A predictive effective field theory: Heavy Quark Effective Theory on the lattice

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NIC, DESY, A Research Centre of the Helmholtz Association

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Heavy Quark Effective Theory

- An effective field theory for
  – low energy: single heavy quark in initial and/or final state
  – energy far below threhold for creation of heavy quark-anti-quark pairs

  typically: **heavy quark $\equiv$ b-quark**

  sometimes: heavy quark $=$ c-quark

- Degrees of freedom
  heavy quark field: 2-component: $P_+\psi_{\rm h} = \psi_{\rm h}$ ($P_\pm = \frac{1\pm\gamma_0}{2}$)
  light quark fields: $u, d, s, (c)$
  gluons

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An EFT:

non-perturbative in

α_s

order by order in

$$\frac{\text{energy}}{M_\mathrm{b}}$$

The expansion parameter

- Typical QCD scale in bound states

$$\Lambda_{\mathrm{QCD}} = [0.5\,\mathrm{fm}]^{-1} = 400\,\mathrm{MeV}$$

$$\frac{\Lambda_{\mathrm{QCD}}}{M_{\mathrm{b}}} \approx \frac{1}{10}$$

- Expect a good asymptotic convergence
- Applicable for small external momenta (in the rest frame of the B-hadron)

$$\frac{|\mathbf{p}|}{M_{\mathrm{b}}} \ll 1$$

HQET Lagrangian

[Eichten & Hill; Isgur & Wise; Georgi]

In the rest frame of a b-hadron ("velocity zero")

$$\overline{\psi}_{\rm b}\{D_\mu\gamma_\mu + m_{\rm b}\}\psi_{\rm b}$$

$$\Downarrow$$

$$\mathscr{L}_{\rm h}^{\rm stat} - \frac{1}{2m_{\rm b}}(\mathcal{O}_{\rm kin} + \mathcal{O}_{\rm spin}) + \mathscr{L}_{\text{anti-quark}}$$

$$E^{\rm QCD} = E^{\rm HQET} + m_{\rm b}$$

$$\mathscr{L}_{\rm h}^{\rm stat} = \overline{\psi}_{\rm h}\, D_0\, \psi_{\rm h}\,, \qquad P_+\psi_{\rm h} = \psi_{\rm h}\,, \quad \overline{\psi}_{\rm h} P_+ = \overline{\psi}_{\rm h}\,, \quad P_\pm = \frac{1\pm\gamma_0}{2}$$

$$\mathcal{O}_{\rm kin}(x) = \overline{\psi}_{\rm h}(x)\, \mathbf{D}^2\, \psi_{\rm h}(x)$$

$$\mathcal{O}_{\rm spin}(x) = \overline{\psi}_{\rm h}(x)\, \boldsymbol{\sigma}\cdot\mathbf{B}(x)\, \psi_{\rm h}(x)$$

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- Searches of physics beyond the SM in *Flavour physics* have seen no significant sign.
  Uncertainties are probably too large $\longrightarrow$ precision physics.

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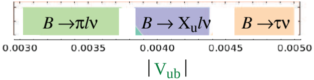
G. Isidori – *Quark flavour mixing with right-handed currents* Euroflavour2010, Munich

Motivation

Exp. side: RH currents provide a natural solution to the "V_{ub} puzzle"

$$\mathrm{B}(B \to \pi\, l\nu) \propto |V_{ub}|^2$$
$$\mathrm{B}(B \to \tau\nu) \propto |V_{ub}|^2$$
$$\mathrm{B}(B \to X_u l\nu) \propto |V_{ub}|^2$$

$\xrightarrow[\text{SM}]{\text{Within}}$



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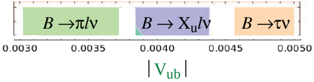
Within SM →

Lattice → ME ME



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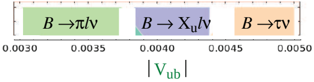

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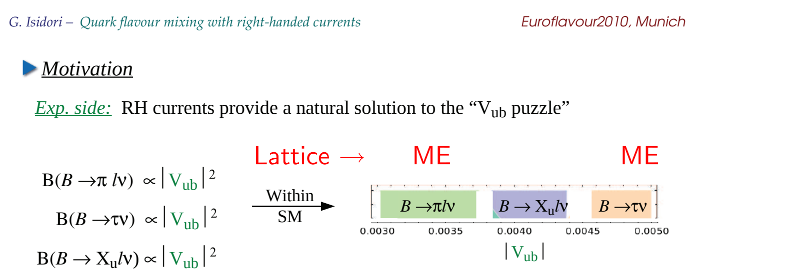
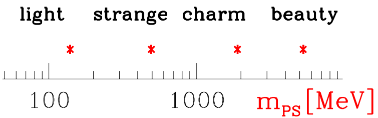


