Calculation of Multi-Particle Processes in QCD

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Plan

- Procedure to calculate matrix elements in QCD using an iterative algorithm based on the Dyson-Schwinger equation
- Improvement in computational efficiency
- Monte Carlo summation instead of summation over all possible colour and helicity configurations
- Some examples and results
- Summary and Outlook

In collaboration with C. G. Papadopoulos

QCD Process

Given process: $u\bar{u} \rightarrow u\bar{u}s\bar{s}4g$

With some cuts:

$$p_T = \sqrt{p_x^2 + p_y^2} \qquad \Delta R_{ij} = \sqrt{\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2} \qquad \eta = -\ln\tan(\theta/2)$$

we ask to compute the cross section for a given CMS energy

<u>Steps:</u>

- Find the number of Feynman graphs
- Write them down
- Compute them to get the amplitude
- Sum over all color and helicity configurations
- Square the amplitude
- Integrate over the phase space

Number of Graphs

P. Draggiotis, R. Kleiss Eur. Phys. J. C23, 701 (2002)

QCD with 1 fermion pairs

N=8	N=9	N=10	N=11	N=12	N=13
15495	231280	4016775	79603720	1773172275	43864374400

QCD with 3 (identical) fermion pairs

N=8	N=9	N=10	N=11	N=12	N=13
4362	59424	946050	17258640	355273170	8151299520

roughly grows like **n!**

Dyson-Schwinger Equations

- Starting point Dyson-Schwinger equations
- Give recursively n-point Green functions in terms of 1-,2- ... (n-1)-point functions

$$\langle 0 \, | T\phi(x_1)\ldots\phi(x_n)| 0 \, \rangle = \frac{\int d\phi \;\; (\phi(x_1)\ldots\phi(x_n))e^{iS}}{\int d\phi e^{iS}} \qquad S = \int d^4x \, \mathcal{L}$$

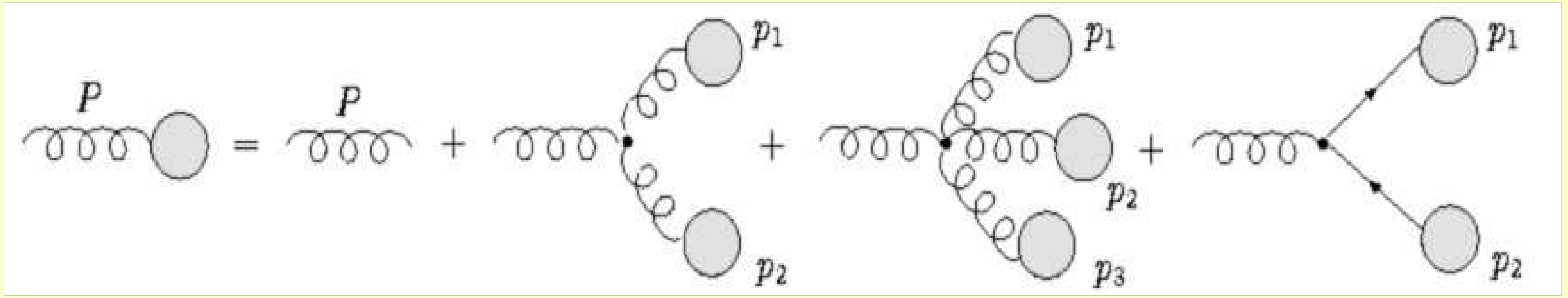
- Quantum equations of motion for Green's functions = DS equations

$$\boxed{\langle 0 \, | \left(\frac{\delta}{\delta\phi(x)} \int d^4x' \, \mathcal{L} \right) T\phi(x_1)\ldots\phi(x_n)| 0 \, \rangle = \sum_{i=1}^{n} \langle 0 \, | T\phi(x_1)\ldots(i\delta(x-x_i))\ldots\phi(x_n)| 0 \, \rangle}$$

Recursion Equation

- Gluons (diagrammatically):

A. Kanaki, C. G. Papadopoulos Comp. Phys. Commun. 132, 306 (2000)
P. Draggiotis, R. Kleiss, C. G. Papadopoulos Eur. Phys. J. C24,447 (2002)



- sub-amplitude with an off-shell gluon of momentum P
- Contributions from three- and four-vertices
- Fermion-antifermion vertex
- Blobs denote sub-amplitudes with the same structure

Recursion Equation

- Gluon (equation with suppressing colour):

