Scaling of Askaryan Pulses
@
ARENA

17 May, 2005

D. Seckel, Univ. of Delaware

Topics

- Review RF pulses from showers
- Scaling of universal shape
- Application to AVZ
- Hadronic showers
- Different media
- Phases & pulse shapes
- Other issues
- Summary

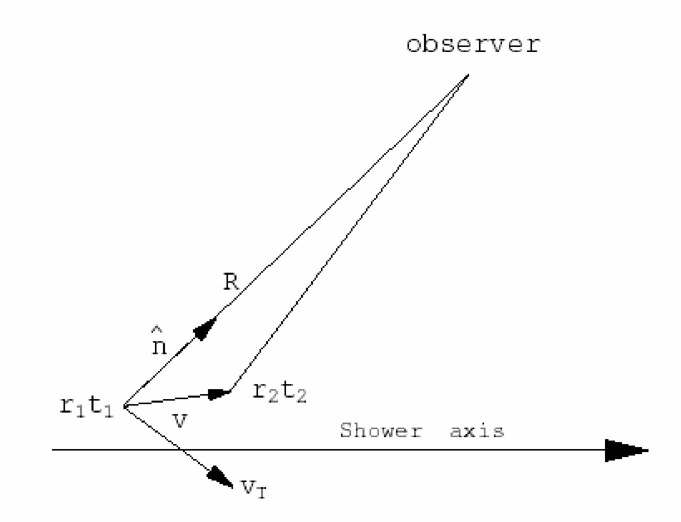
Radiation from single particles

$$R\vec{E}_\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R\vec{E}(t) e^{i\omega t} \mathrm{d}t \;;$$

$$R\vec{E}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R\vec{E}_\omega e^{-i\omega t} \mathrm{d}\omega \;.$$

ZHS: 2

0.5



$$\vec{E}(\vec{x},t) = \frac{q}{c}\left[\frac{\hat{n}\times\{(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\}}{(1-\vec{\beta}\cdot\hat{n})^3 R}\right]_{\mathrm{ret}}$$

$$R\vec{E}_\omega(\vec{x}) = \frac{1}{\sqrt{2\pi}}\left(\frac{q}{c}\right) e^{ikR} e^{i\omega(t_1-\hat{n}\cdot\vec{r}_1/c)} \vec{\beta}_\perp \frac{(e^{i\omega\delta t(1-\hat{n}\cdot\vec{\beta})}-1)}{1-\hat{n}\cdot\vec{\beta}}$$

Sum over particles/Integrate over shower

$$\hat{n} \times (\hat{n} \times \vec{\beta}) = -\vec{\beta}_\perp$$

$$R\vec{E}_\omega(\vec{x}) = \frac{1}{\sqrt{2\pi}}\left(\frac{q}{c}\right) e^{ikR} e^{i\omega(t_1 - \hat{n}\cdot\vec{r}_1/c)}\, \vec{\beta}_\perp \frac{\left(e^{i\omega\delta t(1-\hat{n}\cdot\vec{\beta})} - 1\right)}{1 - \hat{n}\cdot\vec{\beta}}$$

$$i\omega t$$

$$\vec{E} = \sum_i \vec{E}_i \Rightarrow i\,\nu\,\mathrm{Sin}[\theta] \int d^3x\, dt\, \rho(\vec{x}, t)\, e^{i\,\phi(\theta, \vec{x})}$$

Shower Front + Light Cone

$$E(\theta, \nu) = i\,\nu\,\mathrm{Sin}[\theta] \int dy\, dz\, \rho(y, z)\, e^{i\,\phi(\nu,\theta,y,z)}$$