- More precise & reliable lattice calculations are needed to check whether such puzzles are for real or others are there.
- **HQET is a great help**

Lattice QCD is a challenge

multiple scale problem
always difficult
for a numerical treatment



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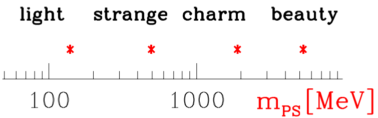

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$$\Lambda_{\rm UV} = a^{-1}$$
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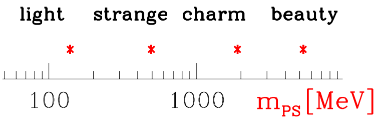

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$$L^{-1} \ll m_\pi\,,\ \ldots\ ,m_{\rm D}\,,m_{\rm B} \ll a^{-1}$$

$$\mathrm{O}(\mathrm{e}^{-Lm_\pi}) \qquad\qquad m_{\rm D}\,a \lesssim 1/2$$

$$\downarrow \qquad\qquad \downarrow$$

$$L \gtrsim 4/m_\pi \sim 6\,\mathrm{fm} \qquad\qquad a \approx 0.05\,\mathrm{fm}$$

$$L/a \gtrsim 120$$

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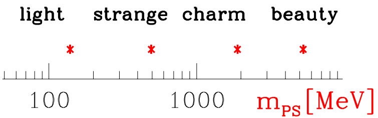

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beauty not accomodated: need HQET, $\Lambda_{\rm QCD}/m_{\rm b}$ expansion

HQET at the quantum level

Path integral with weight (directly on the lattice)

$$W_{\mathrm{HQET}} \equiv \exp(-a^4 \sum_x [\mathscr{L}_{\mathrm{light}}(x) + \mathscr{L}_{\mathrm{h}}^{\mathrm{stat}}(x)]) \times \left\{ 1 + a^4 \sum_x (\omega_{\mathrm{kin}}\, \mathcal{O}_{\mathrm{kin}}(x) + \omega_{\mathrm{spin}}\, \mathcal{O}_{\mathrm{spin}}(x)) \right\}$$

This yields

$$\begin{aligned} \langle \mathcal{O} \rangle &= \langle \mathcal{O} \rangle_{\mathrm{stat}} + \omega_{\mathrm{kin}} a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{\mathrm{kin}}(x) \rangle_{\mathrm{stat}} + \omega_{\mathrm{spin}} a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{\mathrm{spin}}(x) \rangle_{\mathrm{stat}} \\ &\equiv \langle \mathcal{O} \rangle_{\mathrm{stat}} + \omega_{\mathrm{kin}} \langle \mathcal{O} \rangle_{\mathrm{kin}} + \omega_{\mathrm{spin}} \langle \mathcal{O} \rangle_{\mathrm{spin}} \,, \end{aligned}$$

with

$$\langle \mathcal{O} \rangle_{\mathrm{stat}} = \frac{1}{\mathcal{Z}} \int_{\mathrm{fields}} \mathcal{O} \exp(-a^4 \sum_x [\mathscr{L}_{\mathrm{light}}(x) + \mathscr{L}_{\mathrm{h}}^{\mathrm{stat}}(x)])$$

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renormalizable
$\mathscr{L}_{\rm h}^{\rm stat} = \overline{\psi}_{\rm h}\, [D_0 + \delta m]\, \psi_{\rm h}$

The weight is expanded because of renormalizability

HQET at the quantum level

$$\langle \mathcal{O} \rangle \quad = \quad \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} \langle \mathcal{O} \rangle_{\rm kin} + \omega_{\rm spin} \langle \mathcal{O} \rangle_{\rm spin}$$

also local fields in correlation functions need to be expanded:

$$\begin{aligned} \mathcal{O}_{\rm QCD} \quad &= \quad A_0(x)\, A_0^\dagger(0) \\ A_0(x) \quad &\rightarrow \quad A_0^{\rm HQET}(x) = Z_{\rm A}^{\rm HQET}\, [A_0^{\rm stat}(x) + \sum_{i=1}^{2} c_{\rm A}^{(i)} A_0^{(i)}(x)] \end{aligned}$$

$$c_{\rm A}^{(i)} = {\rm O}(1/m)\,, \qquad [A_0^{(i)}(x)] = 4$$

$$\begin{aligned} A_0^{(1)}(x) &= \overline{\psi}_{\rm l}(x) \frac{1}{2} \gamma_5 \gamma_i (D_i - \overleftarrow{D}_i) \psi_{\rm h}(x) \\ A_0^{(2)}(x) &= -\partial_i\, A_i^{\rm stat}\,, \end{aligned}$$

