The $B^*B\pi$ Coupling

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Definition and Relevance of $g_{B^*B\pi}$

- It's essentially the matrix element for the **strong decay** $B^* \to B\pi$:

$$\langle B^0(p)\pi^+(q)|B^{*+}(p')\rangle \equiv -g_{B^*B\pi}(q^2)q_\mu \eta^\mu(p')(2\pi)^4\delta(p'-p-q)$$

- This B-decay is **forbidden** since: $m_{B^*} < m_B + m_\pi$.
  But for D^* mesons $D^* \to D\pi$ is the dominant decay mode,
  and $g_c \approx 0.61(7)$ [CLEO, 2002].

- Related to coupling g of **heavy-light meson χPT** (HMχ)
  (the only LEC at leading order in $1/M$):

$$g \propto \lim_{m_b \to \infty, m_d \to 0} g_{B^*B\pi}$$

  Constrains chiral behaviour of e.g. f_B, B_B, $B \to \pi l \nu$ form factor.
  Talks by N. Garron, B. Blossier on ALPHA HQET programme.

Lattice Calculations via a Form Factor

- States like $|B\pi\rangle$ are a problem on the lattice (**Maiani-Testa**).
- Here, **LSZ reduction** of the pion [de Divitiis et al. (UKQCD) 1998] ($\sim$ Goldberger-Treiman) relates $g_{B^*B\pi}(q^2)$ to:

  $$\langle B^0(p)|A_\mu|B^{*+}(p+q)\rangle = \eta_\mu F_1(q^2) + (\eta\cdot q)(2p+q)_\mu F_2(q^2) + (\eta\cdot q)q_\mu F_3(q^2)$$

  In the static and chiral limits, we only need $F_1(0)$.
- Primarily want leading-order HMχ coupling, so we do a **static calculation** (like most previous attempts). P. Fritzsch's talk discussed **RHQ approach**.
- Technical challenge of calculating lattice **3-point functions**: statistics, excited state contamination...

3pt Function with Summed Insertion

- 3pt. fn.s have **2 time separations** t' and $t-t'$ to make large:

$$C_3(t,t';q,p) = \langle \mathcal{O}_q(t)\mathcal{O}(t')\mathcal{O}_p^\dagger(0)\rangle$$

e.g. taking $C_3(t,t/2;p,p)$ gives corrections $O(e^{-(t/2)\Delta E})$.

- Defining the **summed correlator** [Maiani et al. 1987]:

$$D(t;q,p) \equiv a\sum_{t'=0}^{t} C_3(t,t';q,p)$$

The effective matrix element:

$$\partial_t \frac{D(t;q,p)}{\sqrt{C_2(t;p)C_2(t;q)}}$$

gives corrections $O(te^{-t\Delta E})$.

Static Action and All-to-All Light Propagator

- Use **HYP1, HYP2 static actions** to avoid exponential decay of signal-to-noise in Euclidean time [Della Morte et al. (ALPHA) 2005]. Also nice to see universality, some idea of discretization effects.
- Noisy estimation of **all-to-all** light propagator: U(1) noise, full time-dilution.
- Use of **sequential propagator** for the light quark. Gives the insertion summation (incuding outside timeslices):

$$D'(t;q,p) \equiv a \sum_{t'} C_3(t,t';q,p)$$

Must invert again for each operator insertion (here all 3 A_i).

Creation Operators

- To reduce excited state contamination to tolerable level, must **smear light quark fields** used in creation operators.
- Using **Gaussian smearing** with APE-smeared covariant Laplacian:

$$\psi_l^{(k)}(x) = \left(1 + \kappa_G a^2 \Delta\right)^{R_k} \psi_l(x)$$

we construct a basis of 8 wavefunctions of varying size:

$$r_{phys,k} \approx 2\sqrt{\kappa_G R_k} a$$

varying r_{phys} in the range 0 - $\sim$0.7 fm.

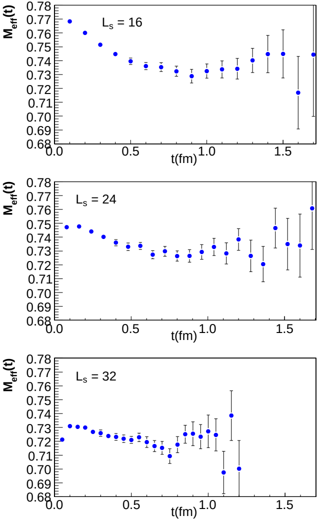
- Ultimately we will perform a **GEVP analysis**.
For now we just pick out the best smearing
(discussion and examples in the PRELIM. $N_f = 2$ results section).