Density of Excess
Projected Track Length

Separation of Variables & Form Factor

$$E(\theta, \nu) = i\,\nu\,\mathrm{Sin}[\theta] \int d y\, d z\, \rho(y, z)\, e^{i\,\phi(\nu,\theta,y,z)}$$

$$\rho(y, z) = f_0\, f_y(y)\, f_z(z)$$

$$\phi(\theta, y, z) = \frac{2\pi\nu}{c}\left(z\left(1 - \frac{\cos\theta}{\beta}\right) + y\,\frac{\sin\theta}{\beta}\right) = \phi_z + \phi_y$$

$$E(\theta, \nu) = i\,\nu\, f_0\, \mathrm{Sin}(\theta_c)\, G_y(\theta, \nu)\, G_z(\theta, \nu)$$

$$G_y(\theta, \nu) = \int d y\, f_y(y)\, e^{i\phi_y(\nu,\theta,y,z)}$$

$$G_z(\theta, \nu) = \int d z\, f_z(z)\, e^{i\phi_z(\nu,\theta,y,z)}$$

Phase Factors and Rescaling FF

$$\phi_z = \frac{2\pi\nu}{c}\, z\,(\sin\delta\tan\theta_c + (1-\cos\delta)) \rightarrow \frac{2\pi\nu}{c}\, z\,\delta\tan\theta_c$$

$$\phi_y = \frac{2\pi\nu}{c}\, y\,(\tan\theta_c\cos\delta + \sin\delta) \rightarrow \frac{2\pi\nu}{c}\, y\tan\theta_c$$

Most MC's use this

$$\begin{aligned} G_y(\theta,\nu) &= \int dy\, f_y(y)\, e^{i\phi_y(\nu,\theta,y,z)} \\ &= \int dy\, f_y(y)\, e^{i a_y(\theta)\nu y}; \qquad a_y(\theta) = \frac{2\pi}{c}(\tan\theta_c\cos\delta + \sin\delta) \\ &= G_y(0, a_y\nu) \equiv G_y(a_y\nu) \end{aligned}$$

$$G_z(\theta,\nu) = G_z(a_z\nu); \qquad a_z(\theta) = \frac{2\pi}{c}(\sin\delta\tan\theta_c + (1-\cos\delta))$$

Normalization and Determination of f, G_y, G_z

$$E(\theta, \nu) = i\nu f_0 \,\mathrm{Sin}(\theta_c)\, G_y(a_y \nu)\, G_z(a_z \nu)$$

$$G_y(0) = 1, \;\; G_z(0) = 1$$

$$a_z(\delta = 0) = 0$$

$$f_0 = \operatorname*{Lim}_{\nu \to 0} \frac{|E(0,\nu)|}{\nu \mathrm{Sin}(\theta_c)}$$

$$G_y(\nu) = \frac{|E(0,\nu)|}{\nu f_0 \,\mathrm{Sin}(\theta_c)}$$

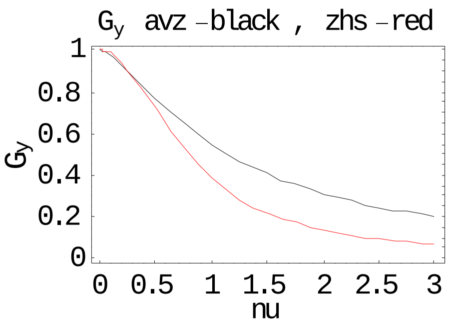
$$\left(G_z(a_z \nu) = \frac{-i E(\theta,\nu)}{\nu f_0 \,\mathrm{Sin}(\theta)\, G_y(a_y \nu)} \right)\Bigg|_{\theta}$$

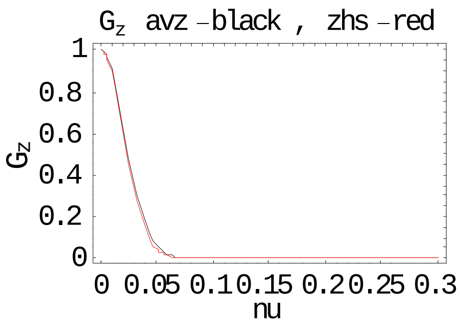
AVZ, ZHS, (G4 later)

$$E(\nu, 0, \text{ice}) = 2.53 \times 10^{-7} \frac{\nu}{\nu_0} \frac{1}{1.+\left(\frac{\nu}{\nu_0}\right)^{1.44}},$$

$$G_y(\nu) = \frac{1}{1.+\left(\frac{\nu}{\nu_0}\right)^{1.44}}, \quad f_0 = \frac{2.53 \times 10^{-7}}{\nu_0 \,\mathrm{Sin}[\theta_{c0}]} \text{ (units?).}$$

AVZ Use a Gaussian, and give the half-width of the Cerenkov cone at 1 GHz as 3.72 °for a 1 PeV (3.1 m) shower. Determine G_z as FT of gaussian