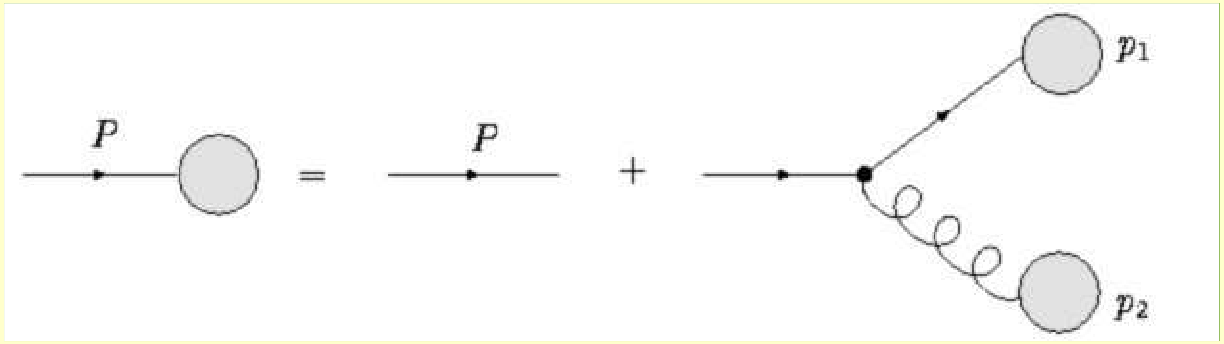
$$
\begin{aligned}
A^{\mu}(P) = \sum_{i=1}^{n} \delta(P-p_i) A^{\mu}(p_i) \; &+ \; ig_s \sum_{p_1+p_2} \Pi^{\mu}_{\nu} \bar{\psi}(p_1) \gamma^{\nu} \psi(p_2) \sigma(p_1,p_2) \\
&+ \frac{ig_s}{2} \sum_{p_1+p_2} \Pi^{\mu}_{\varrho} V^{\varrho\nu\lambda}(P,p_1,p_2) A_{\nu}(p_1) A_{\lambda}(p_2) \sigma(p_1,p_2) \\
&- \frac{g_s^2}{6} \sum_{p_1+p_2+p_3} \Pi^{\mu}_{\sigma} G^{\sigma\nu\lambda\varrho} A_{\nu}(p_1) A_{\lambda}(p_2) A_{\varrho}(p_3) \sigma(p_1,p_2)
\end{aligned}
$$

with gluon propagator
and sign function

$$
\boxed{\Pi^{\mu}_{\nu} = -\frac{ig^{\mu}_{\nu}}{P^2}}
$$

Recursion Equation

- Quark



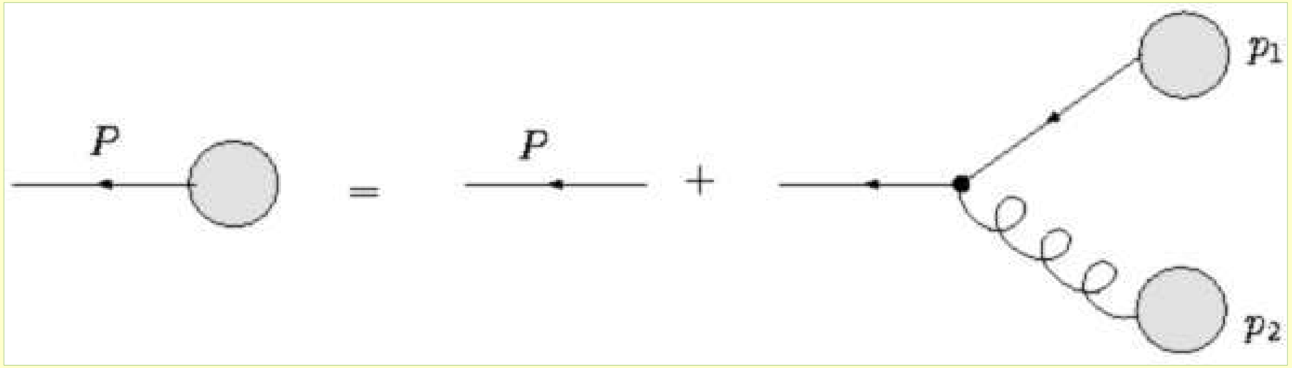
$$\psi(P)=\sum_{i=1}^{n}\delta(P-p_i)\psi(p_i) + ig_s\sum_{p_1+p_2} S \ A^{\mu}(p_1)\gamma_{\mu}\psi(p_2)\sigma(p_1,p_2)$$

