Modelling of non-Gaussian tails of multiple Coulomb scattering in track fitting with a Gaussian-sum filter

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Outline

- **Introduction**
- **A Gaussian-sum filter for treatment of non-Gaussian tails of multiple scattering**
- **Results from a simulation study**
- **Summary and outlook**

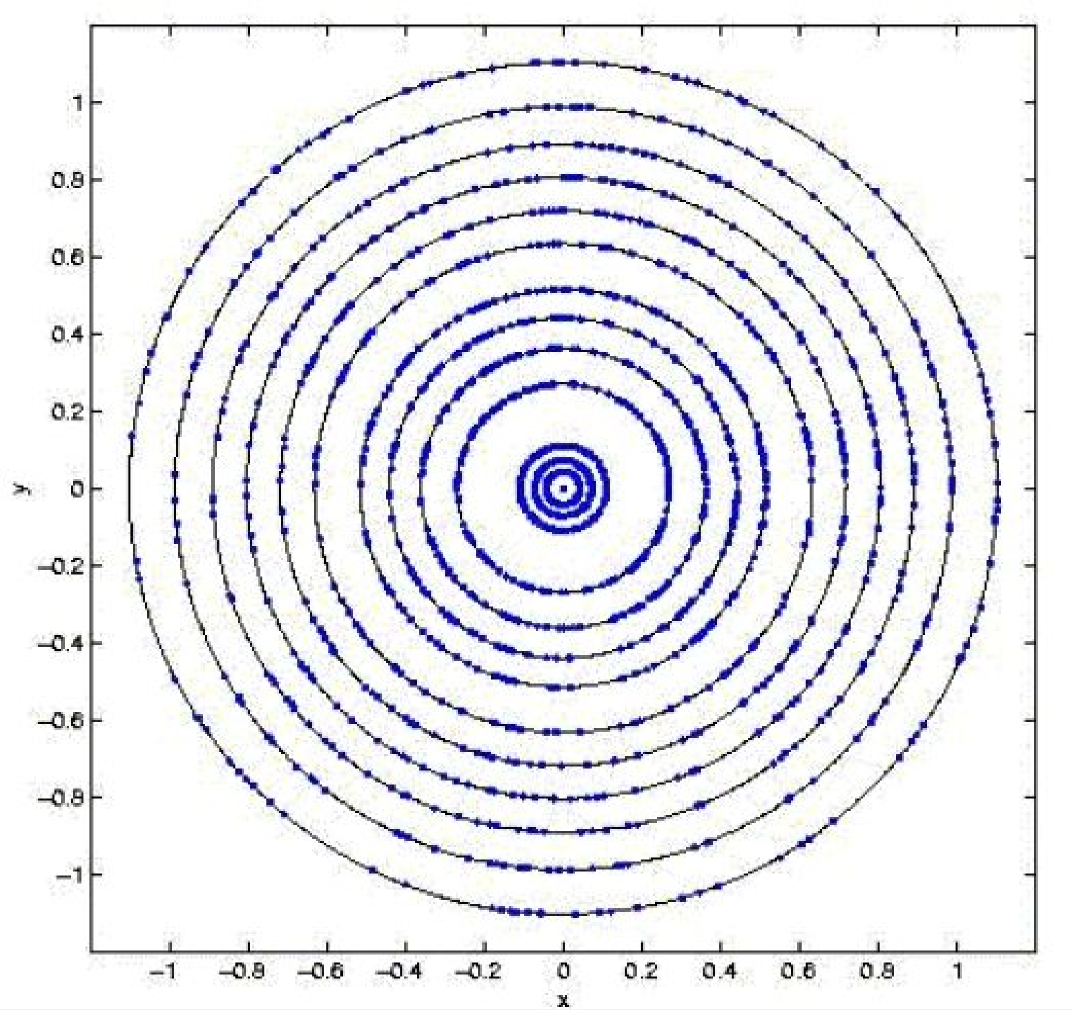
Introduction

- Track reconstruction is traditionally divided into two separate subtasks:
  - track finding
  - track fitting
- Track finding:
  - division of set of hits in a tracking detector into subsets
  - each subset contains hits believed to originate from the same particle

Introduction

- Track fitting:
  - starts out with the hits inside one subset as provided by the track finder
  - aims to optimally estimate a set of track parameters from the information from the hits
  - this information is often a set of measurements of hit positions

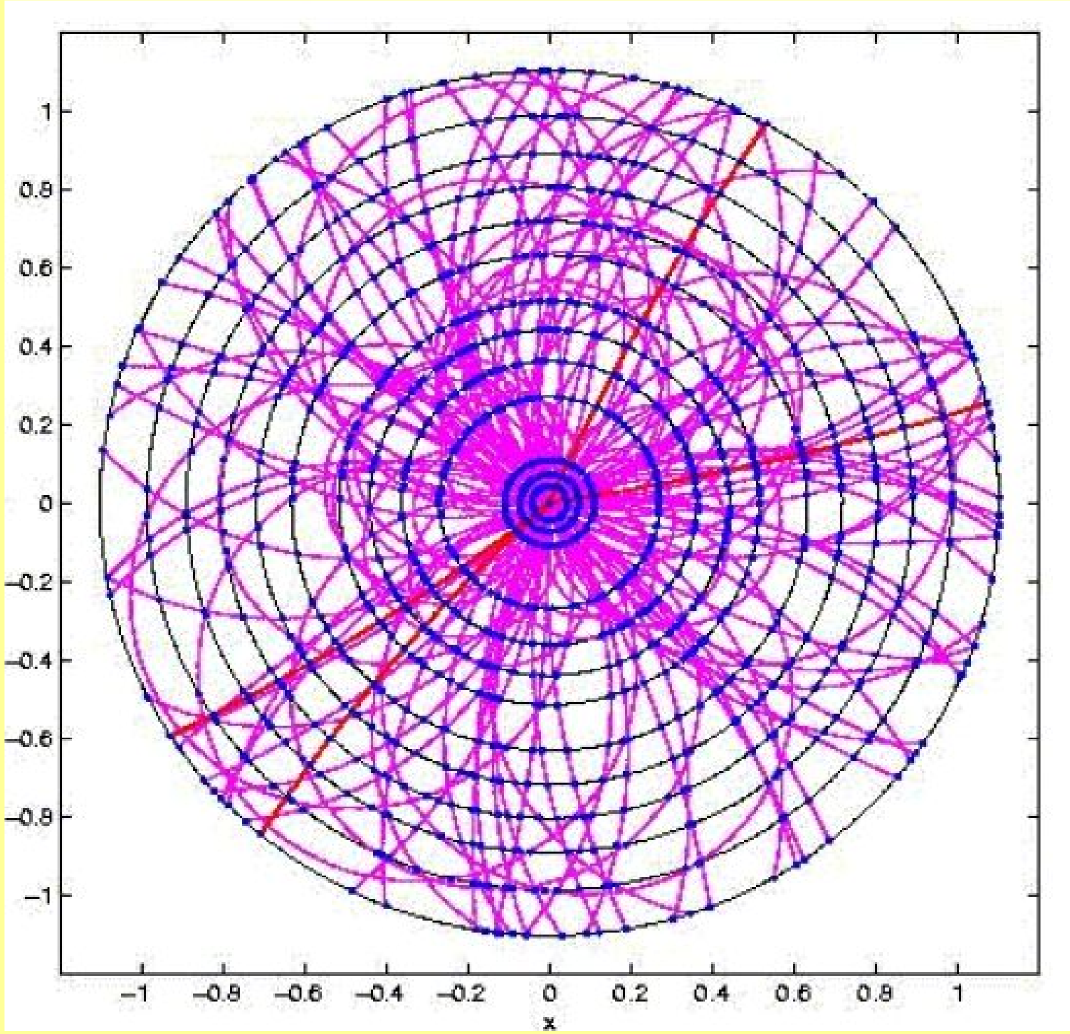
Introduction



Tracking detector with cylindrical layers

Input to track finding is all or parts of the measurements in the detector at a given instance