Scaling vs AVZ

Field calculation is integral over shower profile

$$E = \nu \, \mathrm{Sin}[\theta] \int d\,y\,d\,z\,\rho(y,z)\,e^{i\,\phi(\theta,y,z)}$$

Separation of shower profile

$$\rho(y,z) = f_0\,f_y(y)\,f_z(z)$$

Separation of phase factors

$$\phi(\theta,y,z) = \frac{2\pi\nu}{c}\left(z\left(1-\frac{\cos\theta}{\beta}\right)+y\,\frac{\sin\theta}{\beta}\right) = \phi_z + \phi_y$$

$$\phi_z = \frac{2\pi\nu}{c}\,z\,(\sin\delta\tan\theta_c + (1-\cos\delta))$$

$$\phi_y = \frac{2\pi\nu}{c}\,y\,(\tan\theta_c\cos\delta + \sin\delta)$$

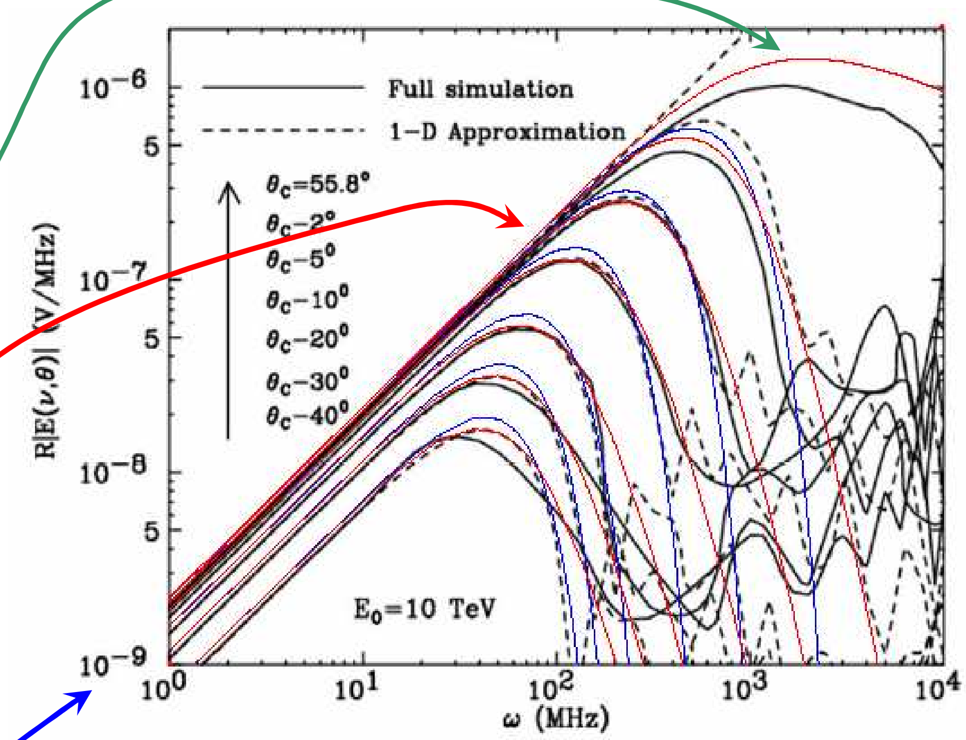
Separation of form factors

$$E(\nu,\delta) = f_0\,\nu\,\mathrm{Sin}[\theta]\,G_y(\nu_y)\,G_z(\nu_z)$$

With scaled frequencies

$$\nu_y = \nu\left(\mathrm{Cos}\,\delta + \frac{\mathrm{Sin}\,\delta}{\tan\theta_c}\right) \simeq \nu\left(1+\frac{\delta}{\tan\theta_c}\right)$$

$$\nu_z = \nu\,l\left(\mathrm{Sin}\,\delta + \frac{(1-\mathrm{Cos}\,\delta)}{\tan\theta_c}\right) \simeq \nu\,l\,\delta$$



- Adapted from Alvarez, Vazquez, Zas
- "Full sim" is approx a
- Blue – Gaussian for f(z), AVZ approx c for G_y
- Red – Griessen for f(z)