With propagator

$$S=\frac{iP_{\mu}\gamma^{\mu}}{P^2}$$

Recursion Equation

- Antiquark



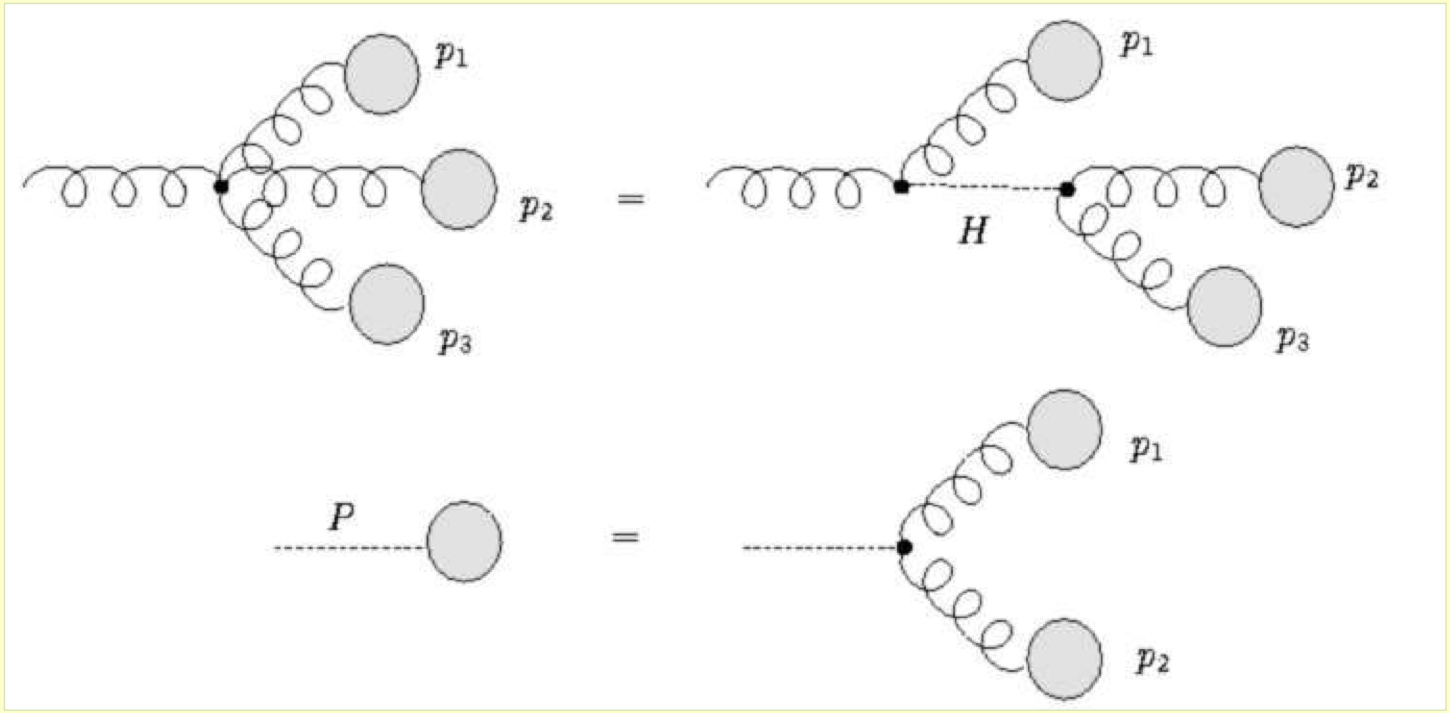
$$\bar{\psi}(P)=\sum_{i=1}^{n}\delta(P-p_i)\bar{\psi}(p_i) \; + \; ig_s\sum_{p_1+p_2}\bar{\psi}(p_2)A^{\mu}(p_1)\gamma_{\mu}\tilde{S}\sigma(p_1,p_2)$$

With propagator

$$\tilde{S}=\frac{-iP_{\mu}\gamma^{\mu}}{P^2}$$

Recursion Equation

- Auxiliary field $H_{\mu\nu}$ - reduce computational complexity



- Elimination of the four-vertex and the new H-gluon-gluon vertex

Recursion Equation

- Recursion equation for gluon is changed (four-vertex part only)

$$A^\mu(P)=\sum_{i=1}^{n}\delta(P-p_i)A^\mu(p_i)\ +\ ig_s\sum_{p_1+p_2}\Pi^\mu_\nu\bar{\psi}(p_1)\gamma^\nu\psi(p_2)\sigma(p_1,p_2)$$
$$+\ \frac{ig_s}{2}\sum_{p_1+p_2}\Pi^\mu_\varrho V^{\varrho\nu\lambda}(P,p_1,p_2)A_\nu(p_1)A_\lambda(p_2)\sigma(p_1,p_2)$$
$$-\ g_s\sum_{p_1+p_2}\Pi^\mu_\sigma X^{\sigma\nu\lambda\varrho}A_\nu(p_1)H_{\lambda\varrho}(p_2)\sigma(p_1,p_2)$$

- New equations for auxiliary field and H-gluon-gluon vertex

$$H_{\mu\nu}(P)=-\ \frac{g_s}{4}\sum_{p_1+p_2}X^{\mu\nu\lambda\varrho}A_\lambda(p_1)A_\varrho(p_2)\sigma(p_1,p_2) \qquad X^{\mu\nu\lambda\varrho}=g^{\mu\lambda}g^{\nu\varrho}-g^{\nu\lambda}g^{\mu\varrho}$$