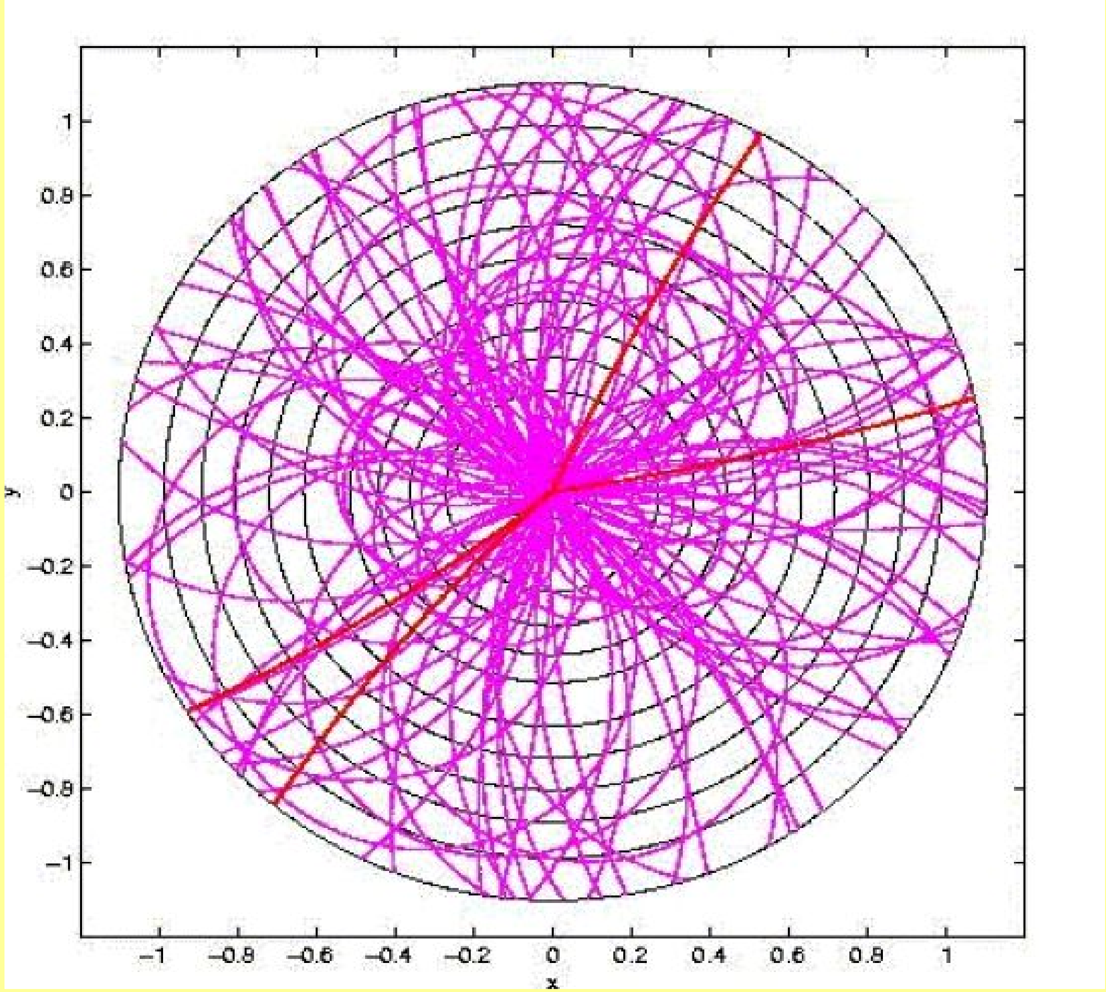
Introduction



A successful track finder identifies a set of potential tracks as indicated in the figure

Hits along these tracks are given to the track fitter for parameter estimation and final validation of track candidate

Introduction



After the track fit one usually forgets about the hits and only cares about a compact representation of the tracks

Introduction

- This talk will be about a specific algorithm for track fitting
  - aims to treat effects of multiple Coulomb scattering in a very precise way
- The talk after lunch will attack the track finding problem
  - some new adaptive algorithms have been developed and compared to the Kalman filter

A Gaussian-sum filter for multiple scattering

- The Kalman filter has since many years been the default algorithm for track fitting:
  - it is a least-squares estimator and therefore optimal when probability densities involved in fitting procedure are Gaussian
  - due to recursive nature of the filter inversion of (potentially) big covariance matrix of measurements is avoided
  - optimal estimates of track parameters are available at any detector layer, enabling material effects to be treated locally

A Gaussian-sum filter for multiple scattering

- When some of the probability densities involved during reconstruction are not Gaussian, the Kalman filter is not necessarily optimal anymore
- Plausible that a non-linear estimator which better takes the actual shape of the distribution into account can do better
- One such estimator is the Gaussian-sum filter (GSF)
  - adequate when densities can be modelled by Gaussian mixtures, i.e weighted sums of single Gaussians

A Gaussian-sum filter for multiple scattering

- The state vector of the GSF also becomes a Gaussian mixture, implying that the GSF estimate of track parameters is a set of parameter vectors, covariance matrices and weights
- The algorithm resembles a set of Kalman filters running in parallel
- Each of the filters or components has a weight attached
- If a single quantity is needed for the final estimate, the mean value and covariance matrix of the mixture is often used

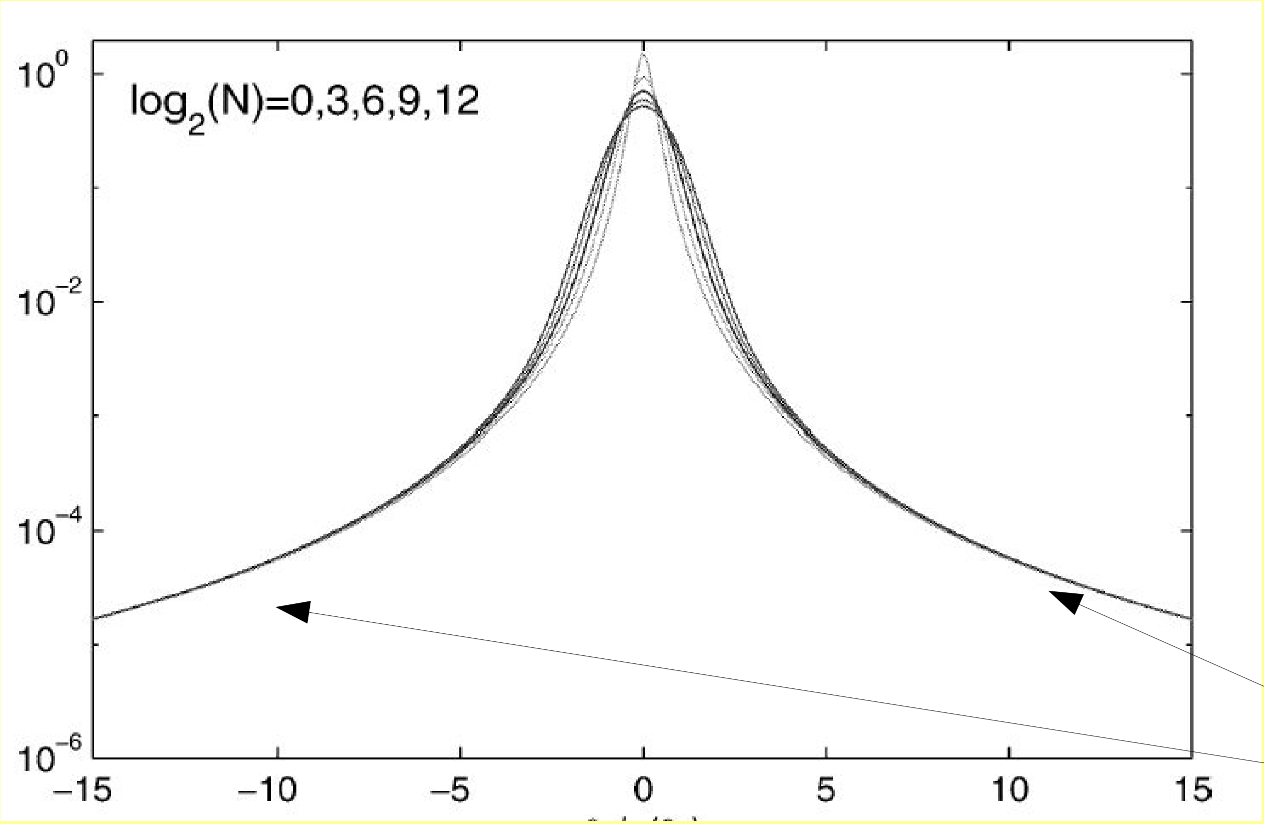
A Gaussian-sum filter for multiple scattering

- The GSF has been successfully applied several times in the past years:
  - treatment of bremsstrahlung energy loss in track fitting in CMS tracker at CERN (Adam, Frühwirth, Strandlie and Todorov, CHEP2003)
  - vertex fitting of tracks originating from a GSF track fit (Frühwirth and Speer, ACAT2003)
  - track finder by modelling distribution of several competing measurements as Gaussian mixture (Strandlie and Frühwirth, ACAT2005)

A Gaussian-sum filter for multiple scattering

- This work presents an implementation of a GSF for treatment of non-Gaussian tails of multiple scattering
- It uses a Gaussian-mixture approximation of the multiple scattering deflection angle (Frühwirth and Regler, NIM A (2001))
- Plots on following slides from Frühwirth and Regler (with permission)

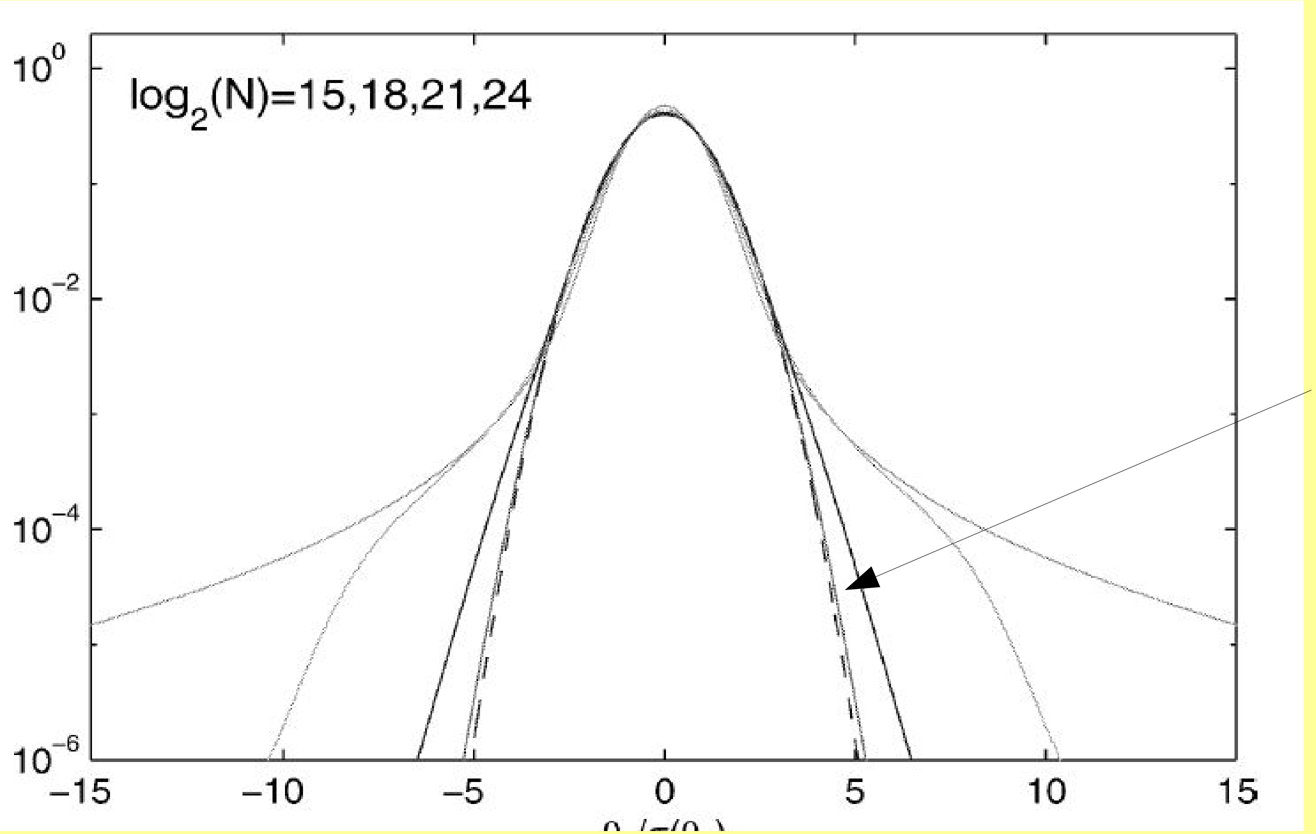
A Gaussian-sum filter for multiple scattering