High frequency

- Large phases from individual particles…slow to average
- Initial particle track may be long, and be noticed for $E_s < 1$ TeV
- Not clear to me that AVZ form is not contaminated (form was derived at 10 TeV)

Phases

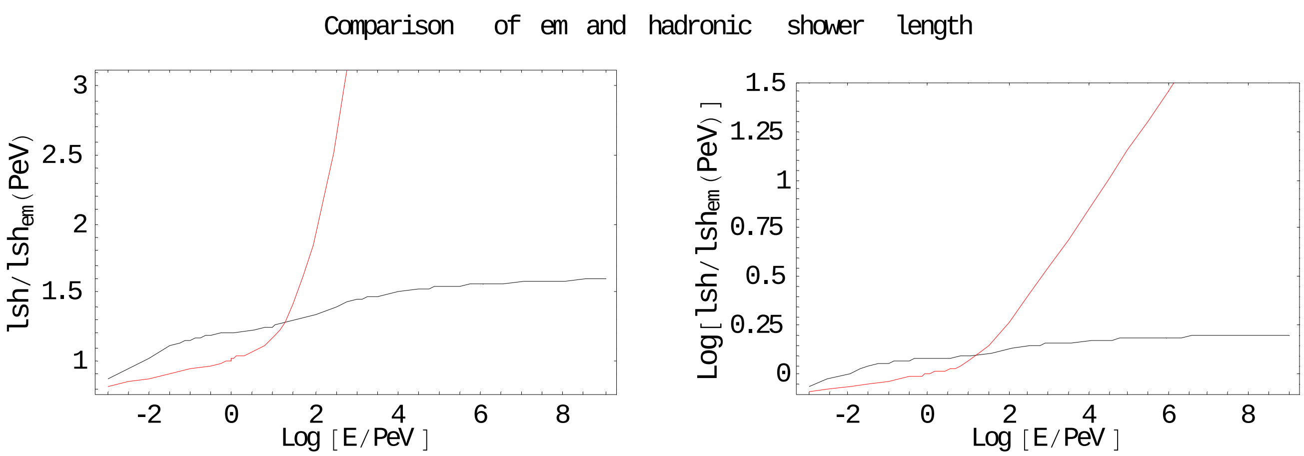
- later

Scaling with energy and type. (AZ)

$$f_0: \quad l_{tot} = \frac{E}{dE/dx}$$

$$G_y: \quad R_m \text{ doesn't change}$$

$$G_z: \quad l_{sh}(i, E)$$



1. LPM effect (Note: limited at ~10^{20} eV by γp interactions (BR,K))
2. At PeV proton showers are longer
3. Hadronic shower size asymptotes at about 93% of e-shower

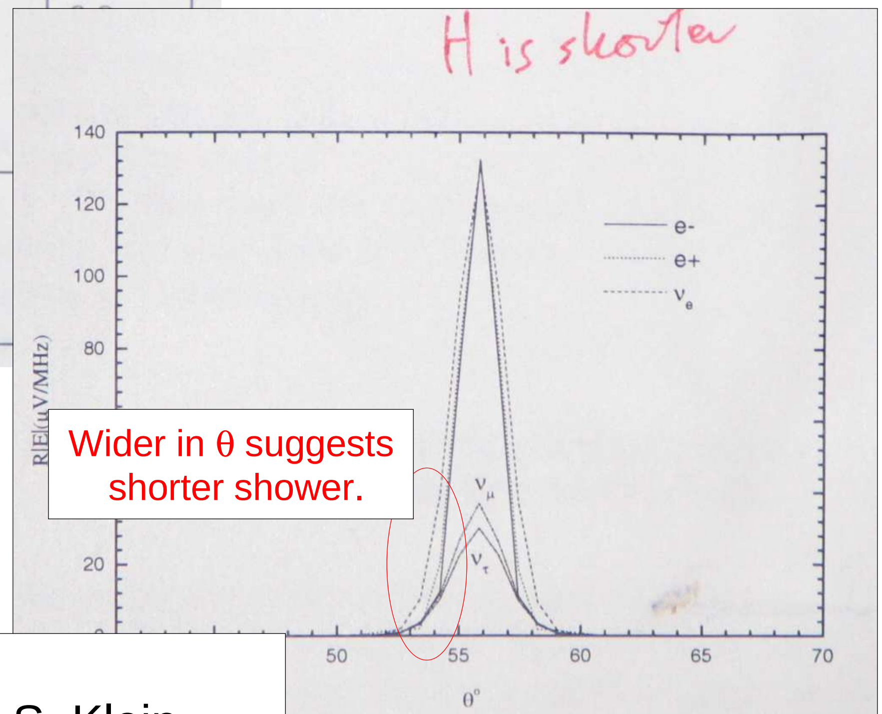
Hadronic showers II. (HM)