HQET at the quantum level

Example

$$C_{\mathrm{AA,R}}^{\mathrm{QCD}}(x_0) = Z_{\mathrm{A}}^2\, a^3 \sum_{\mathbf{x}} \left\langle A_0(x) A_0^\dagger(0) \right\rangle_{\mathrm{QCD}}$$

its HQET expansion (with energy shift)

$$\begin{aligned} C_{\mathrm{AA}}^{\mathrm{QCD}}(x_0) \quad &= \quad \mathrm{e}^{-m\,x_0} (Z_{\mathrm{A}}^{\mathrm{HQET}})^2 \Big[C_{\mathrm{AA}}^{\mathrm{stat}}(x_0) + c_{\mathrm{A}}^{(1)}\, C_{\delta\mathrm{AA}}^{\mathrm{stat}}(x_0) \\ &\qquad + \omega_{\mathrm{kin}}\, C_{\mathrm{AA}}^{\mathrm{kin}}(x_0) + \omega_{\mathrm{spin}}\, C_{\mathrm{AA}}^{\mathrm{spin}}(x_0) \Big] \end{aligned}$$

with

$$\begin{aligned} C_{\delta\mathrm{AA}}^{\mathrm{stat}}(x_0) \quad &= \quad a^3 \sum_{\mathbf{x}} \langle A_0^{\mathrm{stat}}(x) (A_0^{(1)}(0))^\dagger \rangle_{\mathrm{stat}} + a^3 \sum_{\mathbf{x}} \langle A_0^{(1)}(x)\, (A_0^{\mathrm{stat}}(0))^\dagger \rangle_{\mathrm{stat}} \\ C_{\mathrm{AA}}^{\mathrm{kin}}(x_0) \quad &= \quad a^3 \sum_{\mathbf{x}} \langle A_0^{\mathrm{stat}}(x) (A_0^{\mathrm{stat}}(0))^\dagger \rangle_{\mathrm{kin}} \\ C_{\mathrm{AA}}^{\mathrm{spin}}(x_0) \quad &= \quad a^3 \sum_{\mathbf{x}} \langle A_0^{\mathrm{stat}}(x) (A_0^{\mathrm{stat}}(0))^\dagger \rangle_{\mathrm{spin}} \end{aligned}$$

Expansion of energies...

$$
\begin{aligned}
m_{\rm B} &= -\lim_{x_0\to\infty} \widetilde{\partial}_0 \ln C_{\rm AA}^{\rm QCD}(x_0) = \ldots \\
&= m_{\rm bare} + E^{\rm stat} + \omega_{\rm kin} E^{\rm kin} + \omega_{\rm spin} E^{\rm spin}\,,
\end{aligned}
$$

$$
\begin{aligned}
E^{\rm stat} &= -\lim_{x_0\to\infty} \widetilde{\partial}_0 \, \ln C_{\rm AA}^{\rm stat}(x_0) \\
E^{\rm kin} &= -\frac{1}{2}\langle B|\mathcal{O}_{\rm kin}(0)|B\rangle_{\rm stat} \\
E^{\rm spin} &= -\frac{1}{2}\langle B|\mathcal{O}_{\rm spin}(0)|B\rangle_{\rm stat}\,,
\end{aligned}
$$

... and matrix elements

$$
\begin{aligned}
\mathcal{A}(B \to \tau\nu) &\propto \langle 0|A_\mu^\dagger(0)|B\rangle \propto \\
F_{\rm B}\sqrt{m_{\rm B}} &= \lim_{x_0\to\infty} \left\{2\exp(m_{\rm B}x_0)\, C_{\rm AA}^{\rm QCD}(x_0)\right\}^{1/2} \\
&= Z_{\rm A}^{\rm HQET}\,\Phi^{\rm stat} \lim_{x_0\to\infty} \Big\{1 + \tfrac{1}{2}x_0\left[\omega_{\rm kin}E^{\rm kin} + \omega_{\rm spin}E^{\rm spin}\right] \\
&\quad + \tfrac{1}{2}c_{\rm A}^{(1)} R_{\delta A}^{\rm stat}(x_0) + \tfrac{1}{2}\omega_{\rm kin}\frac{C_{\rm AA}^{\rm kin}(x_0)}{C_{\rm AA}^{\rm stat}(x_0)} + \tfrac{1}{2}\omega_{\rm spin}\frac{C_{\rm AA}^{\rm spin}(x_0)}{C_{\rm AA}^{\rm stat}(x_0)}\Big\} \\
\Phi^{\rm stat} &= \lim_{x_0\to\infty} \left\{2\exp(E^{\rm stat}x_0)\, C_{\rm AA}^{\rm stat}(x_0)\right\}^{1/2} = \langle B|A_0^{\rm stat}(0)|0\rangle
\end{aligned}
$$