Distribution of deflection angle of multiple scattering in standard measure for various numbers of scattering centres

Long, non-Gaussian tails!!

A Gaussian-sum filter for multiple scattering

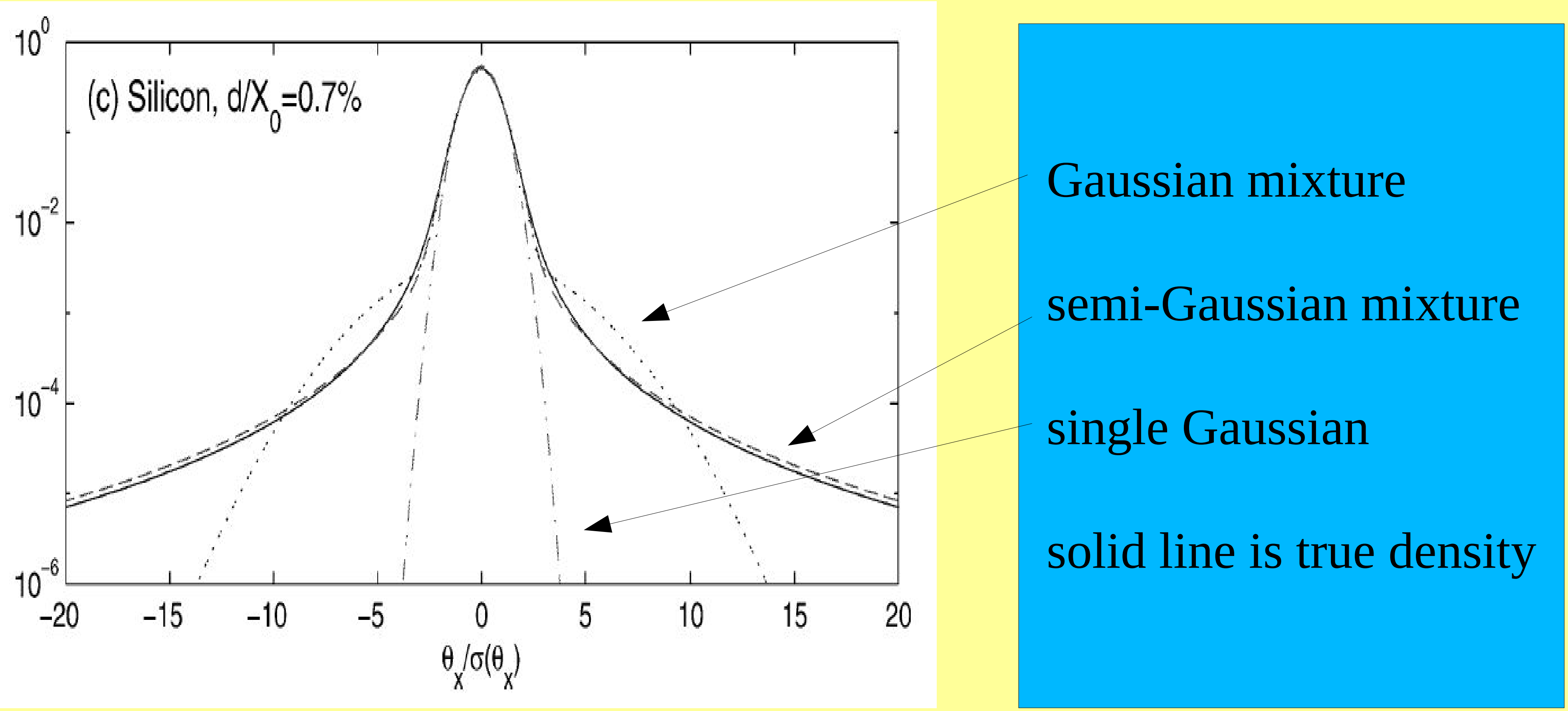


eventually converging towards Gaussian distribution

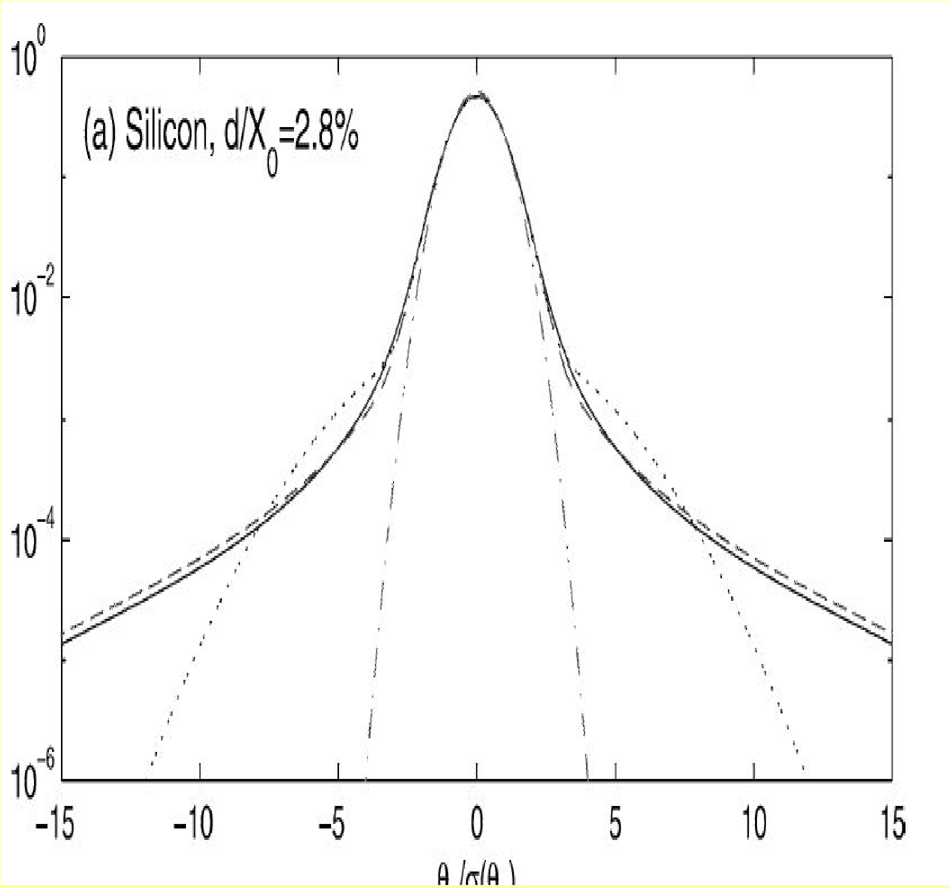
A Gaussian-sum filter for multiple scattering

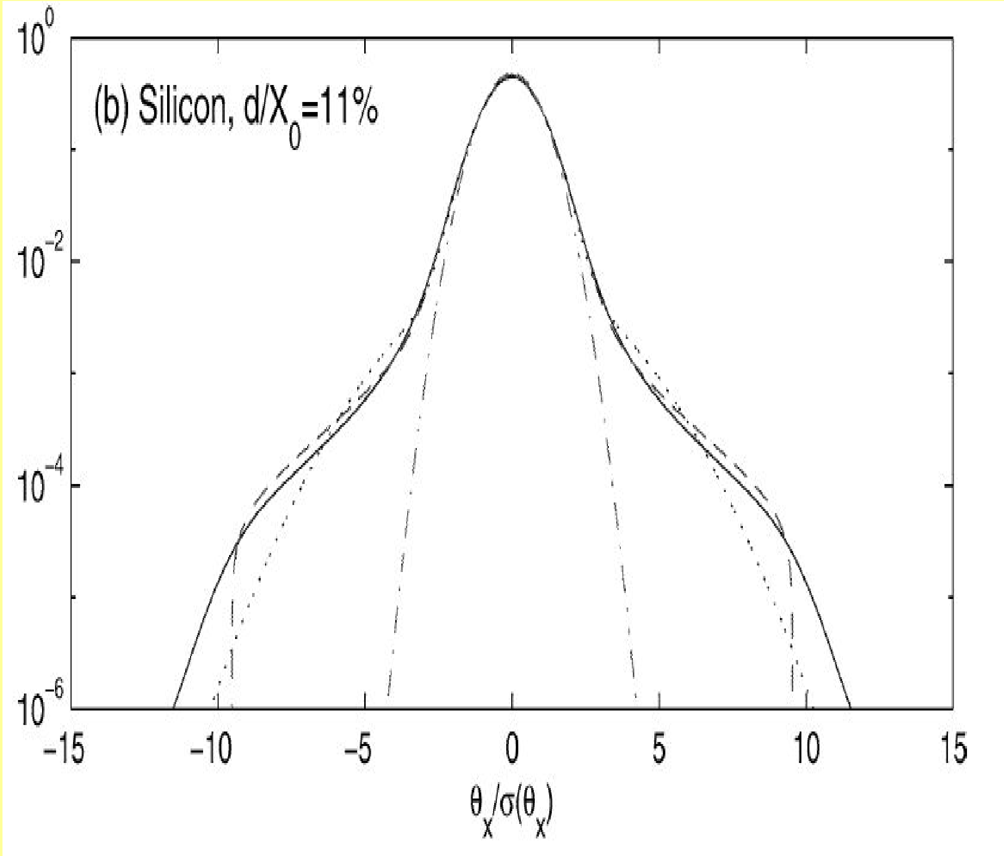
- Frühwirth and Regler showed that a two-component Gaussian-mixture approximates quite well the true density of multiple scattering
- A parametrization of the mixture variances and weights as a function of radiation length is available (means are always zero)
- A two-component, semi-Gaussian mixture approximation of the multiple scattering density (for simulation purposes in thin scatterers) has also been derived (Frühwirth and Liendl, NIM A (2001))