E	name	Δr_z	$R\|E\|1$	w	$b(w)*1e5$	$R\|E\|2$
10^2	e^-	$1.33E5$	13	0.90	9.9	13
90	π^-	$1.04E5$	9.2	0.85	8.8	9.1
10^2	π^+	$1.19E5$	11	0.86	9.0	11
90	p	90007	6.9	0.80	7.5	[illegible]
10^2	ν_e	$1.26E5$	12	0.87	9.3	[illegible]
10^2	ν_μ	39982	3.5	0.84	8.5	[illegible]
10^2	ν_τ	36800	3.0	0.82	8.0	[illegible]
10^3	e^-	$1.33E6$	132	0.90	9.9	[illegible]
	ν_e	$1.32E6$	131	0.89	9.7	[illegible]
	ν_μ	$4.08E5$	37	0.86	9.0	[illegible]
	ν_τ	$3.67E5$	30	0.82	8.1	[illegible]

Excess charge

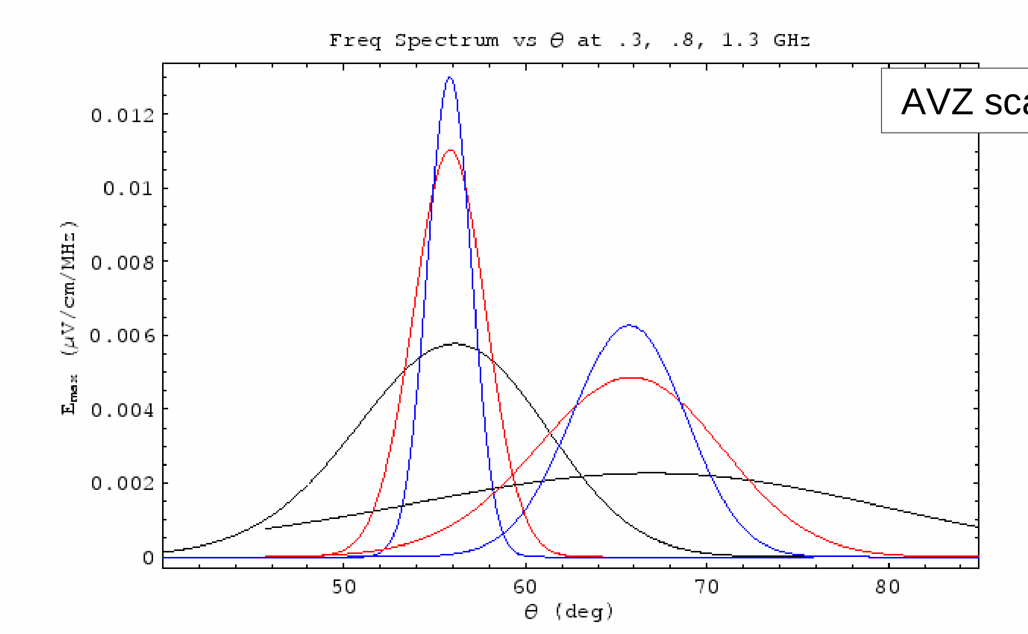
E-field @ 1 GHz

Contained charge @1/GHz



- Size similar to AZ
- Same R_M ? (Affects G_y) – also see S. Klein
- Difference between proton and jet (G_z)?
- What about charm (~10%)?

