Energy dependence of the QCD coupling constant with four quark flavours

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Lattice QCD, a brief summary

- 1974: K. Wilson ⇒ discrete hypercubic spacetime lattice [Wilson (1974) Phys.Rev.D,2445]
- quarks on sites, gluons on links between lattice sites
- QCD action: $S_{\mathrm{QCD}} = S_{\mathrm{F}} + S_{\mathrm{G}}$
- Wilson's plaquette gauge action
- Wilson fermions
- $O(a)$ improvement according to Symanzik:
  $D \longrightarrow D + c_{\mathrm{sw}} \frac{ia}{4} \sigma_{\mu\nu} \mathcal{F}_{\mu\nu}$ [Sheikholeslami & Wohlert(1985)Nucl.Phys.B259]

Schrödinger functional setup

- definition: $Z[C', C] = e^{-\Gamma} = \int_{T\times L^3} \mathscr{D}[U,\psi,\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$ [Lüscher et al. (1992) Nucl.Phys.B384]
- boundary conditions
  - spatial direction: periodic for gauge fields and periodic up to a phase θ for quark fields
  - temporal direction: Dirichlet boundary conditions are imposed for $x_0 = 0, T$
- definition of coupling in SF scheme: $\frac{\partial \Gamma[B(\eta)]}{\partial \eta}\Big|_{\eta=0} = \frac{k}{\bar{g}^2}$ where $\Gamma[B(\eta)] = -\ln Z[C', C]$ is the effective action, k is a normalization constant [Lüscher et al.(1993) Nucl.Phys.B413]

Determination of $c_{\rm sw}$ for $N_f = 4$: General strategy

- general idea: $\mathcal{O}(a)$ terms violate chiral symmetry
- unrenormalized PCAC relation:

$$\frac{1}{2}(\partial_\mu + \partial^*_\mu)\langle (A_{\rm I})^a_\mu \mathcal{O}\rangle = 2m\langle P^a(x)\mathcal{O}\rangle$$

where

$$(A_{\rm I})^a_\mu = A^a_\mu + a \cdot c_{\rm A}\frac{1}{2}(\partial_\mu + \partial^*_\mu)P^a$$

with

$$A^a_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$$

and

$$P^a(x) = \bar{\psi}(x)\gamma_5\frac{\tau^a}{2}\psi(x)$$

Definition of mass

- unrenormalized PCAC quark mass

$$m(x_0) = \frac{\frac{1}{2}(\partial_0^* + \partial_0) f_A(x_0) + a \cdot c_A \partial_0^* \partial_0 f_P(x_0)}{2 f_P(x_0)}$$

at $x_0 = \frac{T}{2}$

- $m(x_0) = r(x_0) + a \cdot c_A s(x_0)$, where

$$r(x_0) = \frac{\frac{1}{2}(\partial_0^* + \partial_0) f_A(x_0)}{2 f_P(x_0)} \quad s(x_0) = \frac{\partial_0^* \partial_0 f_P(x_0)}{2 f_P(x_0)}$$

- c_A independent mass $M(x_0, y_0)$ [Lüscher et al.(1997)Nucl.Phys.B491:323]

$$M(x_0, y_0) = m(x_0) - s(x_0) \frac{m(y_0) - m'(y_0)}{s(y_0) - s'(y_0)} \text{ at } (x_0, y_0) = \left(\frac{3}{4} T, \frac{1}{4} T\right)$$

Improvement requirement (from now: $a = 1$)

- second mass $M'\left(\frac{3}{4}T, \frac{1}{4}T\right)$ with f'_{X}
- f_X and f'_X are related to each other through a time reflection
- improvement requirement:

$$\Delta M = M\left(\frac{3}{4}T, \frac{1}{4}T\right) - M'\left(\frac{3}{4}T, \frac{1}{4}T\right) = \Delta M^{(0)} \approx 0$$

at quark mass

$$M\left(\frac{1}{2}T, \frac{1}{4}T\right) = 0,$$

where $\Delta M^{(0)}|_{M=0, c_{\mathrm{sw}}=1} = 0.000277$ at $L = 8$ (tree level cutoff eff.)

