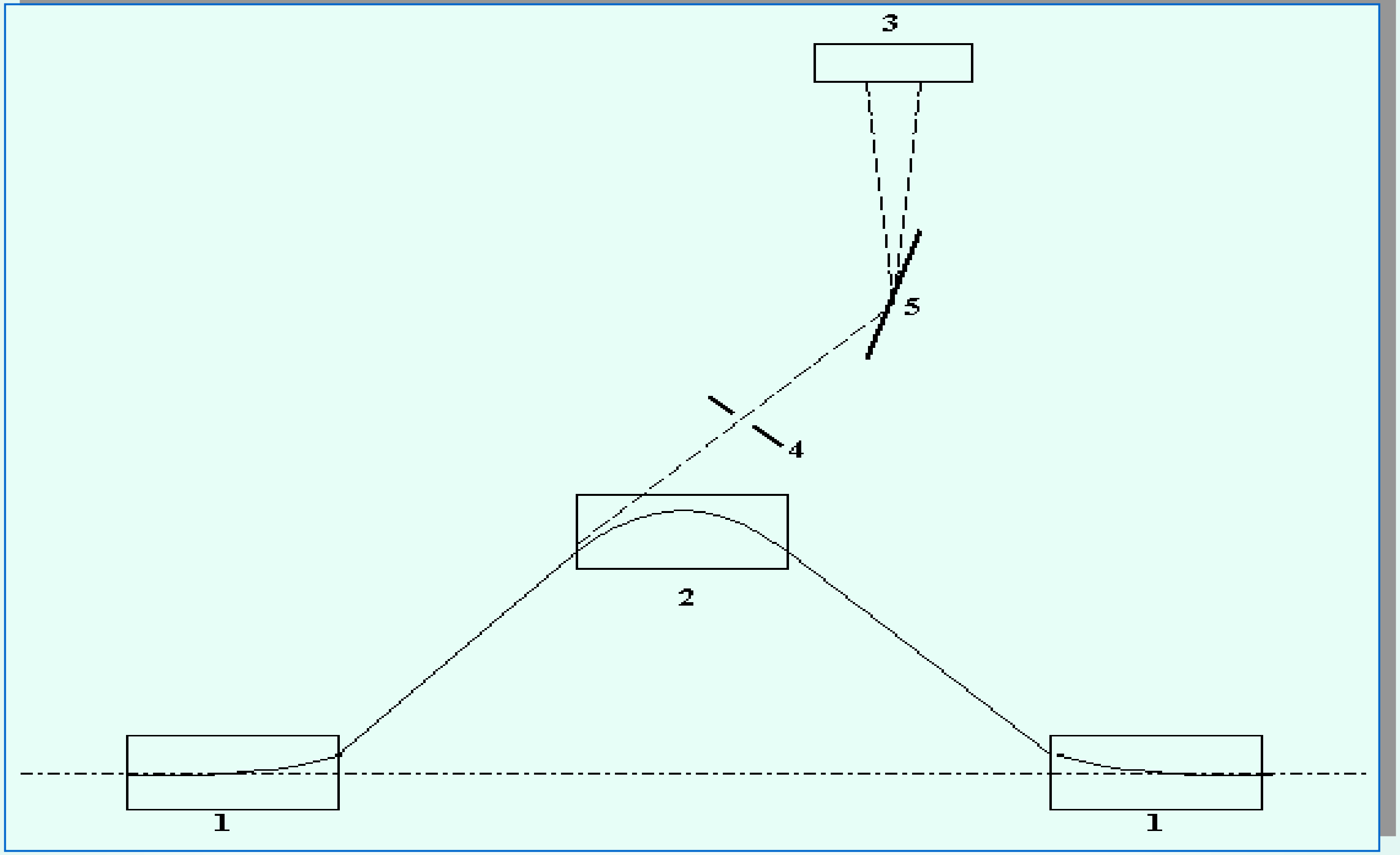
$$n_{\gamma} \cong \frac{\Delta\varepsilon_{SR}}{\varepsilon_{cr}} \cong 1.5$$ **- the number of photons radiated by one electron.**

Electron energy resolution in magnetic spectrometer

$$\frac{\Delta\varepsilon}{\varepsilon} \cong \frac{n_{\gamma}^{1/2}\varepsilon_{cr}}{\varepsilon} \cong \frac{\sqrt{\Delta\varepsilon_{SR}\varepsilon_{cr}}}{\varepsilon} \cong 10^{-4}$$

- the fluctuation of average electron energy (the energy spread, straggling) at SR radiation.