HQET at the quantum level

$$C_{\mathrm{AA}}^{\mathrm{QCD}}(x_0) = \mathrm{e}^{-m\,x_0}(Z_{\mathrm{A}}^{\mathrm{HQET}})^2 \left[C_{\mathrm{AA}}^{\mathrm{stat}}(x_0) + c_{\mathrm{A}}^{(1)}\, C_{\delta\mathrm{AA}}^{\mathrm{stat}}(x_0) + \omega_{\mathrm{kin}}\, C_{\mathrm{AA}}^{\mathrm{kin}}(x_0) + \omega_{\mathrm{spin}}\, C_{\mathrm{AA}}^{\mathrm{spin}}(x_0) \right]$$

- Parameters in the effective theory

$$(\omega_1, \ldots \omega_5) = (m_{\mathrm{bare}} = m + \delta m\,,\ \ln(Z_{\mathrm{A}}^{\mathrm{HQET}})\,,\ c_{\mathrm{A}}^{(1)}\,,\ \omega_{\mathrm{kin}}\,,\ \omega_{\mathrm{spin}})$$

$\omega_i = \omega_i(g_0, aM_{\mathrm{b}})$ bare parameters

g_0: bare QCD coupling

M_{b}: RGI b-quark mass

(other quark masses not written)

- Renormalization

  keep M_{b} fixed change $g_0 \to 0$ (i.e. $a \to 0$):

  all divergences (logarithmic and power) absorbed in ω_i

HQET at the quantum level

Finite parts of the parameters ω_i need to be determined

- from experiments

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- from non-perturbative QCD:

Matching

$$\boxed{\Phi_i^{\mathrm{HQET}}(\{\omega_i(g_0, aM_{\mathrm{b}})\}) = \Phi_i^{\mathrm{QCD}}(M_{\mathrm{b}})}$$

keep full predictivity if this can be achieved

Issues in a non-perturbative treatment

- power divergences
  $$\frac{g_0^{2L}}{a^n} \sim \frac{1}{\log(a\Lambda_{\rm QCD})\, a^n}$$
  need NP subtraction
  $n = 1, 2$

  e.g.
  $$(\mathcal{O}_{\rm kin})_{\rm R}(z) = Z_{\mathcal{O}_{\rm kin}}\left(\mathcal{O}_{\rm kin}(z) + \frac{c_1}{a}\,\overline{\psi}_{\rm h}(z)D_0\psi_{\rm h}(z) + \frac{c_2(g_0)}{a^2}\,\overline{\psi}_{\rm h}(z)\psi_{\rm h}(z)\right)$$

- power corrections
  $$(\alpha(m))^L \overset{m\to\infty}{\gg} \frac{\Lambda_{\rm QCD}}{m}$$
  need NP leading terms to define power corrections

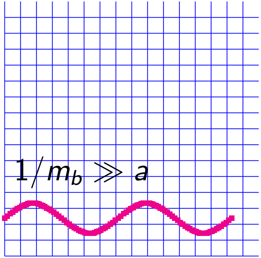
It is in general not enough to compute Wilson coefficients in perturbation theory

Non-perturbative matching of HQET and QCD [Heitger, S., 2001]

- The trick: start in small volume, $L = L_1 \approx 0.5\,\mathrm{fm}\,, \quad a = 0.01\,\mathrm{fm}$

Φ_k finite volume masses, decay constants ...

QCD

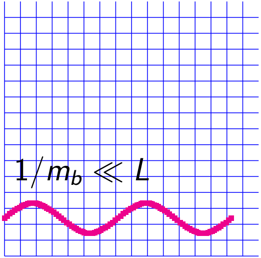


$$\boxed{\Phi_k^{\mathrm{QCD}} = \Phi_k^{\mathrm{HQET}}}$$

$k = 1, 2, \ldots, N_{\mathrm{HQET}}$

$N_{\mathrm{HQET}} =$
\# of parameters

HQET



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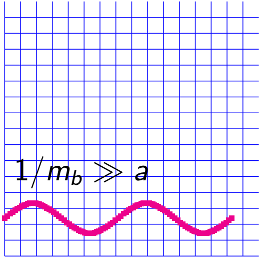

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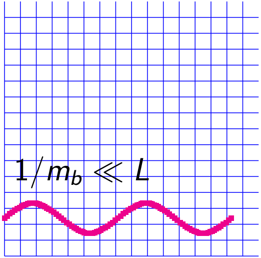
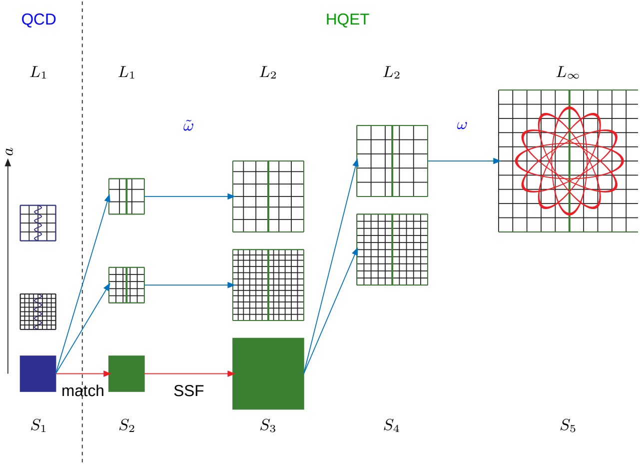