Scaling with medium



AVZ scaled

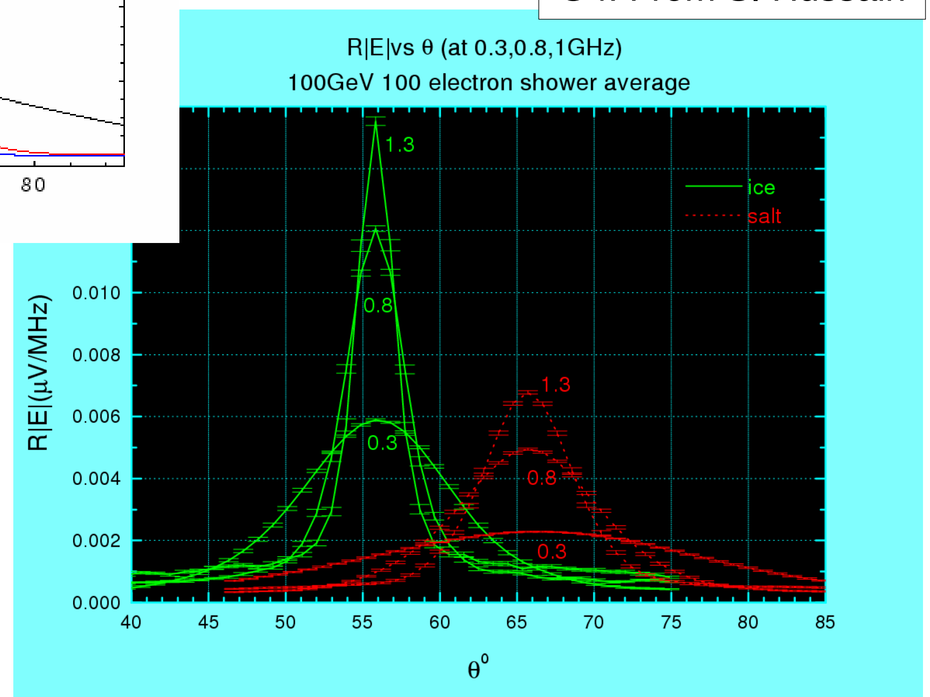
Yield of projected track length

$$f_0 : \quad l_{\rm tot} = \frac{\alpha_i E}{dE/dx}$$

$$G_y : \quad R_M \sim \frac{E_s}{\rho \, dE/dx}$$

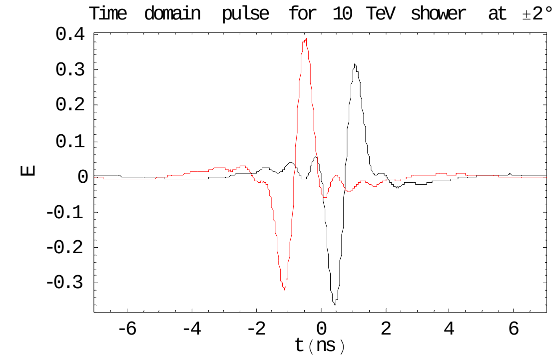
$$G_z : \quad l_{\rm sh} \sim x \sim \frac{X}{\rho}$$

G4: From S. Hussain

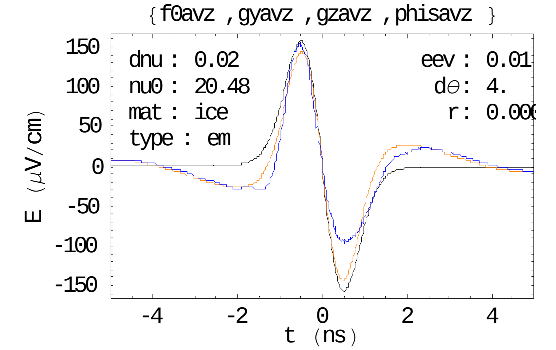


Phases & Pulse shapes

Hussain: G4 10 TeV shower, reconstructed complex $E(\omega)$ at $\theta = (-4,-2,0,2,4)$, FFT => E(t)



Comparison to AVZ scaled at θ=4. AVZ pulse shape symmetric bipolar. Comparison shown with and Without filter