Quenched Test

- We did a **precision test** of the method, with **continuum limit**, using the ALPHA quenched HQET lattices and s quark.
- 3 spacings, fixed physical volume $L \approx 1.5$ fm, and $T = 2L$:

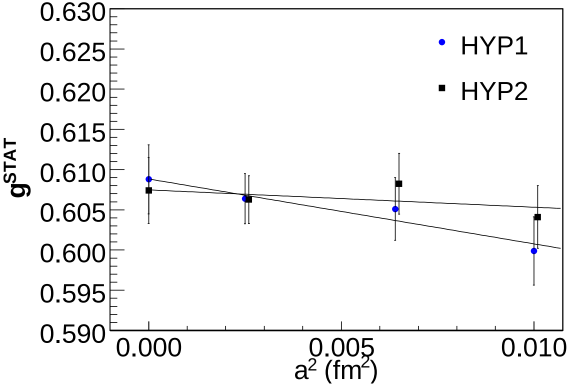
	N_{conf}	N_{noise}
$16^3 \times 32$, $a \approx 0.10$ fm	100	200
$24^3 \times 48$, $a \approx 0.08$ fm	100	48
$32^3 \times 64$, $a \approx 0.05$ fm	100	32

Quenched Test: Precision and Plateaus



Quenched Test: Continuum Limit

- No discernible spacing dependence at this (0.5%) level. Fitting **linearly in $\mathbf{a^2}$**: HYP1: 0.6088(43) HYP2: 0.6078(41)



CLS Lattices

- **Configurations** generated, shared within **CLS community effort**: Berlin, CERN, Zeuthen, Madrid, Mainz, Rome, Valencia, Wuppertal
- **Action**: $N_f = 2$ Wilson, non-perturbatively $O(a)-$improved.
- **Algorithm**: DD-HMC [Lüscher, 2005]
- Used the following CLS datasets so far:

	Name	L	m_π (MeV)	$m_\pi L$	N_{conf}	N_{noise}
$\beta = 5.2$, $a \approx 0.08$ fm	A4	32	≈365	4.8	160	8
$\beta = 5.3$, $a \approx 0.07$ fm	E4	32	≈560	6.2	157	16
	E5	32	≈420	4.7	400	4
	F6	48	≈290	5.0	80	2
	F7	48	≈250	4.2	301	4

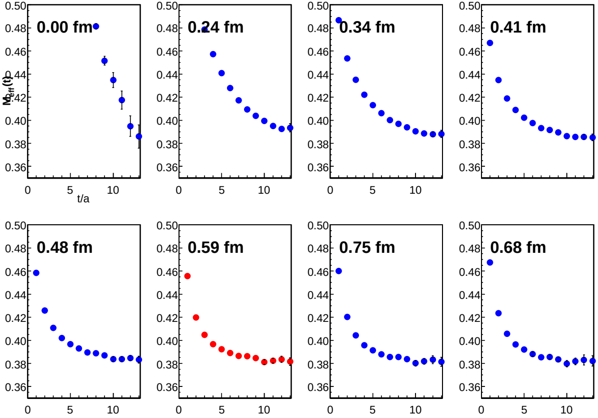
- **Scale setting** is PRELIMINARY. See B. Leder's talk for method via $r_0 = 0.5$ fm.

Simulation and Measurement Details

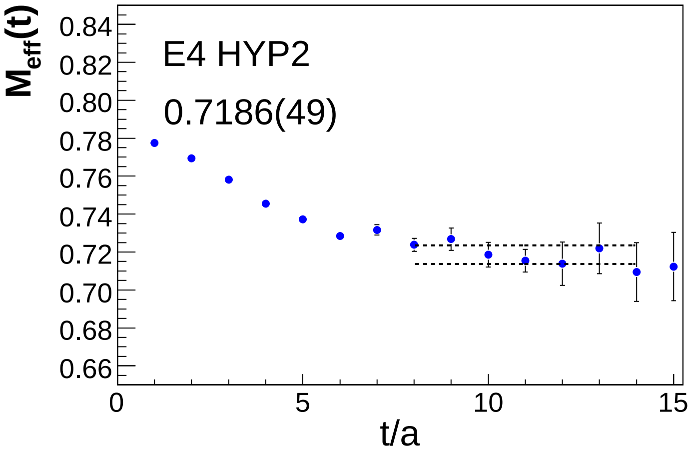
- Code based on DD-HMC.
- **Longer trajectories** ($\tau = 2.0, 4.0$) [Meyer et al. (ALPHA) 2006] can reduce autocorrelations (while keeping the acceptance, and reversibility violations, acceptable).
- Fuller **autocorrelation analysis** eventually (F. Virotta's talk). **Binning** is used for now. Autocorrelations seem tolerable.
- **Many inversions per config**, e.g.: $4 \times T \times N_{noise} = 1536$ for F6, F7. The dfl_sap_gcr solver in DD-HMC is generally fast. Construction of dfl subspace worth more effort here than in HMC...