SR at low photon energy



Scheme of SR spectrum measurements

ancillary magnet, 2- spectrometer magnet, 3- SR- detector, 4- vertical slit, 5- crystal monochromator.

The critical SR energies from ancillary and sp

Vertical angle SR distribution

$$\theta_{sr} \approx 0.5 \cdot \left(\frac{\lambda}{R} \right)^{1/3}$$ ≈ **200 μrad at λ≈5000 A and R=5 km.**

$$\frac{I_s}{I_a} = \frac{B_s \cdot \psi_s \cdot S\left(\frac{\varepsilon}{\varepsilon_{cr_s}} \right)}{B_a \cdot \psi_a \cdot S\left(\frac{\varepsilon}{\varepsilon_{cr_a}} \right)}$$

where ψ_a and ψ_s are horizontal angles at which is measured SR from ancillary and spectrometer magnets, S is normalized function for total SR power density

$$S = \begin{cases} \frac{27\sqrt{2}}{16\sqrt{3\pi}} \sqrt{\frac{\varepsilon}{\varepsilon_{cr}}} \cdot \exp\left(-\frac{\varepsilon}{\varepsilon_{cr}} \right) & \varepsilon >> \varepsilon_{cr} \\ \frac{4}{3} \left(\frac{\varepsilon}{\varepsilon_{cr}} \right)^{1/3} & \varepsilon << \varepsilon_{cr}. \end{cases}$$

SR radiation at low photon energy

At a small photon energy (ε ≈ 1-10 keV, εcr ≈ 30 MeV) of

$$\varepsilon/\varepsilon_{cr_s} << 1 \,, \quad \varepsilon/\varepsilon_{cr_s} << 1$$

the ratio of SR fluxes from ancillary and spectrometer magnets is equal to

$$\frac{I_s}{I_a} = \frac{B_s \cdot \psi_s \cdot \left(\frac{\varepsilon}{\varepsilon_{cr_s}}\right)^{1/3}}{B_a \cdot \psi_a \cdot \left(\frac{\varepsilon}{\varepsilon_{cr_a}}\right)^{1/3}} = \frac{\psi_s}{\psi_a} \cdot \left(\frac{B_s}{B_a}\right)^{2/3}$$

The NMR magnetic field measurements with a relativistic accuracy of $\Delta B/B \approx 10^{-5}$ **and SR flux measurements with accuracy of** $\Delta I/I \approx 10^{-4}$ **permit to get information about accuracy of horizontal angles**

$$\frac{\Delta\psi_s}{\psi_s} \approx \frac{\Delta\psi_a}{\psi_a} \approx 10^{-4}$$

The accurate measurements of horizontal angles let us to get information about deflection angle resolution

$$\frac{\Delta\alpha}{\alpha} \approx \frac{\Delta\psi}{\psi} \approx 10^{-4}$$

The measurement of FWHM of SR spot

$$\theta_{sr} \approx 0.5 \cdot \left(\frac{\lambda}{R}\right)^{1/3}$$ ≈ **15 μrad at λ≈1.2 A (E=10 keV) and R=5 km.**

Spot size of kev SR

$$d = L \cdot \theta_{sr}$$ ≈ **3 mm at L=200 m**

Electron energy resolution

$$\frac{\Delta E_e}{E} = \frac{\Delta\lambda}{\lambda} + 3\frac{\Delta d}{d} + 3\frac{\Delta L}{L}$$

$$\frac{\Delta\lambda}{\lambda} \approx 10^{-4}$$ **photon wave length resolution with filter and monochromator,**

$$\frac{\Delta d}{d} \approx 3 \cdot 10^{-5}$$

$$\frac{\Delta d}{d} \cong \frac{1}{N_\gamma^{1/2}}$$

$N_\gamma \approx 10^9$ photons.