$\rightarrow$ HQET-parameters from QCD-observables in small volume
– at small lattice spacing $\quad L^{-1} \ll m_{\mathrm{b}} \ll a^{-1}$
power divergences subtracted non-perturbatively

The HQET strategy: first view

[Heitger, S., 2001]

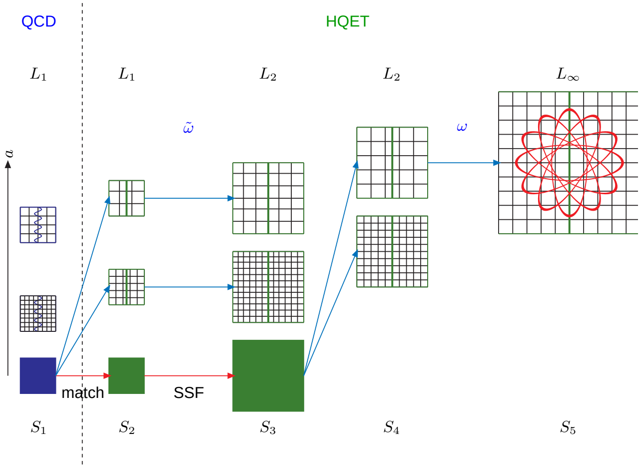


$\omega_k(a_1)$ $\omega_k(a_2)$ coefficients in effective Lagrangian

$a_1 = 0.025 \ldots 0.05\mathrm{fm}$ $a_2 = 0.05 \ldots 0.1\mathrm{fm}$

The HQET strategy: second view

[Heitger, S., 2001]

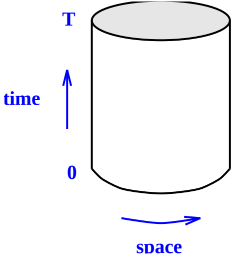


$\Phi_i^{\mathrm{HQET}}(L_1)$ $\Phi_i^{\mathrm{HQET}}(L_2)$ large V physics

observables

Schrödinger functional toolbox for finite volume

$T \times L^3$, Euclidean
Dirichlet boundary conditions in time
[Lüscher,Narayanan,Weisz,Wolff, 92; Sint, 93; ALPHA Collaboration, 92-...]

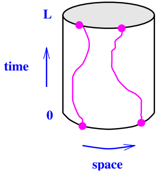


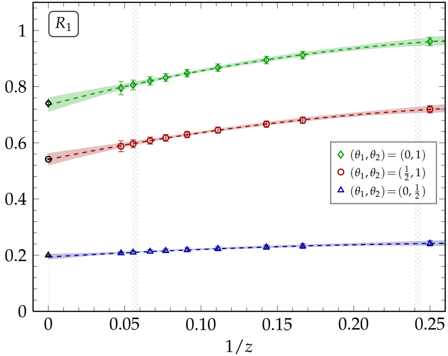
- no zero-modes
- boundary fields for gauge invariant quark correlation functions
- running coupling ...

Tests of HQET [Heitger, Jüttner, S., Wennekers, 2004; Fritzsch & Heitger, 2010]

Example: SF boundary-to-boundary correlators $N_\mathrm{f} = 2$ dynamical fermions

$$R_1 = \tfrac{1}{4}\left(\ln\left(\frac{f_1(\theta_1)k_1(\theta_1)^3}{f_1(\theta_2)k_1(\theta_2)^3}\right)\right)$$





$z = LM$

$L \approx 0.5\,\mathrm{fm}$

spin averaged

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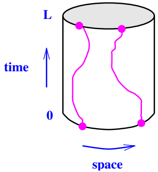
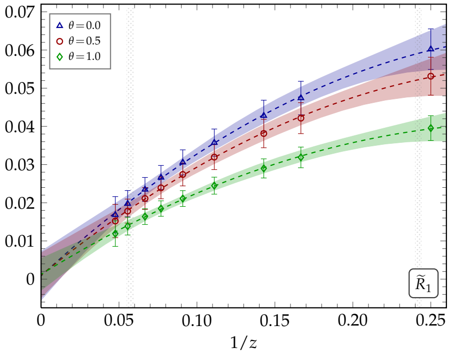
Example: SF boundary-to-boundary correlators $N_\mathrm{f} = 2$ dynamical fermions

$$\widetilde{R}_1 \quad = \quad \frac{3}{4} \ln\left(\frac{f_1}{k_1}\right) \propto \omega_{\mathrm{spin}}$$