Building Amplitude

- Off-shell fields – building blocks of any process
- Used iteratively, at each step two (three) momenta are combined
- Initial conditions for the external particles:
  - Gluon: $A^{\mu}_{a}(p_i)=\epsilon^{\mu}_{\lambda}(p_i)\,\delta_{aa_i}$
  - Quarks: $\psi_k(p_i)=u(p_i)\,\delta_{kk_i}$ $\qquad\psi_k(p_i)=\bar{u}(p_i)\,\delta_{kk_i}$
  - Antiquarks: $\bar{\psi}_k(p_i)=\bar{u}(p_i)\,\delta_{kk_i}$ $\qquad\bar{\psi}_k(p_i)=u(p_i)\,\delta_{kk_i}$
- After n-1 steps:

$$\mathcal{A}(p_1,p_2,\ldots,p_n) = \left\{ \begin{array}{l} \hat{A}^{\mu a}(P_i)\,A^{a}_{\mu}(p_i) \\ \hat{\bar{\psi}}(P_i)\,\psi(p_i) \\ \bar{\psi}(p_i)\,\hat{\psi}(P_i) \end{array} \right.$$

Systematic Approach

- Systematic approach to build the amplitude from the initial momenta (binary representation)
- Process with n external particles with momenta p_i^μ
- Define momentum

$$P^\mu = \sum_{i=1}^{n} m_i p_i^\mu \qquad m_i = 0,1$$

- Binary vector $\vec{m} = (m_1, \ldots, m_n)$ can be uniquely represented by the integer

$$m = \sum_{i=1}^{n} 2^{i-1} m_i \qquad 0 \leqslant m \leqslant 2^n - 1$$

Systematic Approach

- All momenta replaced by the corresponding integers

$$J^{\mu}(P) \rightarrow J^{\mu}(m)$$

- Ordering of integers in binary representation by levels $l = \sum_{i=1}^{n} m_i$
- All external momenta are of level 1
- Amplitude corresponds to the unique level n number $2^n - 1$

$$\mathcal{A} = J(1) \cdot J(2^n - 2)$$

Ordering dictates path of the computation
– from level-1 to level-2 and so on up to the amplitude

Example

$\bar{u}(1)u(2)\rightarrow\bar{d}(4)d(8)g(16)$

Level	Current	$\vec{m}$
Level 1:	$\bar{\psi}(1)$	$(0,0,0,0,1)$
	$\psi(2)$	$(0,0,0,1,0)$
	$\psi(4)$	$(0,0,1,0,0)$
	$\bar{\psi}(8)$	$(0,1,0,0,0)$
	$A(16)$	$(1,0,0,0,0)$
Level 2:	$A(12)=\bar{\psi}(8)\psi(4)$	$(0,1,1,0,0)$
	$\psi(18)=A(16)\psi(2)$	$(1,0,0,1,0)$
	$\psi(20)=A(16)\psi(4)$	$(1,0,1,0,0)$
	$\bar{\psi}(24)=\bar{\psi}(8)A(16)$	$(1,1,0,0,0)$

Level 3:

$$\psi(14)=A(12)\psi(2) \qquad (0,1,1,1,0)$$

$$A(28)=\bar{\psi}(8)\psi(20)+\bar{\psi}(24)\psi(4) \qquad (1,1,1,0,0)$$

Level 4:

$$\psi_0(30)=A(28)\psi(2)+A(16)\psi(14)+A(12)\psi(18) \qquad (1,1,1,1,0)$$

Level 5:

$$\mathcal{A} = \bar{\psi}(1)\psi_0(30) \qquad (1,1,1,1,1)$$

- Total amount of currents $\displaystyle\sum_{k=1}^{n-1}\binom{n}{k}=2^n-2$

Cost Function

- How many operations are needed to compute the n-point amplitude
- Basic steps:
  - How many sub-amplitudes at each level k
  - How many ways of splitting k to two numbers of levels k_1, k_2
  - Sum over all levels

$$O(n) = \sum_{k=1}^{n-1} \binom{n}{k} \sum_{l=1}^{k-1} \binom{k}{l} = \sum_{k=1}^{n-1} \binom{n}{k} (2^k - 2) = 3^n - 3 \cdot 2^n + 3$$

- Asymptotically number of operation grows like $\boxed{3^n}$ instead of $\boxed{n!}$

Colour and Helicity

- Sum over colour and helicity configurations
- $8^{n_g} \times 3^{n_q} \times 3^{n_{\bar{q}}}$ colour configurations
- $2^{n_g} \times 2^{n_q} \times 2^{n_{\bar{q}}}$ helicity configurations

  $n_g, n_q, n_{\bar{q}}$ number of gluons, quarks and antiquarks
- Many colour and helicity configurations lead to zero amplitude

> - Any representation can be used as long as we end up with the correct sums on the average