The number of $\varepsilon \approx$ 10 keV photons counted by detector

$$N_\gamma \approx N_{\gamma hard} \cdot \left(\frac{\varepsilon}{\varepsilon_{cr}}\right)^{1/3} \cdot \frac{\Delta\varepsilon}{\varepsilon} \approx 10^2 \text{ ph/bunch}$$

at $\Delta\varepsilon / \varepsilon \approx 10^{-4}$, $N_{\gamma hard} \approx 3 \cdot 10^7$ ph/bunch

Number of bunches

$$n_{bunch} \approx 10^7$$

Time of measurements

$\tau \approx$ 1-3 msec

Information about average electron energy

Electron energy resolution in SR detector placed on short distance from spectrometer

The SLC SR detector the space resolution of one detector channel is of

$$d \cong 30 \text{ µm}$$

Short distance between spectrometer and detector $L_{S\text{-}d}$

$$L_{S-d} = \gamma d \cong 25 \text{ m}.$$

The SR spot size on detector is about

$$\delta \cong L_{s-d}\theta_{SR} \approx 20 \text{ µm}$$

SR spot size is comparable with detector channel size

$$\delta \approx d$$

The detector energy resolution is determinate by the space resolution of detector channel *d* and deflection angle in the magnetic spectrometer α

$$\frac{\Delta\varepsilon}{\varepsilon} \cong \frac{\Delta\alpha}{\alpha} \cong \frac{d}{L_{S-d}\alpha} \cong \frac{1}{\gamma\alpha} \cong 2\cdot 10^{-3}$$

at $\gamma \cong 8\cdot 10^5$ **and** $\alpha = 0{,}6$ **µrad**

Center gravity measurements of SR signal

Large magnetic spectrometer - SR detector distance

$$L_{S-d} \cong 200 \text{ m.}$$

The SR spot size

$$\delta \cong \frac{L_{S-d}}{\gamma} \cong 200 \ \mu\text{m}$$

$$d \cong 30 \ \mu\text{m}$$

The SR detector consists of 10 channels to measure the dependence of SR distribution on horizontal coordinate in detector

$$\delta \approx 10d$$

The number of hard SR photons ($\varepsilon \approx \varepsilon_{cr} \approx$ 30 MeV) counted by detector

$$N_{\gamma\,hard} \cong \frac{\Delta\varepsilon_{SR}}{\varepsilon_{cr}} \cdot \frac{1}{\gamma\alpha} \cdot N_e \cong \frac{N_e}{\gamma\alpha} \cong 3 \cdot 10^7 \text{ ph./bunch}$$

where $N_e = 10^{10}$ **is the electron number per bunch.**

The number of $\varepsilon \approx$ 10 keV photons counted by detector

$$N_\gamma \approx N_{\gamma hard} \cdot \left(\frac{\varepsilon}{\varepsilon_{cr}}\right)^{1/3} \cdot \frac{\Delta\varepsilon}{\varepsilon} \approx 10^6 \text{ ph/bunch}$$

at $\Delta\varepsilon \approx \varepsilon \approx$ 10 keV.

The fluctuation of SR intensity in each detector channel

$$\frac{\Delta I}{I} \approx \frac{1}{N_\gamma^{1/2}} \cong 10^{-3}$$

Integral measurements of 1-10 keV SR fluxes

$$N_\gamma \approx N_{\gamma hard} \cdot \left(\frac{\varepsilon}{\varepsilon_{cr}} \right)^{1/3} \cdot \frac{\Delta\varepsilon}{\varepsilon} \approx 10^6 \text{ ph/bunch}$$

at $\Delta\varepsilon \approx \varepsilon \approx$ 10 keV.