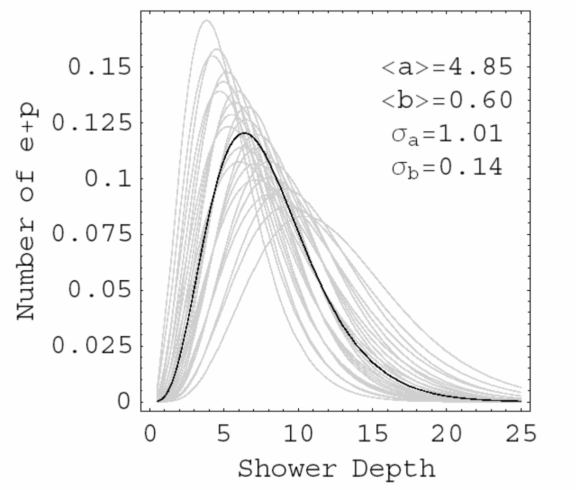


If $\rho(t)$ symmetric, G_z is real, $\phi=0$. $i\nu G_z$ will transform to time derivative of ρ will be bipolar symmetric. If not, then $\phi\neq0$, asymmetric shape. Changes sign with θ.

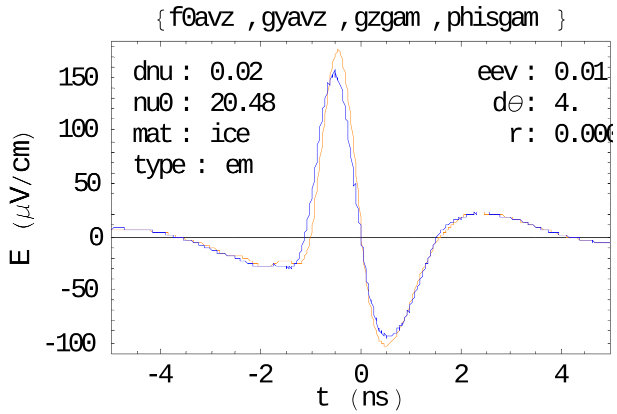
Gamma is a better pulse shape

- Without phases AVZ effectively gaussian shower profile.
- Gamma function is approximation to shower profile. $\rho_z(t) = N\, t^{a-1}\, e^{-bt}$
- Causal, analytic phase, easy to scale.

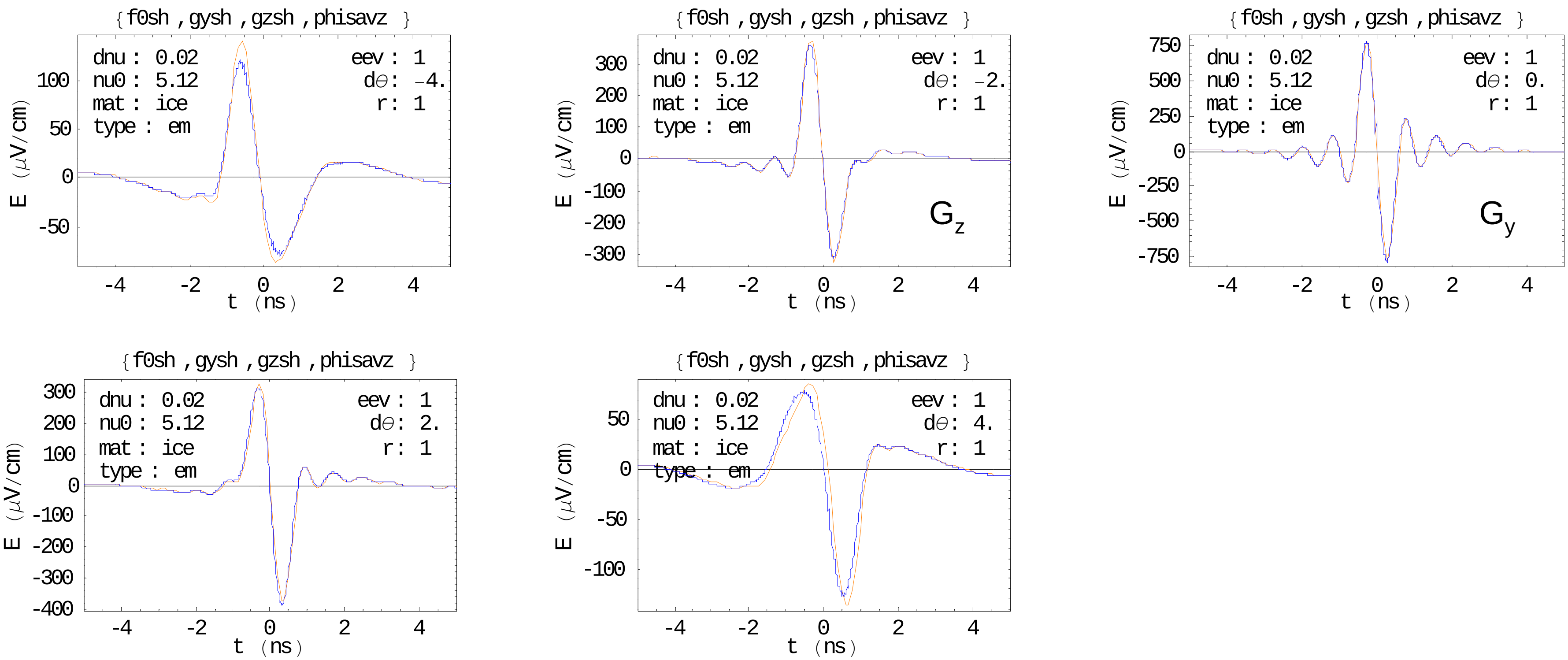
$$g(\omega) = \int_0^\infty dt\, e^{i\omega t}\, N t^{a-1}\, e^{-bt}$$
$$= \left(\frac{1}{1-i\,\frac{\omega}{b}}\right)^a$$
$$= \left(\frac{b}{\sqrt{b^2+\omega^2}}\right)^a e^{ia\,\mathrm{ArcTan}(\frac{\omega}{b})}$$