Colour Treatment

P. Draggiotis, R. Kleiss, C. G. Papadopoulos Phys. Lett. B439, 157 (1998)

- Simplification for gluon fields

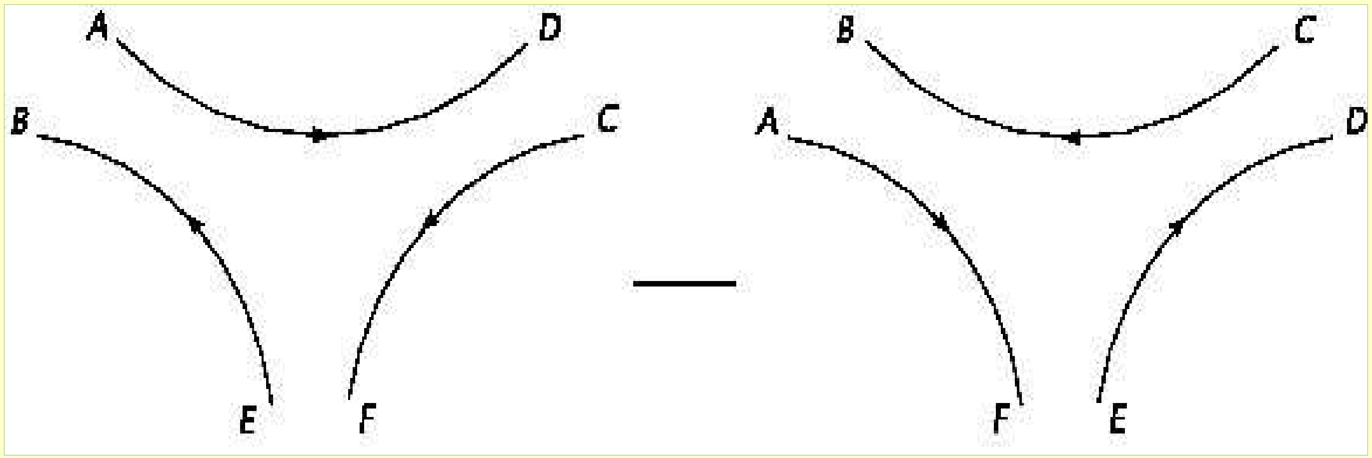
$$\boxed{G_{AB} = \sum_{a=1}^{8} t^{a}_{AB} G^{a}} \qquad A, B = 1, 2, 3$$

- New objects traceless 3x3 matrices in colour space
- Diagonalization of the colour structure of 3-gluon vertex

$$\boxed{f^{abc} t^{a}_{AB} t^{b}_{CD} t^{c}_{EF} = -\frac{i}{4}\left(\delta_{AD}\delta_{CF}\delta_{EB} - \delta_{AF}\delta_{CB}\delta_{ED}\right)}$$

Colour Treatment

- 3-gluon vertex in new representation



- Shows the colour flow in the real physical process
- Gluon represented by $q\bar{q}$ states in colour space
- Colour remains unchanged on an interrupted colour line

Colour Treatment

- Quarks and antiquarks already in this representation
- Additionally representation independent identity is used

$$\boxed{\sum_{a=1}^{8} t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)} \qquad i,j,k,l=1,2,3$$

- Recursion equations modified to reflect the new colour structure

> Next step - make the computation of the colour part of an amplitude more efficient !

Monte Carlo Techniques

- Produce a cross section with sufficient speed

$$\text{Sum over color } 3^{n_q+n_{\bar q}} \longrightarrow \text{Monte Carlo techniques}$$

- Choosing via MC particular colour and anti-colour assignment for quark, antiquark and gluon ($q\bar{q}$ pair)
- Necessary condition for non-vanishing colour assignment

$$N_C \,\mathrm{Type}\, I_i = N_{\bar C} \,\mathrm{Type}\, I_i \qquad i=1,2,3$$

$$3^{n_q+n_{\bar q}} \longrightarrow \sum_{i=0}^{n_q} \sum_{j=0}^{n_q-i} \sum_{k=0}^{n_q-i-j} \left(\frac{n_q!}{i!\,j!\,k!} \right)^2 \delta(i+j+k=n_q)$$

Helicity Treatment

- Summation over helicity configurations of the external partons

  $\longrightarrow$ MC integration over a phase variable

- Polarization vector for gluons:

$$\boxed{\epsilon^{\mu}_{\phi}(p)=e^{i\phi}\epsilon^{\mu}(p,+)+e^{-i\phi}\epsilon^{\mu}(p,-)}$$

- For incoming quarks e.g.:

$$\boxed{u_{\phi}(p)=e^{i\phi}u_{+}(p)+e^{-i\phi}u_{-}(p)}$$

- ϕ - random number $\phi\in(0,2\pi)$

Fermi Sign Function

- Incorporate Grassman variables for fermions (sign change when 2 identical fermions are interchanged)
- Momenta $P_1 \rightarrow (0001),\ P_2 \rightarrow (0010),\ P_3 = P_1 + P_2 \rightarrow (0011)$
- Sign relative to the permutation of 2 momenta is computed $\sigma(P_m, P_l)$
- Operation on two binary strings representing momenta $\sigma(m, l)$

$$\boxed{\sigma(m,l) = (-1)^{\chi(m,l)} \qquad \chi(m,l) = \sum_{i=n}^{2} \hat{m}_i \left(\sum_{j=n}^{i-1} \hat{l}_j \right)}$$