The fluctuation of SR intensity in each detector channel

$$\frac{\Delta \mathrm{I}}{\mathrm{I}} \approx \frac{1}{\mathrm{N}_\gamma^{1/2}} \cong 10^{-3}$$

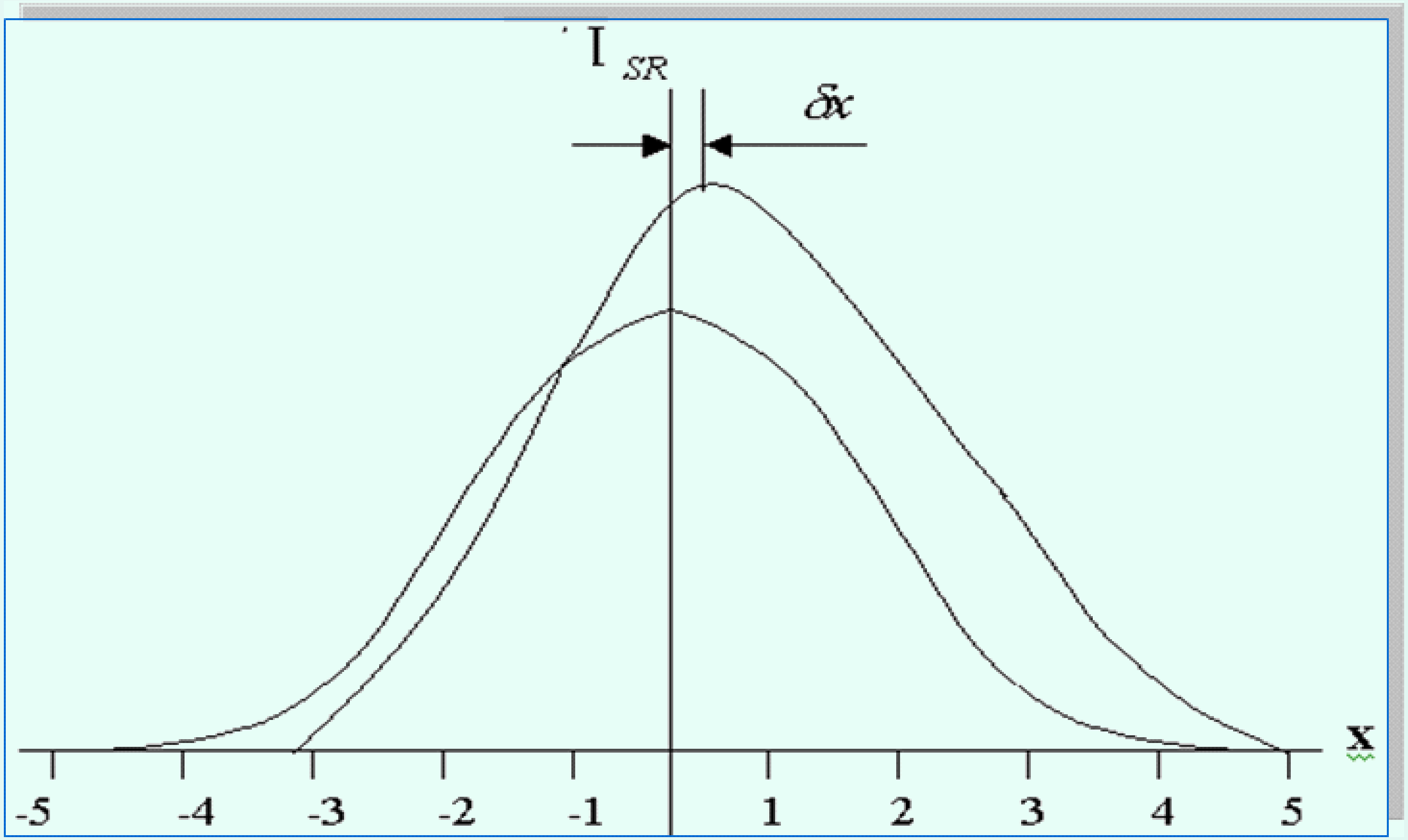
Variation of electron energy produces a variation of SR flux

$$\frac{\Delta I}{I} \approx \frac{\Delta E_e}{E}$$

$$\frac{\Delta I}{I} \approx \frac{\Delta E_e}{E} \quad \frac{\Delta \mathrm{I}}{\mathrm{I}} \approx \frac{1}{\mathrm{N}_\gamma^{1/2}} \cong 10^{-3}$$

SR signal center gravity measurements

An electron energy variation of produces a center gravity displacement for SR horizontal coordinate distribution.