A Gaussian-sum filter for multiple scattering

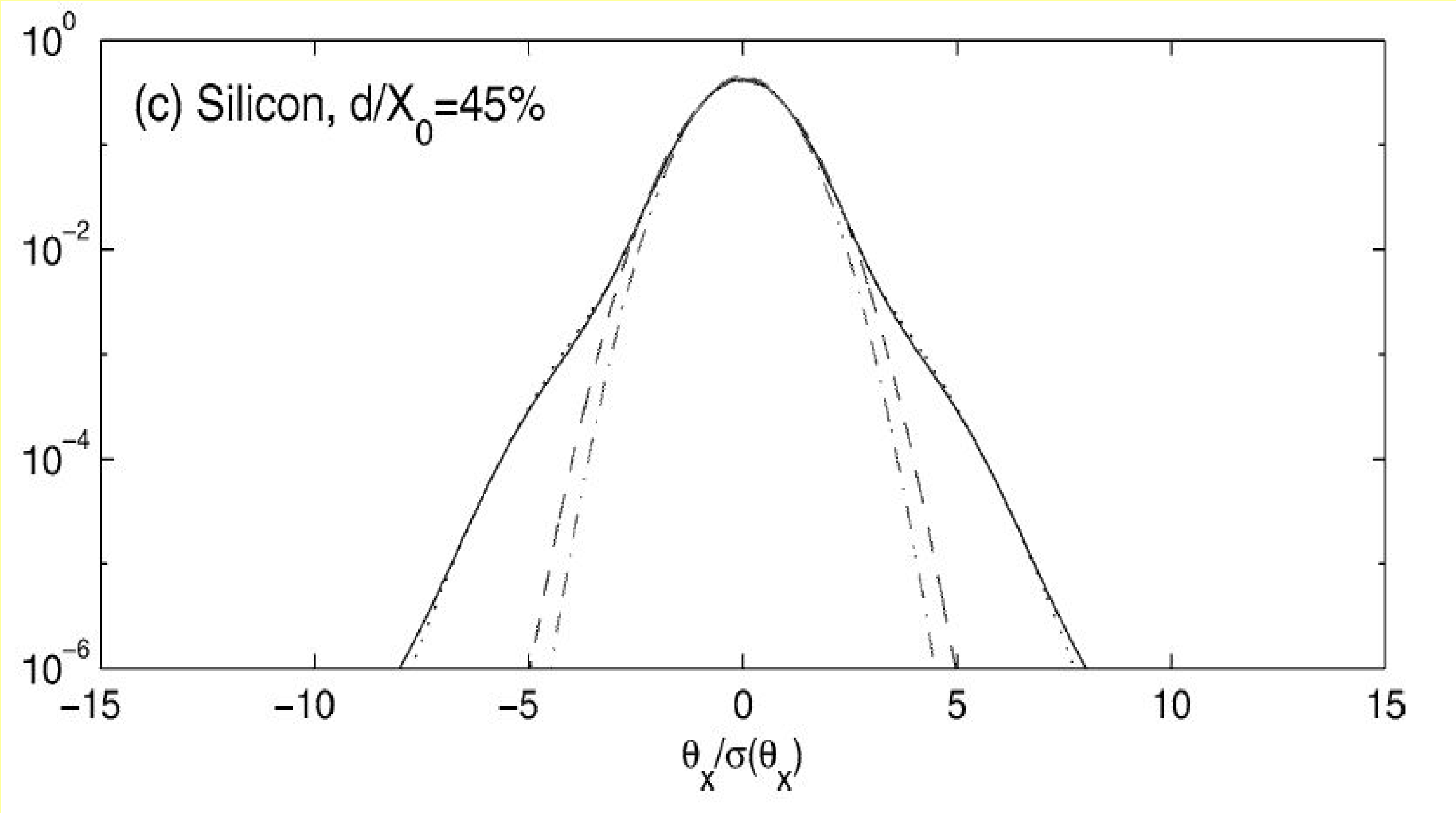


A Gaussian-sum filter for multiple scattering





A Gaussian-sum filter for multiple scattering



A Gaussian-sum filter for multiple scattering

- As the Kalman filter, the GSF proceeds by alternating propagation and update steps
- Propagation of each of the components is done by a standard geometrical propagator, using the relevant track model
- The parameter vector of each of the components is updated as in the Kalman filter

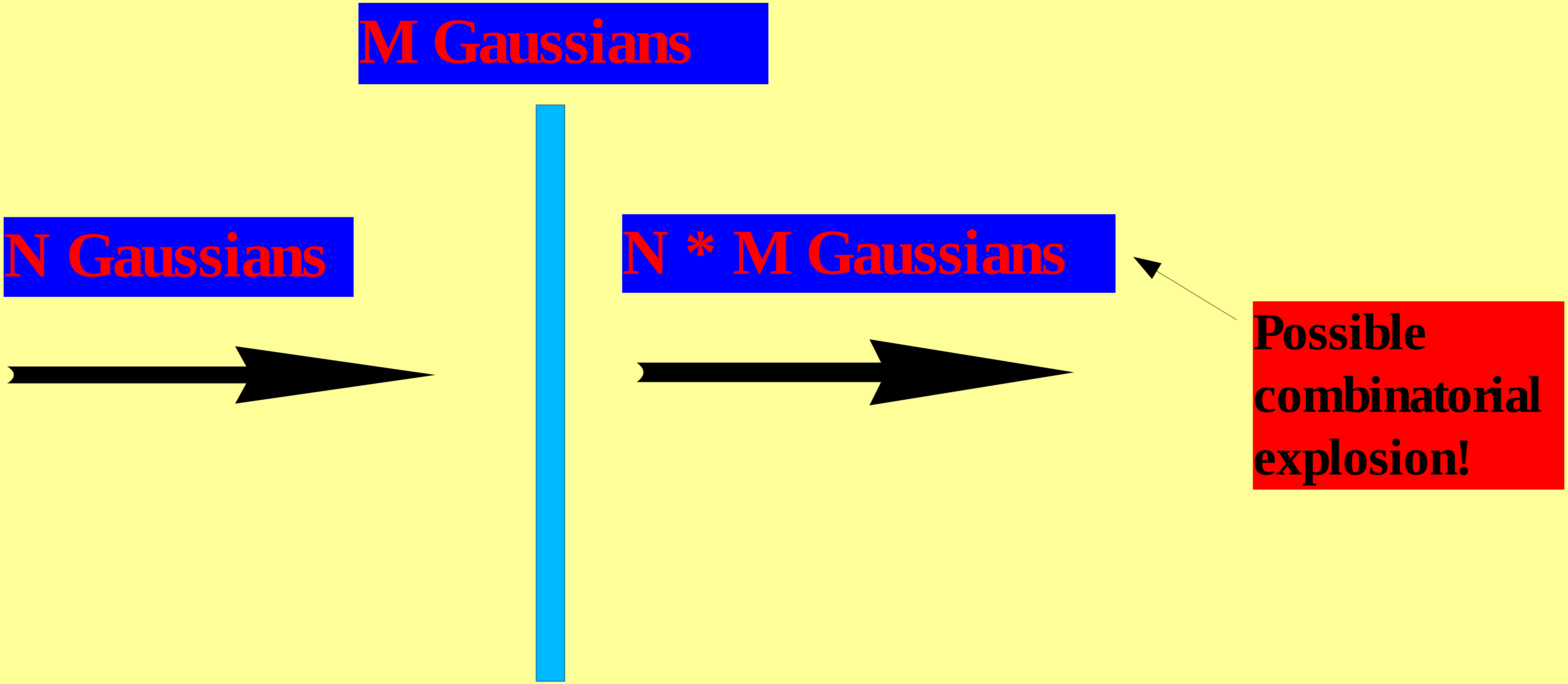
A Gaussian-sum filter for multiple scattering

- The posterior weights are updated by taking the distances between the measurement and all predicted components into account
  - competition between different components
  - components far away from measurement tends to be downweighted by components closer to the measurement
  - adaptive behaviour!