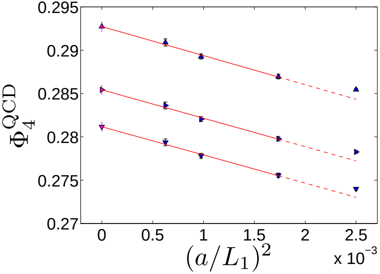
$z = LM$

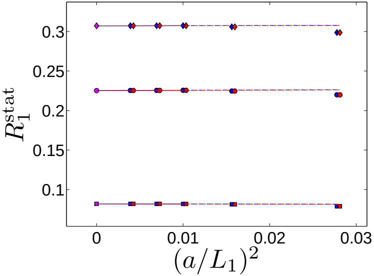
spin symmetry violating

Tests of HQET: the continuum limit

$L_1 = 0.5\text{fm}$: $a = 0.012 \ldots 0.025\text{fm}$:

b-quark can be simulated, continuum limit can be taken





$$\Phi_4 = \mathrm{O}(1) = R_1^{\rm stat} + \omega_{\rm kin} \Phi_4^{(1/m)}$$

three different z three different θ

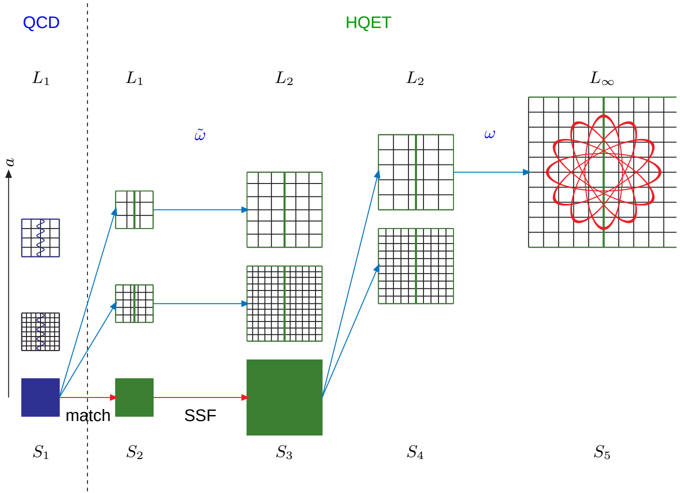
(show $N_{\rm f} = 0$ graphs)

Tests of HQET: conclusion

- HQET confirmed non-perturbatively
- SF correlation functions can be used for matching

$$\boxed{\Phi_i^{\mathrm{HQET}}(\{\omega_i(g_0, aM_{\mathrm{b}})\}) = \Phi_i^{\mathrm{QCD}}(M_{\mathrm{b}})}$$

Full strategy to determine $\omega(M_{\mathrm{b}}, a)$, $a = 0.05\mathrm{fm} \ldots 0.1\mathrm{fm}$



(in the realistic implementation finer resolutions are used)

Non-perturbative determination of parameters [ALPHA Collaboration ,2001 - 2010]

static parameters

$$\omega^{\rm stat} \;=\; (\, m_{\rm bare}^{\rm stat} \,,\, [\ln(Z_{\rm A})]^{\rm stat}\,)^t \,, \qquad N_{\rm HQET} = 2$$

parameters at first order

$$\begin{aligned} \omega^{\rm HQET} \;&=\; (\, m_{\rm bare} \,,\; \ln(Z_{\rm A}^{\rm HQET}) \,,\, c_{\rm A}^{(1)} \,,\, \omega_{\rm kin} \,,\, \omega_{\rm spin}\,)^t \qquad N_{\rm HQET} = 5 \\ \omega^{(1/m)} \;&=\; \omega^{\rm HQET} - \omega^{\rm stat} \end{aligned}$$

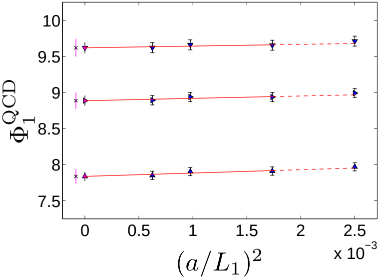
matching: $L_1 \approx 0.5\,\mathrm{fm}$ (low energy scale)

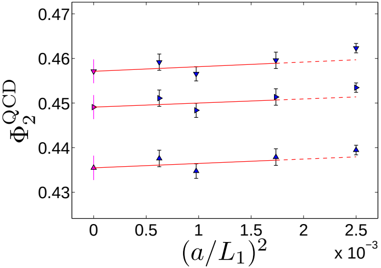
$$\Phi_i(L_1, M, a) = \Phi_i^{\rm QCD}(L_1, M, 0)\,,\;\; i = 1 \ldots N_{\rm HQET}\,.$$