Horizontal distribution of SR in 10 channel detector

The SR center gravity measurements

$$\delta x \cong \frac{4\sigma}{N^{1/2}{}_{\gamma}} \approx 4 \cdot \frac{L_{S-d}}{\gamma N^{1/2}_{\gamma}} \cong 1 \text{ µm}$$

where $L_{S-d} / \gamma \approx 200$ **µm,** $\mathrm{N}_{\gamma} \approx 10^{6}$ **photons/bunch at photon energy of 10 keV.**

SR detector energy resolution at center gravity measurements

The SR center gravity measurement with an accuracy of

$$\delta x \approx 1 \text{ µm}$$

permits one to get an energy resolution of

$$\frac{\delta\varepsilon}{\varepsilon} \approx \frac{\Delta\alpha}{\alpha} \approx \frac{\delta x}{L_{S-d} \cdot \alpha} \cong 1 \cdot 10^{-5}$$

The energy resolution in detector is restricted by a stability of the spectrometer magnetic field and electron energy fluctuation in main spectrometer.

The electron energy resolution is of

$$\frac{\delta\varepsilon}{\varepsilon} \approx 1\cdot 10^{-4}$$

for a spectrometer magnetic field stability of

$$\frac{\Delta \mathrm{B}}{\mathrm{B}} \approx 3\cdot 10^{-5}$$

The electron energy spread produced at SR radiation in the spectrometer magnet restricts the energy resolution

$$\frac{\Delta\varepsilon}{\varepsilon} \cong \frac{n_{\gamma}^{1/2}\varepsilon_{cr}}{\varepsilon} \cong \frac{\sqrt{\Delta\varepsilon_{SR}\varepsilon_{cr}}}{\varepsilon} \cong 10^{-4}$$

Detector

I. A semiconductor strip detector can be used for detection of 10 keV SR at

strip width of 10 μm;

distance between strips of 10 μm;

number of strip channels of 30;

strip thickness of 10 mm;

square root space resolution of 3μm;

total number of 10 keV detected photons of 10^6;

photon intensity per strip of 10^5.

II. The absorption photon energy is measured for each strip and it is written in a dynamic memory FIFO type.

III. Finally the three dimensions spectrum is measured as a function of the number of strips, the photon absorption energy in the strip and the time interval between bunches.

The center gravity of absorption energy distribution for all strips is measured for each electron bunch (for each time interval of 300 ns). The absorption photon energy for each strip corresponds to $E_{str} = N_{\gamma} E_{\gamma} \cong 10^9 \, eV$.

The energy required for production of one electron – hall pair in detector semiconductor is $E_e \cong 3{,}6 \; eV$.

The number of electrons produced in the central strips is estimated as $N_e \cong N_{\gamma} E_{\gamma} / E_e \cong 3 \cdot 10^8$.

This number of electrons produces a 1 V signal on photo-multiplier output for 50 Ohm cable. The noise input for this signal is small. It means the semiconductor strip detector does not restrict the center gravity resolution. The resolution is determinate only by the photon statistic and detector electronics. The amplitude analysis of the events for each bunch is realized, as example, in the CERN CMS project at a time interval of 125 ns.

Conclusion

The application of a 10 channel SR detector with 25 space resolution per channel permits one to reach an energy resolution of $\Delta\varepsilon/\varepsilon \approx 10^{-4}$.

The control about average electron energy during time of $\tau \approx$ 1-3 msec can be realized by SR detector based on SR spectrum measu-rements.