Creation Operator Smearing. Example: A4

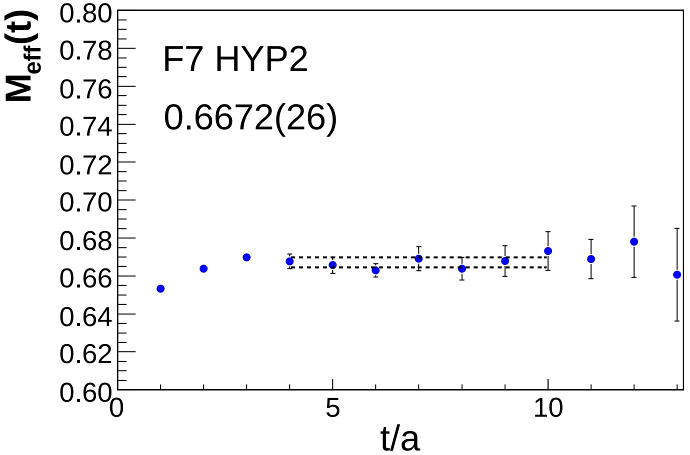
- We select the smearing with best $M_{eff}(t)$ plateau. (fine, from experience with GEVP for 2pt functions).



Effective Matrix Element Plateau. Example: E4.



Effective Matrix Element Plateau. Example: F7.



Renormalization and Chiral Behaviour

- We need: $Z_A(\beta)(1 + b_A(\beta)(m - m_c(\beta)))$
  Non-perturbative Z_A [Della Morte, Sommer, Takeda (ALPHA) 2008],
  one-loop perturbative b_A only,
  and m_c from [Della Morte et al. (ALPHA) 2007],
  all this essentially adds a **0.5% error**.

- We try both a **linear fit** in m_π^2:

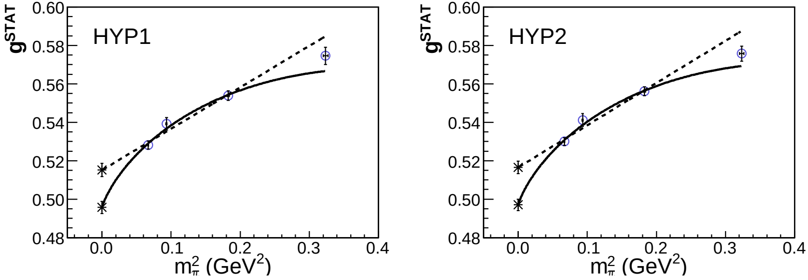
$$g = g_l(1 + c_l m_\pi^2)$$

  and the **HMχ formula** [Fajfer and Kamenik, 2006]:

$$g = g_0 \left[1 - \frac{4g_0^2}{(4\pi f)^2} m_\pi^2 \log(m_\pi)^2 + c_0 m_\pi^2 \right]$$

Chiral Extrapolation at $a \approx 0.07$ fm

- **Excluding heaviest point** (E4: $m_\pi \approx 560$ MeV) from fits:

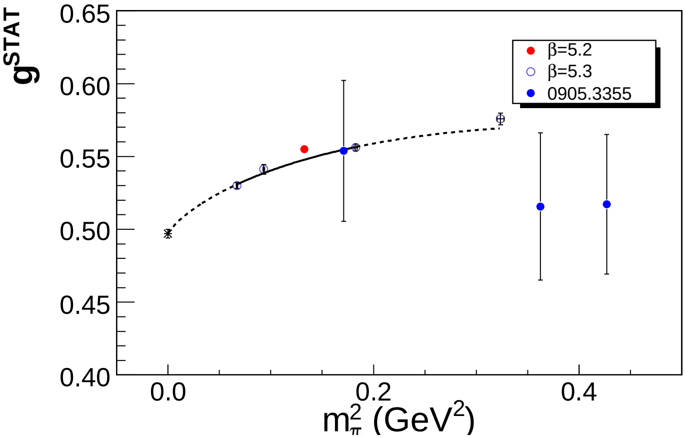


- **PRELIMINARY $\beta = 5.3$ results**:

	linear	log
HYP1	0.5151(34)	0.4957(31)
HYP2	0.5165(33)	0.4970(30)

Status

- Including our (even more PRELIMINARY) $\beta = \mathbf{5.2}$ **point**, and results from the recent publication of **Becirevic et al.**:



Summary

- This seems to be an efficient method.
- TODO:
  - Lattice systematics. **Discretization effects** are highest priority. Need some check of volume effects. Further quark masses desirable.
  - More care with autocorrelations and statistical error estimation.
  - Systematic error from **excited state contamination**. GEVP analysis.
  - An **explicit comparison** with the unsummed 3pt method. Investigation of similar methods for other matrix elements (e.g. between states with different energies)...