- Comparison to G4



Determine G_y, G_z from G4 shower & test



Spectrum at $\theta = (-4,-2,0,2,4)$ from Hussein.

Use $\theta = 0$ to get f_0, G_y

Use $\theta = -2$ to get G_z

$\theta = (-4,2,4)$ from scaling, with phase part of G_z

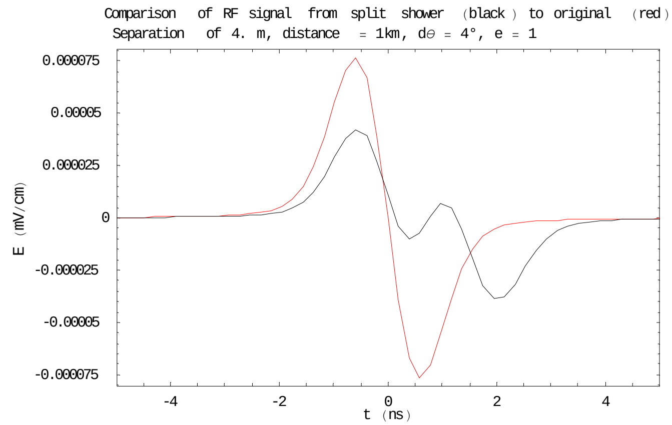
Multiple showers & fluctuations

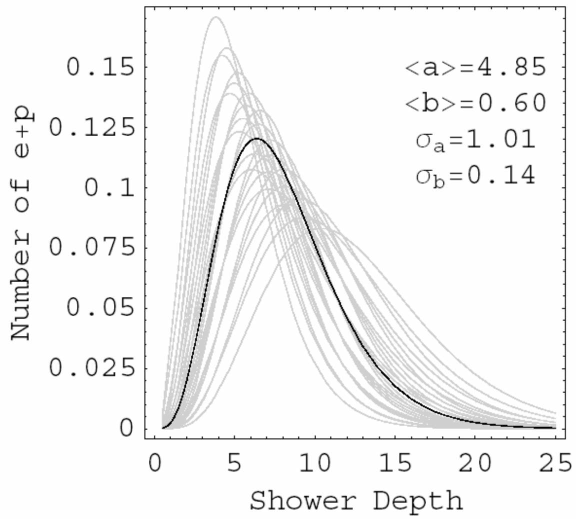
- ν_e CC events

- Multiple interactions

- Model fluctuations by adding subshowers from beginning of shower ?

- Fluctuations from distribution a,b ?





Acoustic (in progress)

- Same as radio…. except
  - scalar vs vector
  - total track length
  - v/c is smaller
  - Attenuation is different
  - experiment bandwidth

Summary

- Scaling of RF pulse to universal shape describes much previous and ongoing work.
- Hadronic showers need another look.
- Different media can be understood
- Better shower shape & treatment of phases needed
- Multiple showers, …