[Lüscher et al.(1997)Nucl.Phys.B491:323]

Dependence of $\Delta M - \Delta M^{(0)}$ on $c_{\rm sw}$

- $L = 8$, $T = 2L$, constant abelian matrices C and C', fermion phase $\theta = 0$
- strategy: For each value of g_0^2, we calculated ΔM at $M \approx 0$ for 3-4 values of c_{sw} and interpolate to $\Delta M - \Delta M^{(0)} = 0$

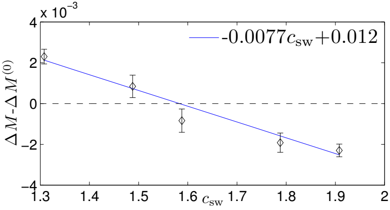
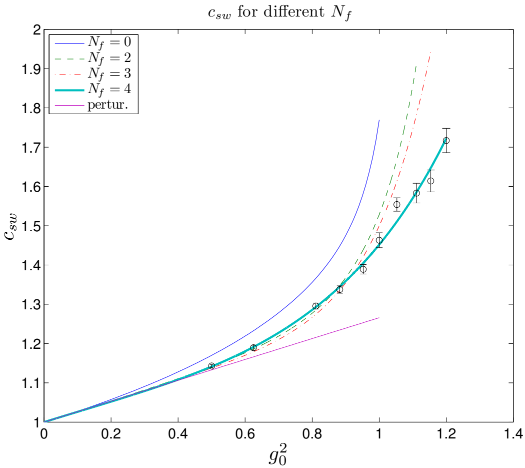


Figure: Determination of $c_{\rm sw}$ at $g_0^2 = 1.11$.

c_{sw} for $N_f = 4$

- $N_f = 0, 2$ ALPHA col.
- $N_f = 3$ CP-PACS & JLQCD col.
- problem: beyond $g_0^2 > 1.2$ no clear sign change in improvement condition



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Step scaling function

- renormalization group equation

$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta = -(b_0 \bar{g}^3 + b_1 \bar{g}^5 + \dots)$$

- b_0 and b_1 universal but higher order coeff. scheme dependent
- integrated form of the β function: the step scaling function (ssf)

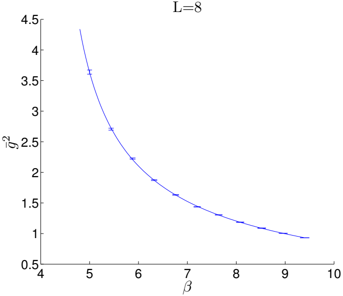
$$\bar{g}^2(1/2L) = \sigma(\bar{g}^2(1/L))$$

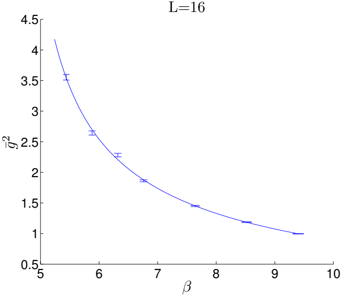
- ssf can be studied by MC simulations:
    - choose a lattice size, for example, $L = 8$
    - adjust $\bar{g}^2(1/L) = u$ and $m = 0$
    - increase the lattice size by a factor 2 and simulate at the same parameters

Our strategy

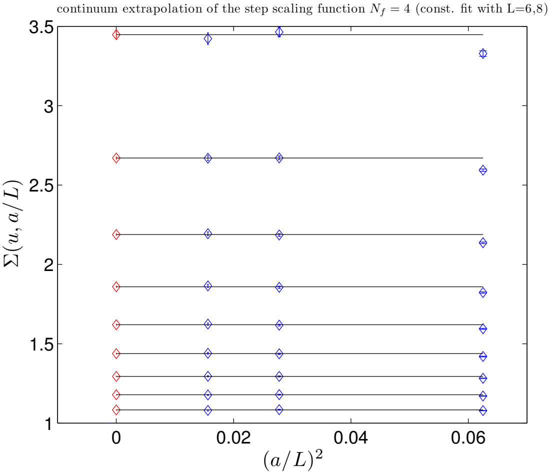
- tuning problem: $\bar{g}^2$ and m has to be tuned at same time
- instead of tuning it is convenient to interpolate a series of simulations [Appelquist et al. (2009) Phy.Rev.D79]
  - perform simulations at some values of g_0^2 (on next slide $\beta = 6/g_0^2$)
  - and interpolate the coupling in between with a fit $\bar{g}^2(g_0^2)$ at fixed L