- Hat over binary string $\longrightarrow$ set 0 when external particle = boson

Results

Preliminary

- For CMS $\sqrt{s}=14\,\mathrm{TeV}$
- Cuts $p_{T_i} > 60\ \mathrm{GeV}, \theta_{ij} > 30^{\circ}, |\eta_i| < 3$
- Non-running coupling constant equal to unity, parton sf CTEQ6

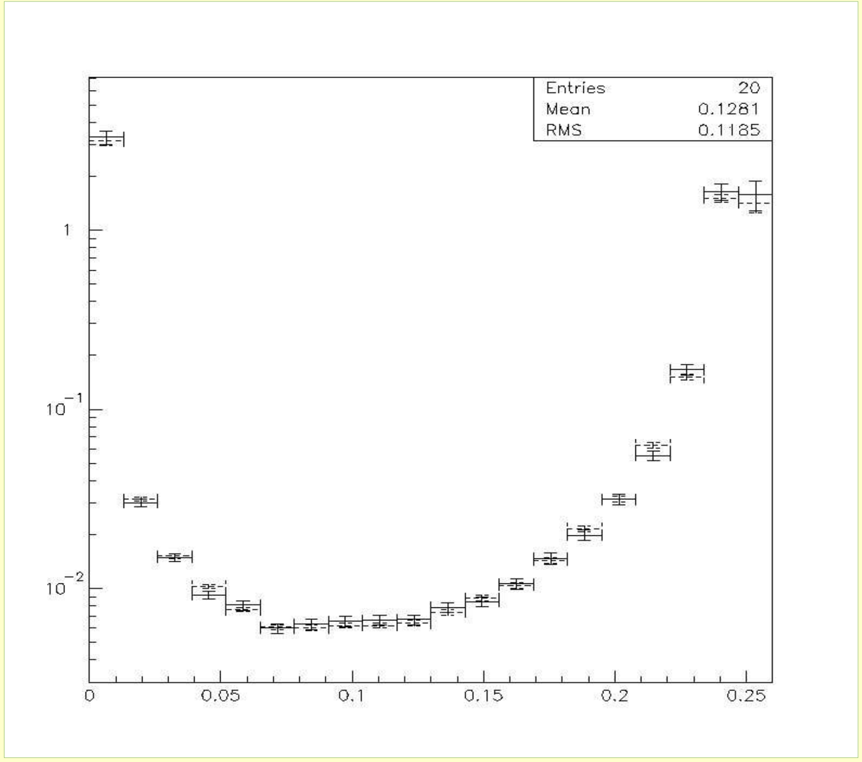
Process	Cross section (nb)	Error (%)
$gg \to gggg$	2.37435	4
$gg \to ggggg$	0.14865	5
$gg \to gggggg$	$0.106563 \cdot 10^{-1}$	10
$u\bar{u} \to gggg$	$0.526137 \cdot 10^{-3}$	3
$u\bar{u} \to ggggg$	$0.366030 \cdot 10^{-4}$	5

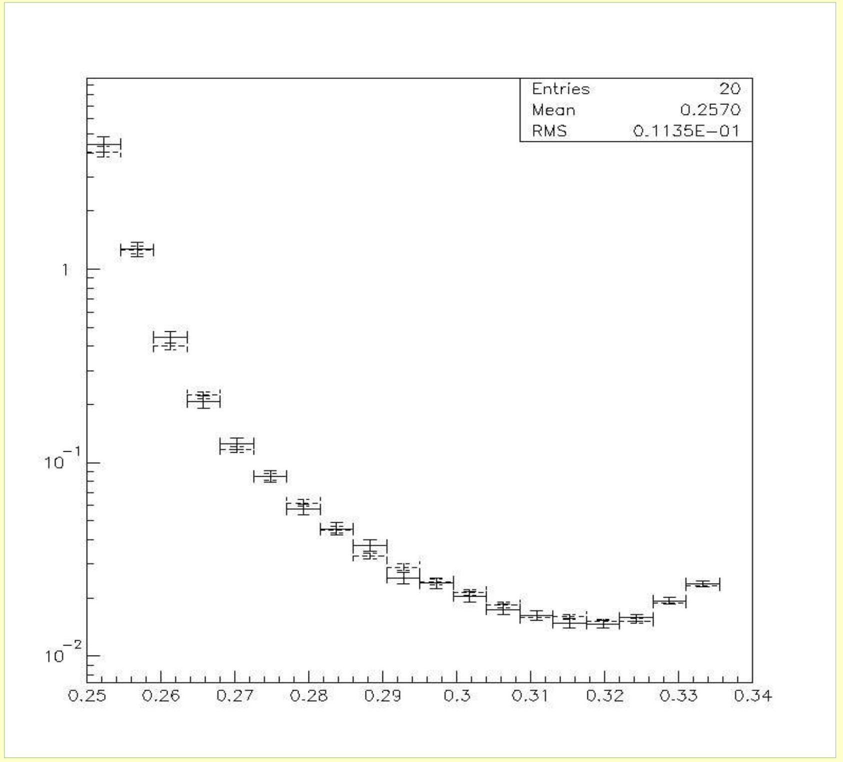
Results

Preliminary

- Distributions for particular colour configurations
- Monte Carlo vs sum over all colour configurations