A Gaussian-sum filter for multiple scattering

- State vector of GSF upon entering a layer with material in general consists of N components
- Inclusion of multiple scattering effects in the layer amounts to a convolution of the probability densities
- The mixture after the convolution consists of 2N components, if multiple scattering is described by a two-component mixture

A Gaussian-sum filter for multiple scattering



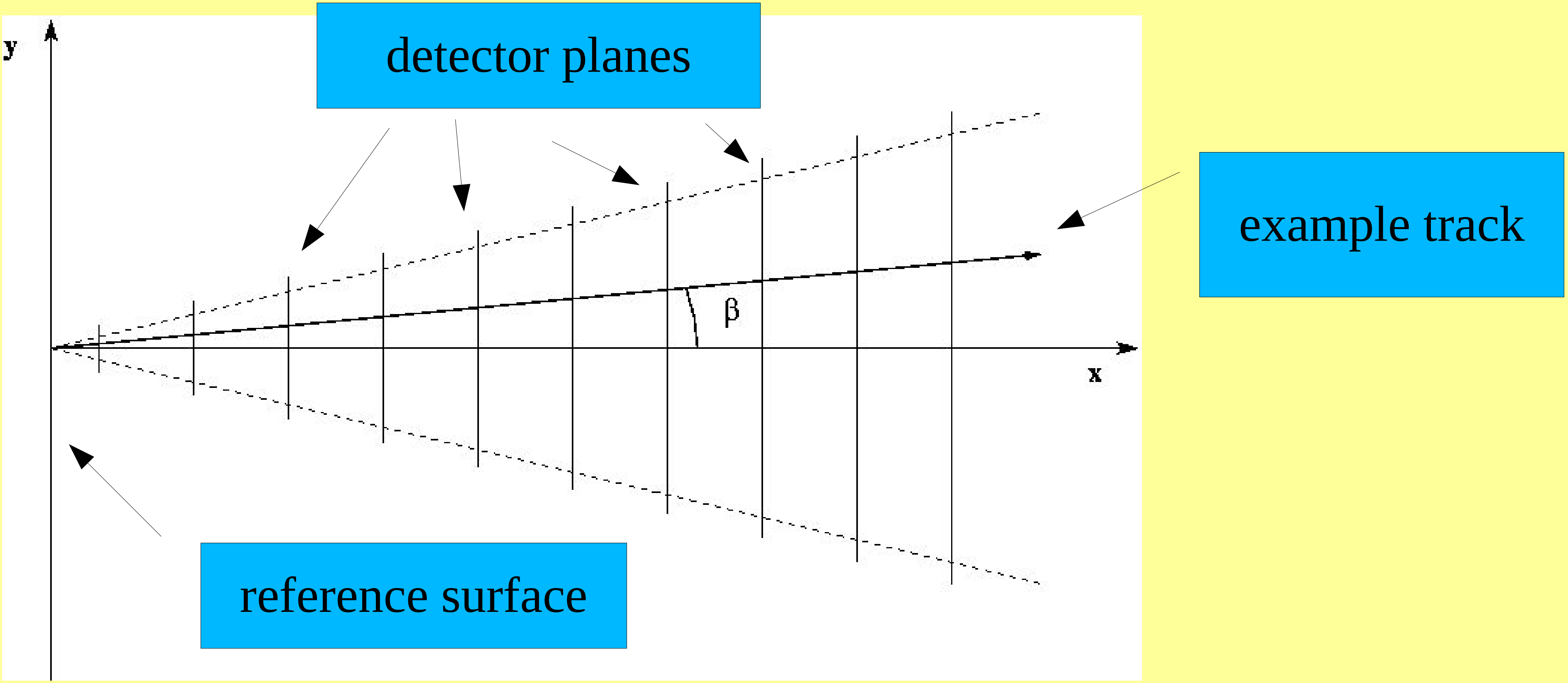
A Gaussian-sum filter for multiple scattering

- Need to invoke procedure for restricting number of components in state vector mixture, yet keeping as much information as possible inherent in original mixture
- Strategy is to successively merge components being close in parameter space until a defined upper limit is reached
- Such a procedure preserves the first two moments of the mixture
  - desired statistical properties of the estimator are kept

Simulation study

- Simulation study with implementation of GSF for treatment of multiple scattering has been performed
- Aim of study has been to be “proof-of-principle” with focus on qualitative features
  - very simple track model (linear) and detector geometry
- Reconstructed parameters are position and angle of inclination at reference surface (“vertex”)

Simulation study



Simulation study

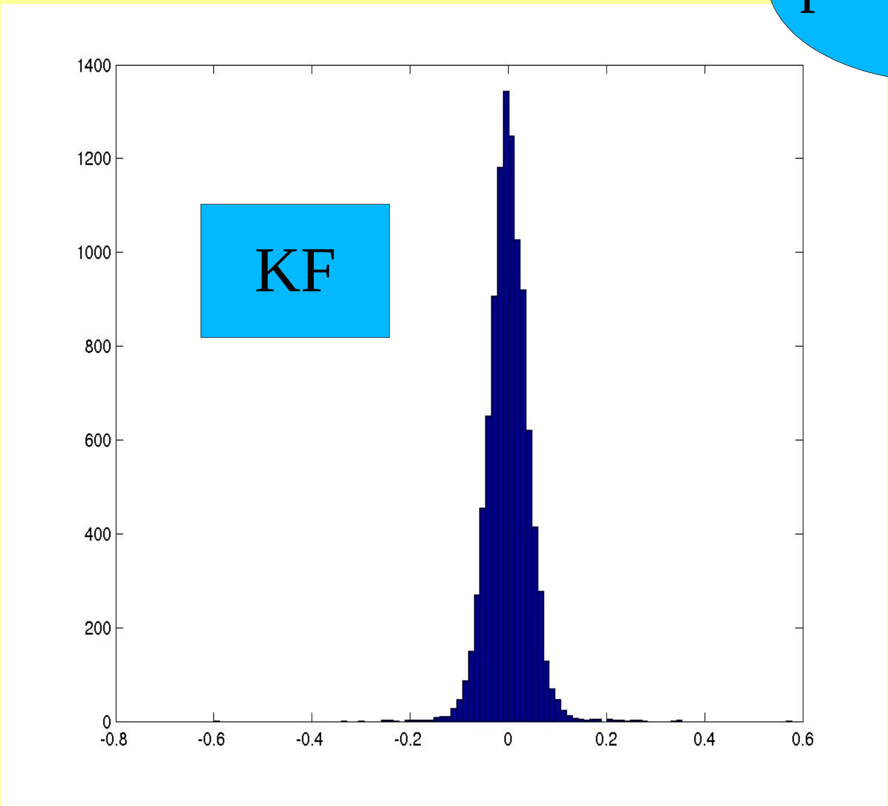
- 10 detector surfaces with 4 cm spacing, thickness 1 % of a radiation length each
- Tracks simulated from reference point in a range of the inclination angle ß (approximately as indicated in figure)
- Momentum range from 0.2 to 100 GeV
- Multiple scattering angle randomly drawn from semi-Gaussian mixture
- Measurement error 30 μm (typical for modern pixel/silicon strip detector elements)

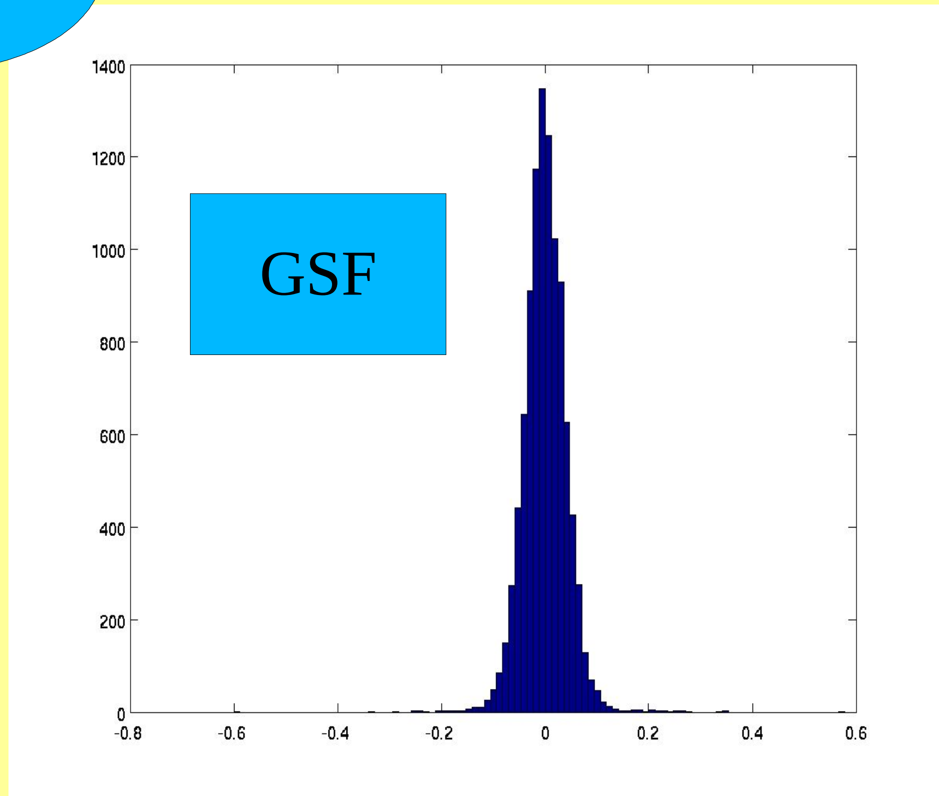
Simulation study

- For a detector layer with thickness 1 % of a radiation length:
  - tail component in Gaussian mixture is four times wider than core component, and tail weight is about 0.035
- Initial fit: global linear regression neglecting material effects to obtain initial parameters at the outermost detector layer
- Kalman filter/GSF running inwards, reconstructed parameters compared to true parameters at reference surface
- For study of GSF resolutions, mean value of state vector mixture at this surface is calculated
- Only correct hits used in reconstruction, assuming perfect track finding