Non-perturbative determination of parameters [ALPHA Collaboration ,2001 - 2010]

$L_1 = 0.5\text{fm}$: $a = 0.012 \ldots 0.025\text{fm}$:

b-quark can be simulated, continuum limit can be taken





$\Phi_1 = L_1\, m_{\rm B}(L_1) = \mathrm{O}(z)$

$\Phi_2 = \ln(L_1^{3/2}\, [F_{\rm B}\sqrt{m_{\rm B}}](L_1) = \mathrm{O}(z^0)$

three different z

Non-perturbative determination of parameters [ALPHA Collaboration ,2001 - 2010]

HQET expansion of Φ_1, Φ_2

$$\begin{aligned}
\Phi_1 &= L_1\, m_\mathrm{B}(L_1) = L\,[m_\mathrm{bare} + \Gamma^\mathrm{stat}] + \mathrm{O}(1/m_\mathrm{b}) \\
\Phi_2 &= \ln(L_1^{3/2}\,[F_\mathrm{B}\sqrt{m_\mathrm{B}}](L_1) = \ln(Z_\mathrm{A}^\mathrm{stat}) + \zeta_\mathrm{A} + \mathrm{O}(1/m_\mathrm{b})
\end{aligned}$$

in general

$$\begin{aligned}
\Phi(L, M, a) &= \eta(L, a) + \phi(L, a)\,\omega(M, a) \\
&= \begin{pmatrix} \Gamma^\mathrm{stat} \\ \zeta_\mathrm{A} \\ \dots \end{pmatrix} + \begin{pmatrix} L & 0 & \dots \\ 0 & 1 & \dots \\ \dots & & \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \dots \end{pmatrix}
\end{aligned}$$

Examples of results: $M_{\rm b}$ [Blossier, Della Morte, Garron, Mendes, Papinutto, Simma, S.]

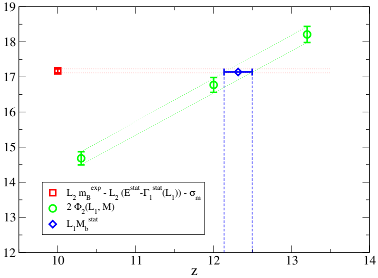
static approximation

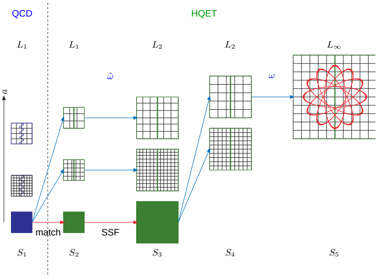
$$m_{\rm B} = \omega_1 + E^{\rm stat} = m_{\rm bare} + E^{\rm stat}$$

$$\lim_{a\to 0}[E^{\rm stat} - \Gamma^{\rm stat}(L_2,a)] \qquad a = 0.1{\rm fm}\ldots 0.05{\rm fm} \qquad [S_4,S_5]$$

$$+ \lim_{a\to 0}[\Gamma^{\rm stat}(L_2,a) - \Gamma^{\rm stat}(L_1,a)] \qquad a = 0.05{\rm fm}\ldots 0.025{\rm fm} \qquad [S_2,S_3]$$

$$+ \frac{1}{L_1}\lim_{a\to 0}\Phi_1(L_1,M_{\rm b},a) \qquad a = 0.025\,{\rm fm}\ldots 0.012\,{\rm fm} \qquad [S_1]\,.$$





Examples of results: M_{b}

	LO (static)	NLO (static + $\mathrm{O}(1/m)$)		
		$(\theta_1,\theta_2)=(0,0.5)$	$(\theta_1,\theta_2)=(0.5,1)$	$(\theta_1,\theta_2)=(0,1)$
$\theta_0=0$	17.1 ± 0.2	17.1 ± 0.2	17.1 ± 0.2	17.1 ± 0.2
$\theta_0=0.5$	17.2 ± 0.2	17.2 ± 0.2	17.2 ± 0.2	17.1 ± 0.2
$\theta_0=1$	17.2 ± 0.2	17.3 ± 0.3	17.3 ± 0.3	17.3 ± 0.3

Table: Dimensionless b-quark mass, $r_0 M_{\mathrm{b}}$, obtained from the B_{s} meson mass, for different values of θ_i.