4g case



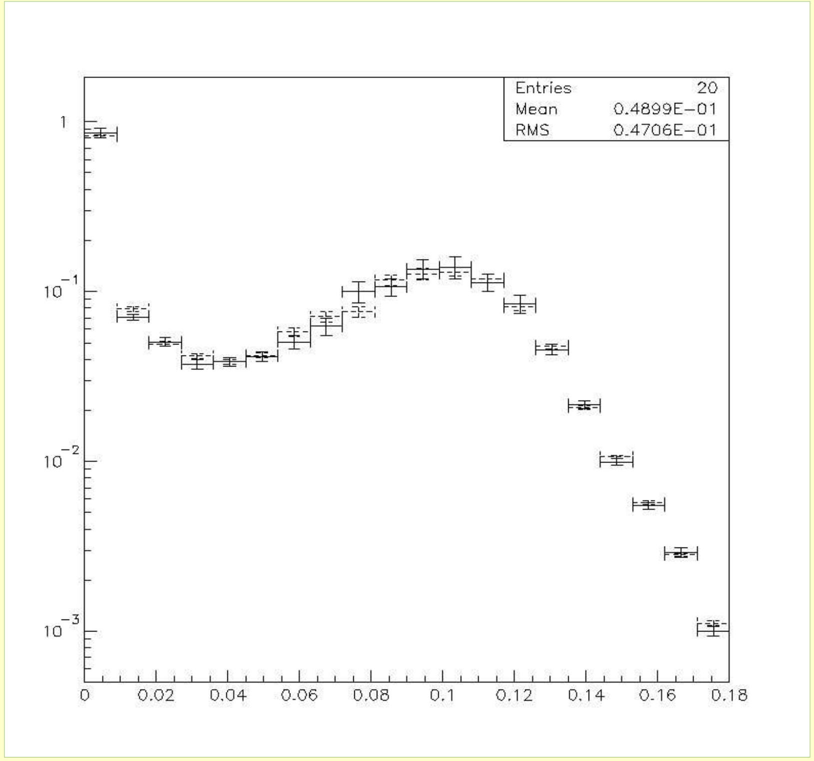


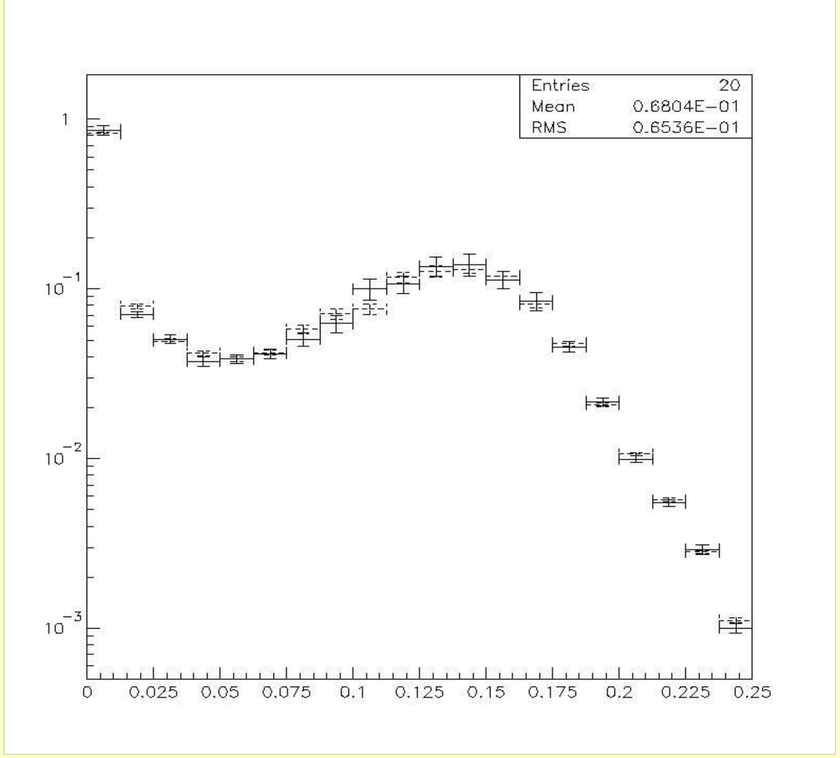
Results

Preliminary

- Distributions for particular colour configurations
- Monte Carlo vs sum over all colour configurations