Simulation study

p=0.2 GeV

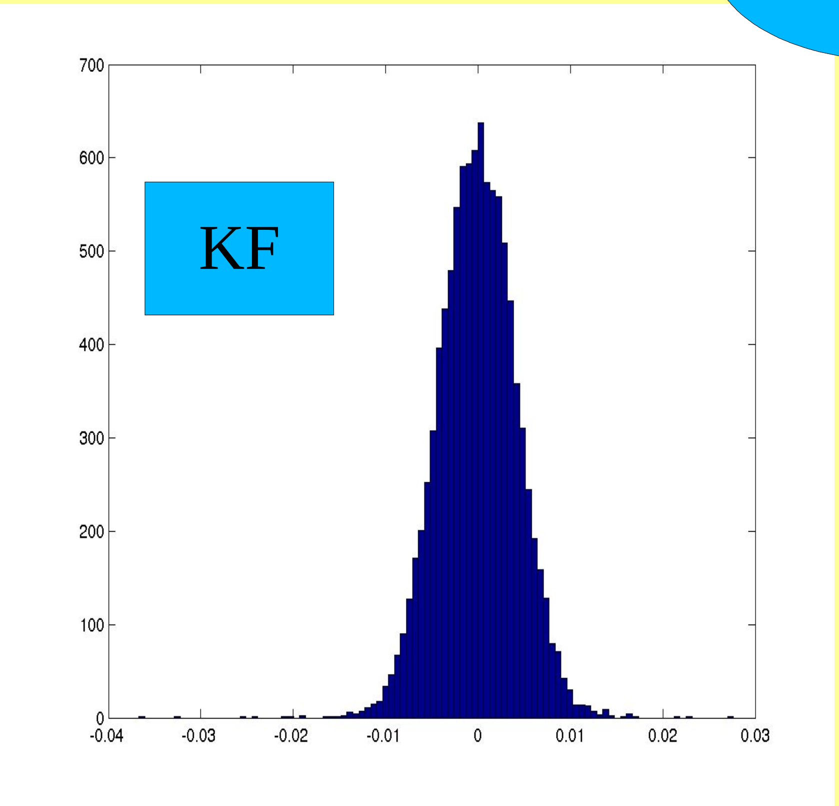


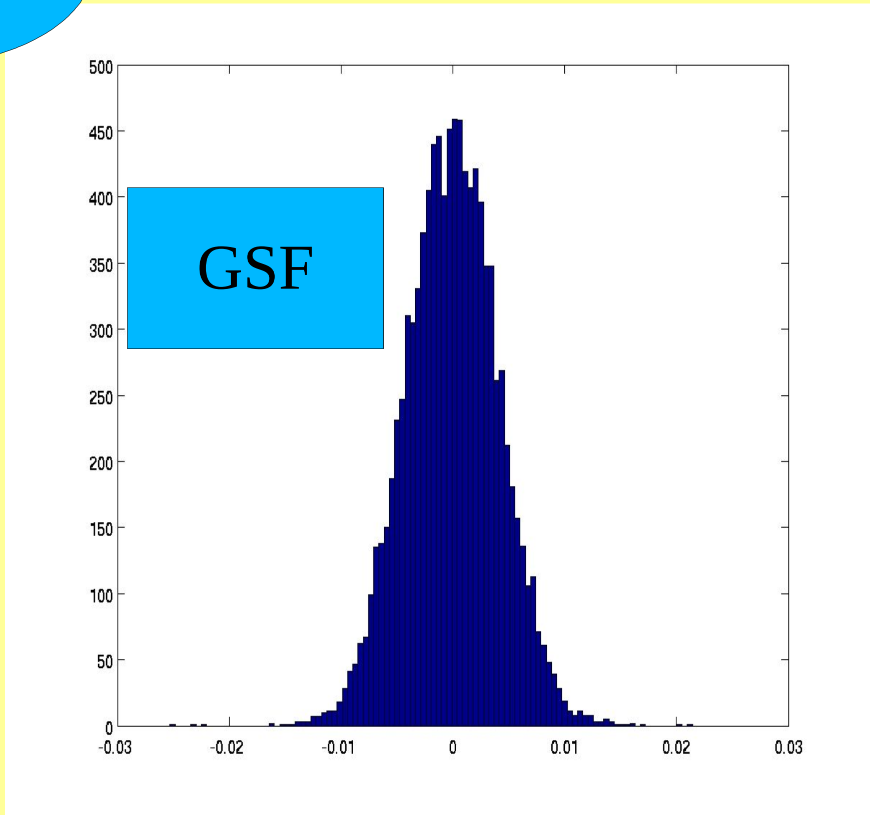


position residuals

Simulation study

p=5 GeV





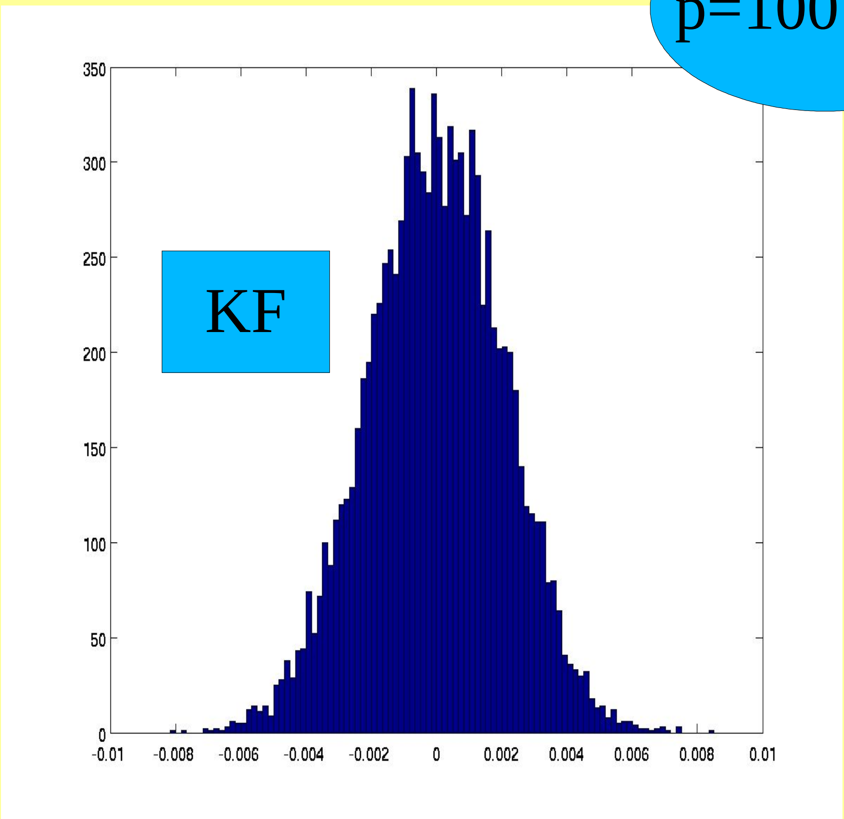
position residuals

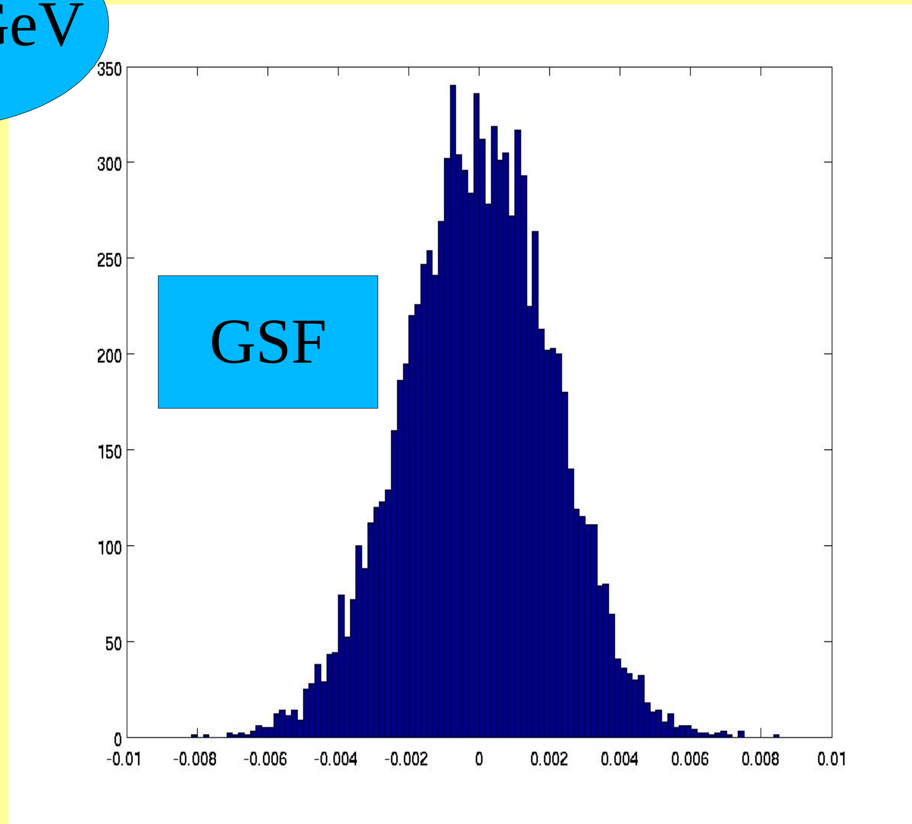
Are Strandlie

ACAT2005, Zeuthen, May 22-27

Simulation study

p=100 GeV





position residuals

Simulation study

p	KF	GSF_2	GSF_4	GSF_8
0.2	1	1	1	1
1	1.0079	1.0079	1	1
5	1.019	1.019	1.00097	1
10	1.0095	1.0095	1.0032	1
100	1	1	1	1

Standard deviations of position residuals, relative to GSF with 8 components kept