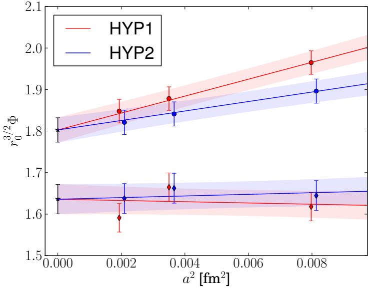
- small $1/m_{\mathrm{b}}$ corrections
- weak dependence on matching conditions

Examples of results: quenched $F_{\mathrm{B_s}}\sqrt{m_{\mathrm{B_s}}}$

[Blossier, Della Morte, von Hippel, Garron, Mendes, Simma, S]

$$F_{\mathrm{B_s}}\sqrt{m_{\mathrm{B_s}}} \;=\; \underbrace{C_{\mathrm{PS}}(M_{\mathrm{b}}/\Lambda)}_{\Uparrow} \; \underbrace{\Phi_{\mathrm{RGI}}}_{\text{number}} \; (1+\mathrm{O}(1/m_{\mathrm{b}}))$$

$$\overset{M_{\mathrm{b}}\to\infty}{\sim} [\log M_{\mathrm{b}}/\Lambda)]^{\gamma_0/2b_0} \text{ from } Z_{\mathrm{A}}^{\mathrm{HQET}}$$



Φ_{RGI} (static limit)

with $1/m_{\mathrm{b}}$: $\Phi^{\mathrm{HQET}} = F_{\mathrm{B_s}}\sqrt{m_{\mathrm{B_s}}}/C_{\mathrm{PS}}^{3-\mathrm{loop}}$

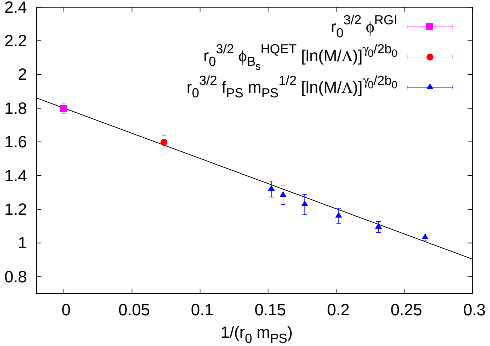
Examples of results: quenched $F_{\mathrm{B_s}}\sqrt{m_{\mathrm{B_s}}}$

	LO (static)	NLO (static + $\mathrm{O}(1/m)$)		
		$(\theta_1,\theta_2) = (0, 0.5)$	$(\theta_1,\theta_2) = (0.5, 1)$	$(\theta_1,\theta_2) = (0, 1)$
$\theta_0 = 0$	233 ± 6	220 ± 9	218 ± 9	218 ± 9
$\theta_0 = 0.5$	229 ± 7	221 ± 9	219 ± 8	219 ± 9
$\theta_0 = 1$	219 ± 6	223 ± 9	221 ± 8	222 ± 8

Table: Pseudo-scalar heavy-light decay constant $f_{\mathrm{B_s}}$ in MeV, for different values of θ_i.

- small $1/m_{\mathrm{b}}$ corrections
- weak dependence on matching conditions

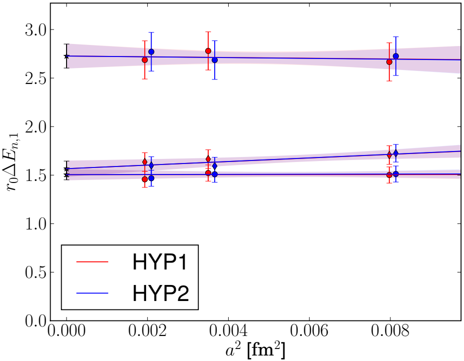
Comparison to relativistic (not so) heavy quarks



Surprisingly consistent picture down to mass of charm

Calls for a direct evaluation also at m_{charm}

Examples of results: quenched level splittings



3s – 1s splitting static

2s – 1s splitting static + $1/m_\mathrm{b}$

2s – 1s splitting static

Static results for splittings are in agreement with [T. Burch et al.]

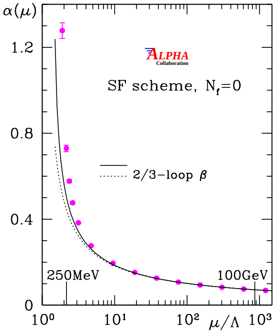
Also ratio of ground state / excited state decay constant

Concluding remarks

- HQET as an effective theory confirmed non-perturbatively after taking numerically the continuum limit
- The continuum limit exists (numerically)
  non-perturbative renormalizability

  of course numerical results are no rig. proof
- First results for $N_\mathrm{f} = 2$ were shown at Lattice 2010
  [Blossier, Bulava, Della Morte, Donnellan, Fritzsch, Garron, Heitger, Mendes, Simma, S.]
  We are at the beginning of phenomenological applications ...
  Flavour Physics / New Physics

More general remarks on QFT on the lattice



Lattice: excellent tool for NP field theory, even EFT

QCD: difficult
many scales
the couplings are strong and weak

tricks: a number of field theoretic + algorithmic tricks have been developed

V: finite volume as a tool plays an important role

SUSY: finite volume may play an important role
$\mathcal{N} = 4$ YM: conformal!
finite volume effects are unavoidable

AdS/CFT: finite volume could play an important role in checking it