5g case





Summary

- Procedure to calculate matrix elements and cross section in QCD using an iterative algorithm based on the Dyson-Schwinger equation is presented
- Free from the task of computing Feynman graphs for a process
- Monte Carlo summation instead of summation over all possible colour and helicity configurations is used (both cases are included in code)
- Based on this algorithm a FORTRAN 95 package has been developed (massive quarks are included)
- At this stage code can compute scattering amplitudes and partonic cross sections (parton SF are included)

Summary

- Good for global quantities like total cross sections and for transverse momenta or invariant mass distributions

- <u>Future interests involve</u>
  - Improve the efficiency of the MC over colour by using self-adaptive MC event generator like eg VEGAS or FOAM
  - Convolution of this code with fragmentation codes like eg HERWIG or PYTHIA to perform realistic simulations of multi-jet processes
  - Efficient phase-space Monte Carlo generator

Notations And Conventions

- Light cone representation

$$V^A = (V^0 + V_z, V^0 - V_z, V_x + iV_y, V_x - iV_y) \qquad A = 1, \ldots, 4$$

- Scalar product

$$V * Z = \frac{1}{2}(V^1 \cdot Z^2 + V^2 \cdot Z^1 - V^3 \cdot Z^4 - V^4 \cdot Z^3)$$

- Polarization vectors

$$\epsilon_-^A = \left(\frac{-p_T}{\sqrt{2}|\vec{p}|}, \frac{p_T}{\sqrt{2}|\vec{p}|}, \frac{(p_x + ip_y)(|\vec{p}| + p_z)}{\sqrt{2}|\vec{p}|p_T}, \frac{(p_x - ip_y)(-|\vec{p}| + p_z)}{\sqrt{2}|\vec{p}|p_T} \right)$$

$$\epsilon_+^A = \left(\frac{p_T}{\sqrt{2}|\vec{p}|}, -\frac{p_T}{\sqrt{2}|\vec{p}|}, \frac{(p_x + ip_y)(|\vec{p}| - p_z)}{\sqrt{2}|\vec{p}|p_T}, \frac{(p_x - ip_y)(-|\vec{p}| - p_z)}{\sqrt{2}|\vec{p}|p_T} \right)$$

Notations And Conventions

- Dirac matrices in chiral representation
- Wave functions which describe massive spinors

$$u_+(p) = \begin{pmatrix} r/c \\ a(p_x + ip_y)/r \\ -mb/r \\ -m(p_x + ip_y)/r \end{pmatrix} \qquad \bar{u}_+(p) = \begin{pmatrix} mb/r \\ m(p_x - ip_y)/r \\ -r/c \\ -a(p_x - ip_y)/r \end{pmatrix}$$

$$u_-(p) = \begin{pmatrix} m(p_x - ip_y)/r \\ -mb/r \\ -a(p_x - ip_y)/r \\ r/c \end{pmatrix} \qquad \bar{u}_-(p) = \begin{pmatrix} a(p_x + ip_y)/r \\ -r/c \\ -m(p_x + ip_y)/r \\ mb/r \end{pmatrix}$$

Notations And Conventions

$$v_+(p)=\begin{pmatrix}-m(p_x-ip_y)/r\\ mb/r\\ -a(p_x-ip_y)/r\\ r/c\end{pmatrix}\qquad \bar{v}_+(p)=\begin{pmatrix}a(p_x+ip_y)/r\\ -r/c\\ m(p_x+ip_y)/r\\ -mb/r\end{pmatrix}$$

$$v_-(p)=\begin{pmatrix}r/c\\ a(p_x+ip_y)/r\\ mb/r\\ m(p_x+ip_y)/r\end{pmatrix}\qquad \bar{v}_-(p)=\begin{pmatrix}-mb/r\\ -m(p_x-ip_y)/r\\ -r/c\\ -a(p_x-ip_y)/r\end{pmatrix}$$

$$a=p_0+|\vec{p}|,\quad b=p_z+|\vec{p}|,\quad c=2|\vec{p}|,\quad r=\sqrt{abc}$$

Notations And Conventions

- Spinors for massless particle

$$u_R(p)=\begin{pmatrix}\sqrt{p_0+p_z}\\(p_x+ip_y)/\sqrt{p_0+p_z}\\0\\0\end{pmatrix}\qquad \bar{u}_R(p)=\begin{pmatrix}0\\0\\-\sqrt{p_0+p_z}\\-(p_x-ip_y)/\sqrt{p_0+p_z}\end{pmatrix}$$

$$u_L(p)=\begin{pmatrix}0\\0\\-(p_x-ip_y)/\sqrt{p_0+p_z}\\\sqrt{p_0+p_z}\end{pmatrix}\qquad \bar{u}_L(p)=\begin{pmatrix}(p_x+ip_y)/\sqrt{p_0+p_z}\\-\sqrt{p_0+p_z}\\0\\0\end{pmatrix}$$