Corresponding table for inclination angle residuals looks very much the same

Simulation study

- KF and GSF equally precise at low and high momenta
- GSF slightly more precise at intermediate momenta
- Resolution at low momenta dominated by scattering in first layer
  - neither KF nor GSF can quantitatively measure such scattering
- Resolution a high momenta dominated by measurement error
  - GSF not able to see structure of multiple scattering angle at such momenta

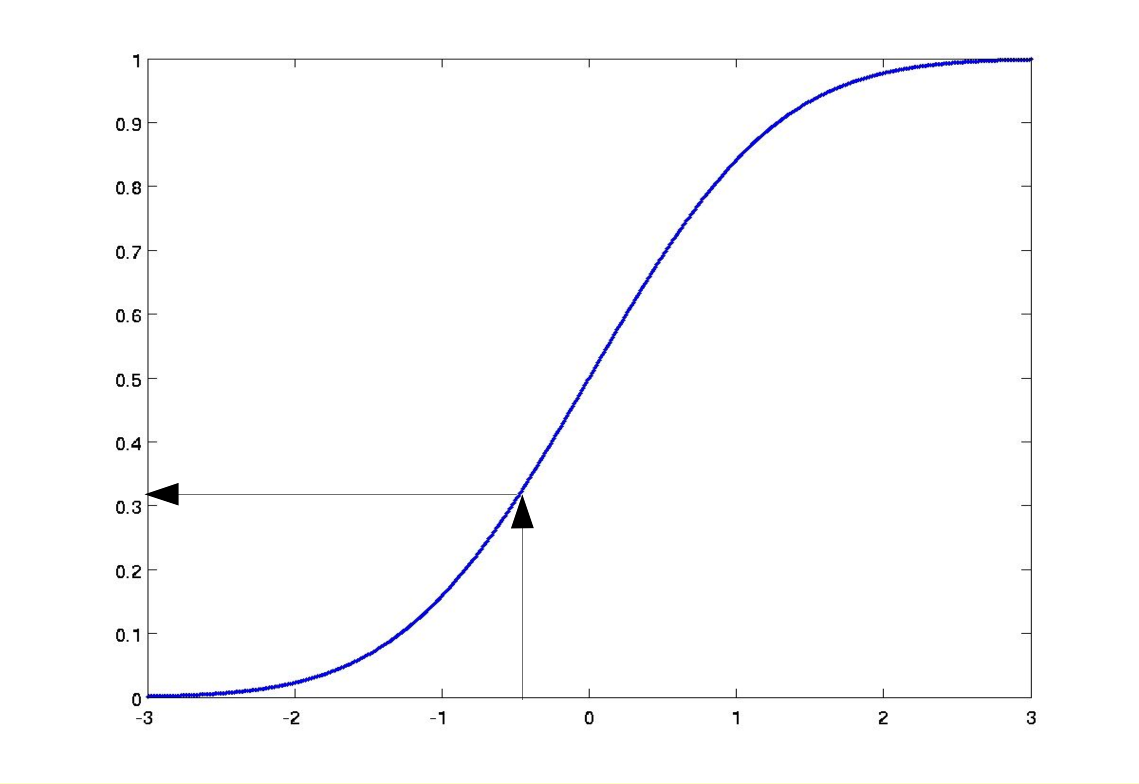
Simulation study

- How well do the estimates of the errors or uncertainties of the track parameters reproduce the actual spread of the parameters?
- Usual practice is to plot a histogram of the cumulative distribution function (cdf) of the chisquare of the track parameters with respect to the true parameters
- If the chisquare really is distributed according to a chisquare distribution, the probability histogram should be reasonably flat

Simulation study

- The same strategy can be used for evaluating a hypothesis about the distribution of any kind of statistic:
  - calculate the value of a test statistic
  - calculate the cdf of the distribution this statistic is believed to obey
    - p-value or probability transform
  - fill up a histogram and check whether it is flat or not

Simulation study



Example: cdf of standard, normal distribution

Quantity believed to obey standard normal distribution has value about -0.5

Histogram entry has value a bit more than 0.3

Simulation study

- KF estimate is single parameter vector and a covariance matrix
  - defining single Gaussian
  - true parameter vector is believed to obey a Gaussian distribution
- GSF estimate is a set of parameter vectors, covariance matrices and weights
  - defining Gaussian mixture
  - true parameter vector is believed to obey a Gaussian-mixture distribution

Simulation study

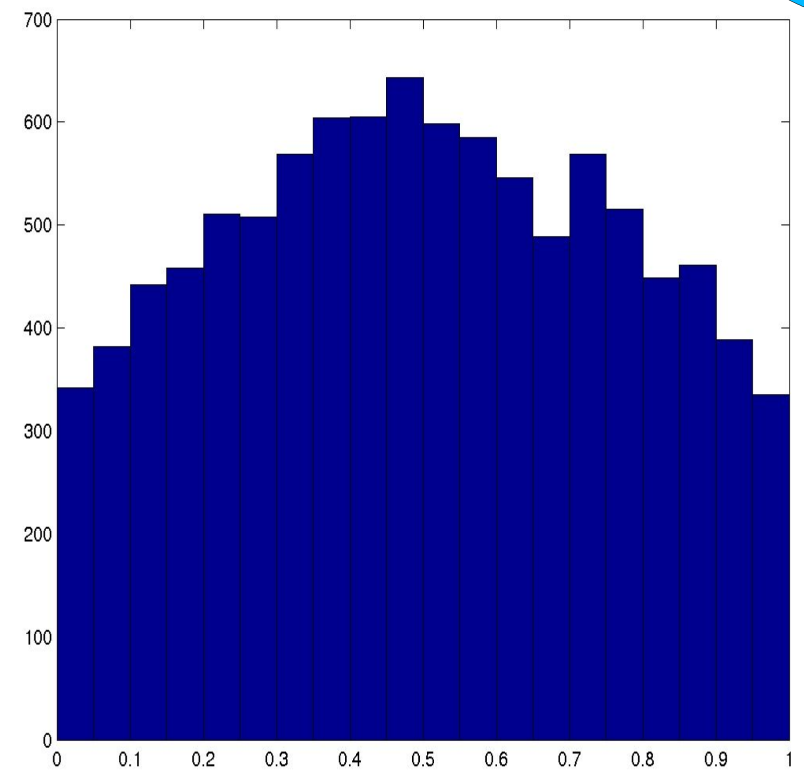
- For an estimated parameter of a reconstructed track, the histogram entry is the integral of the probability distribution below the corresponding true value
  - integral of single Gaussian for the KF
  - integral of Gaussian mixture for GSF
- Next slides show resulting histograms for position parameter
- Again, corresponding histograms for angle of inclination look very similar