Our strategy



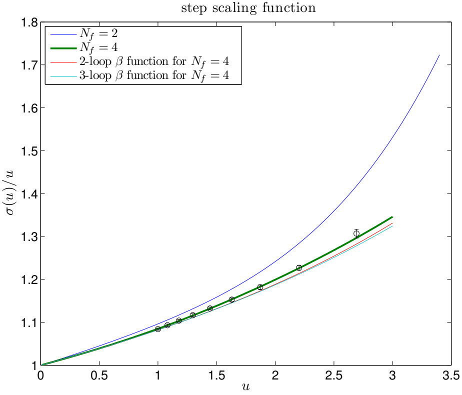


continuum extrapolation of ssf



Step scaling function for $N_f = 4$

- $N_f = 2$ ALPHA col.
- pert. line: solving $-2\ln(2) = \int_u^{\sigma(u)} \frac{dx}{\sqrt{x}\beta(\sqrt{x})}$ with 3-loop β function numerically



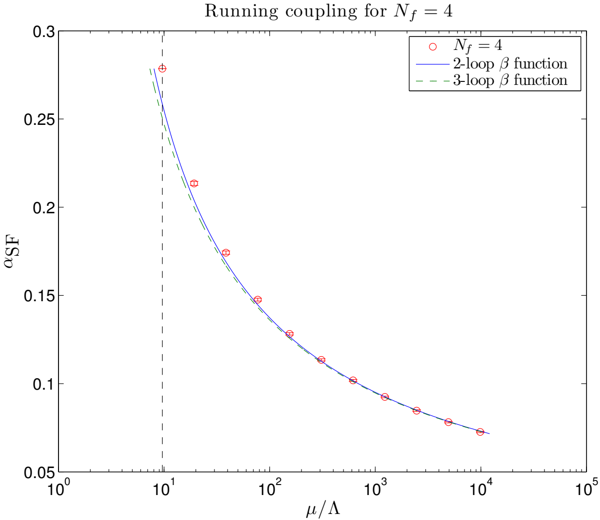
Running coupling

- once ssf is known, evolution of $\alpha_{\rm SF}(\mu) = \frac{\bar{g}^2(\mu)}{4\pi}$, $\mu = 1/L$ can be computed
- strategy: $u_0 = \bar{g}^2(1/L_{\rm max})$, solve $u_k = \sigma(u_{k+1})$, $k = 0, 1, 2, \ldots$
- u_0 corresponds to $\bar{g}^2$ at $\mu = 2^k/L_{\rm max}$
- $L_{\rm max}$ is presently undetermined in MeV in our case, therefore we consider μ/Λ

$\Lambda L_{\max}$ preliminary result

recursion to give $\ln(\Lambda L_{\max})$: 10 steps starting at 3.5000		
u	3-loop, $L = 6/8$	2-loop
3.5000(0.0000)	-1.993(0.000)	-2.083
2.6825(0.0144)	-2.046(0.017)	-2.116
2.1894(0.0120)	-2.086(0.022)	-2.144
1.8545(0.0102)	-2.121(0.026)	-2.171
1.6111(0.0087)	-2.154(0.030)	-2.197
1.4257(0.0074)	-2.185(0.033)	-2.224
1.2798(0.0066)	-2.214(0.036)	-2.249
1.1617(0.0060)	-2.241(0.040)	-2.272
1.0641(0.0056)	-2.265(0.045)	-2.294
0.9821(0.0054)	-2.288(0.051)	-2.315
0.9122(0.0052)	-2.309(0.057)	-2.334

Running coupling with $N_f = 4$ (preliminary)



Summary & Outlook

- Summary
  - our framework: Schrödinger functional (SF) scheme, coupling in SF
  - determination of improvement coefficient $c_{\rm sw}$ for $N_f = 4$: idea, strategy and our result
  - the role of step scaling function (ssf), its numerical determination and our result for $N_f = 4$
  - extraction of the running coupling of QCD from ssf with four flavors of Wilson quarks in SF scheme
  - significant deviations from perturbative running
- Outlook
  - step scaling function: larger lattices and higher statistical precision
  - scale setting: $L_{\rm max}$ in MeV through 2+1+1 simulations
  - determination of Λ parameter in, for example, $\overline{MS}$ scheme ; perturbative change to 5 flavours; $\Rightarrow \alpha_{\overline{MS}^{(5)}}(m_Z)$