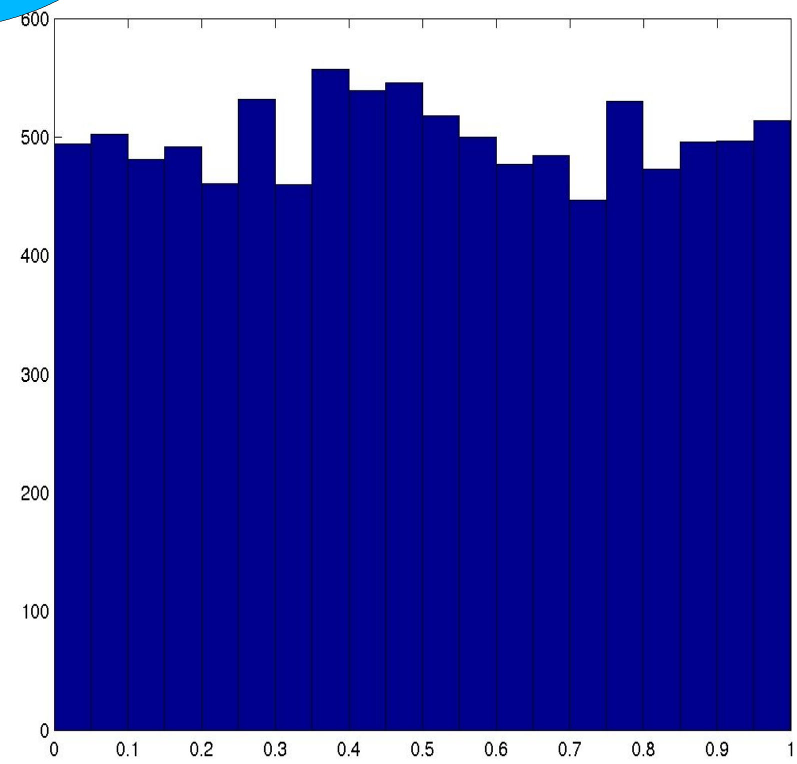
Simulation study

KF

p=0.2 GeV

GSF, keeping
2 components



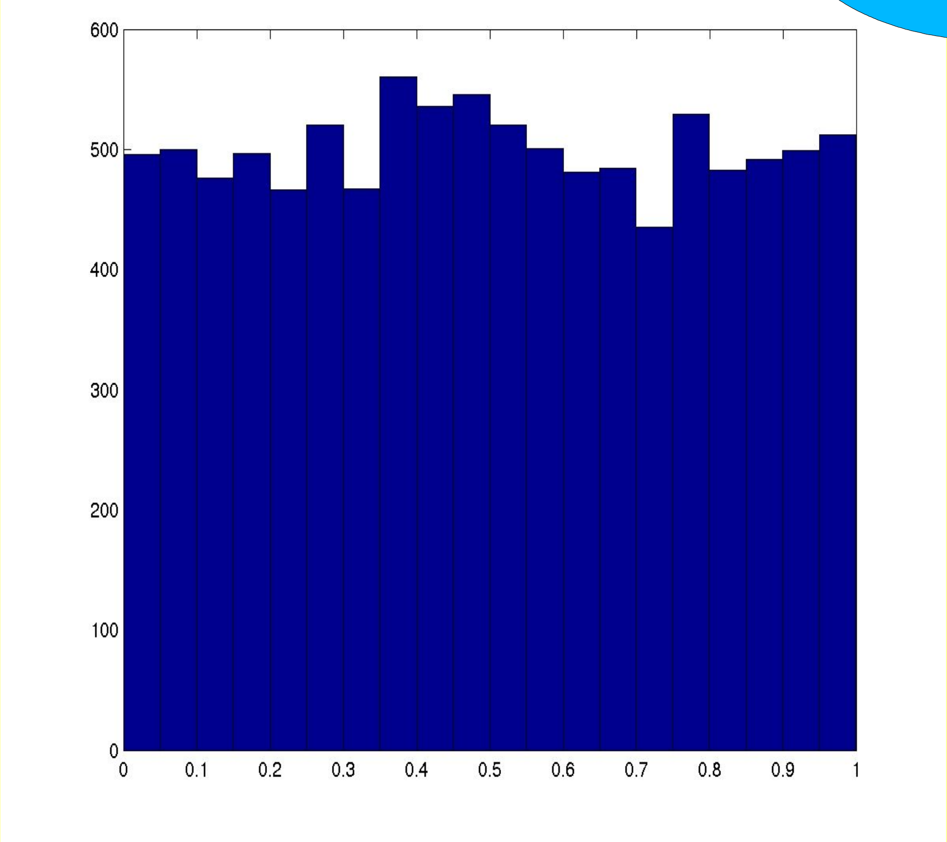


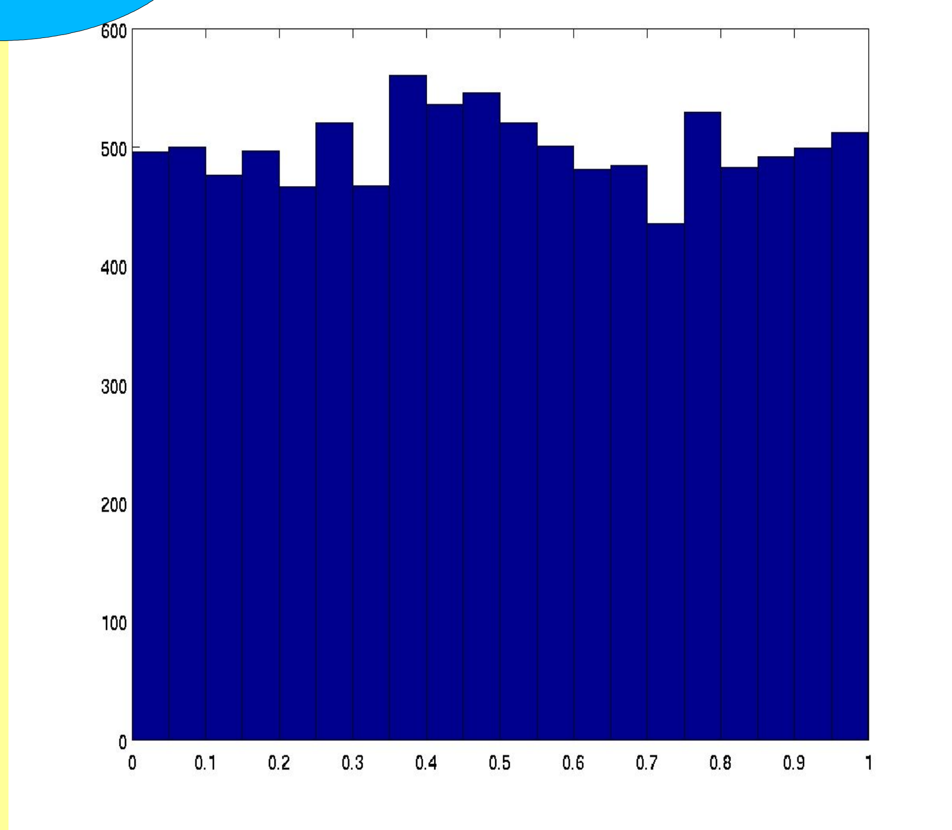
Simulation study

GSF, keeping
4 components

p=0.2 GeV

GSF, keeping
8 components



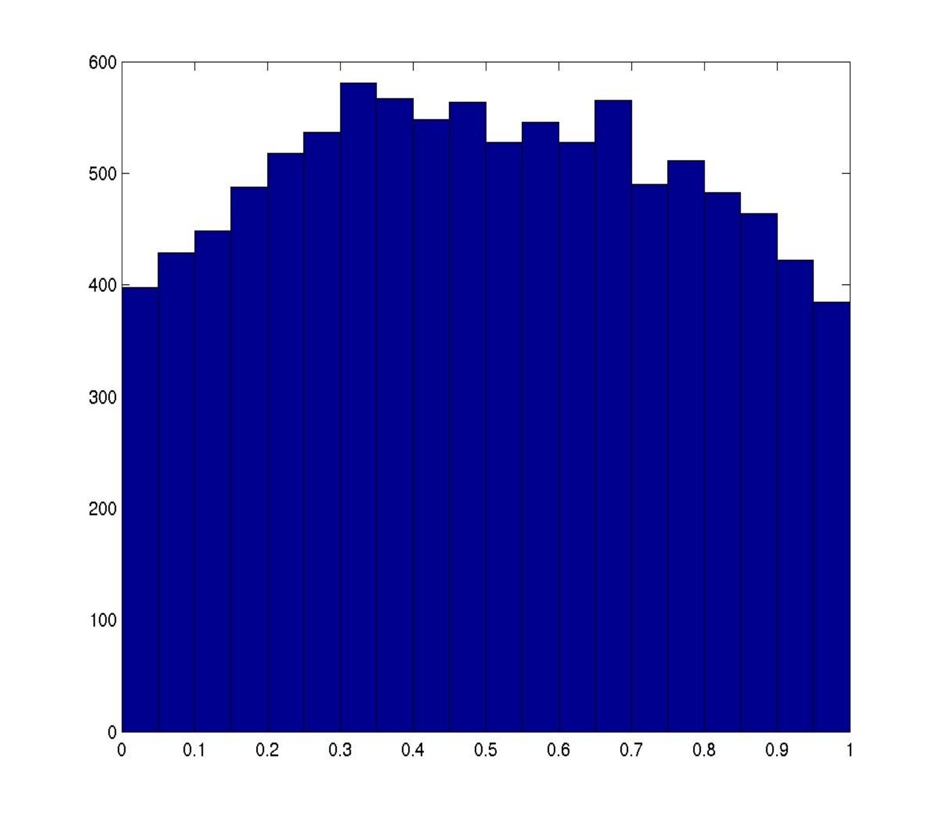


Simulation study

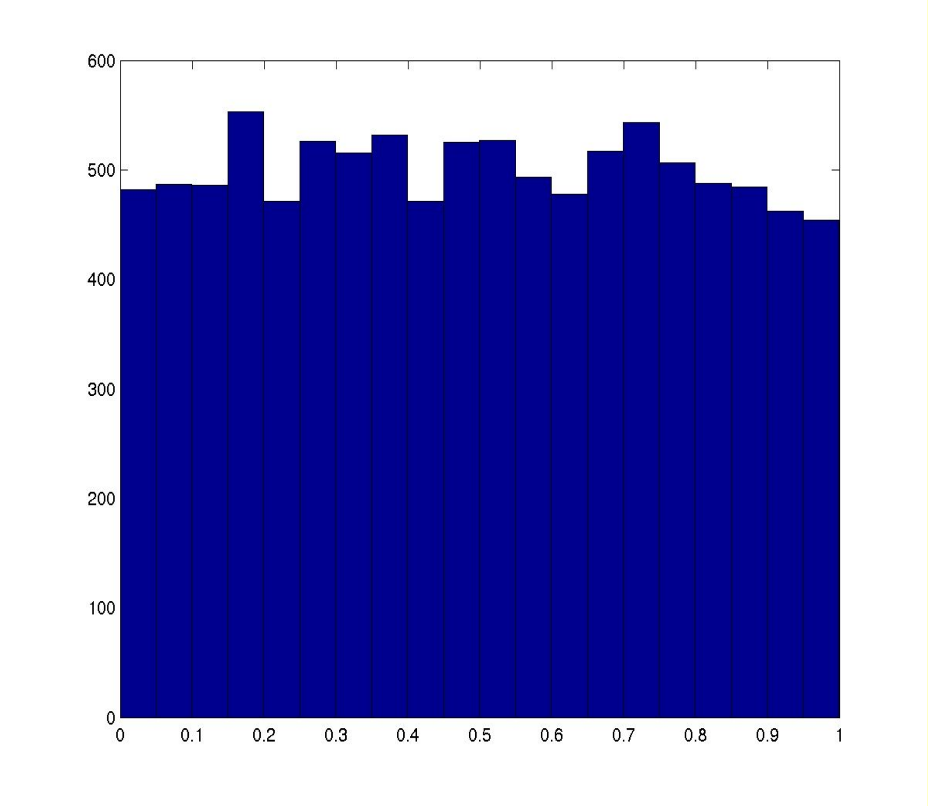
- Probability histogram for GSF is more or less flat when keeping only two components
- This feature is valid for all investigated momenta
- KF histogram deviates significantly from flatness
- Next slides show KF histograms at successively higher momenta

Simulation study

p=1 GeV

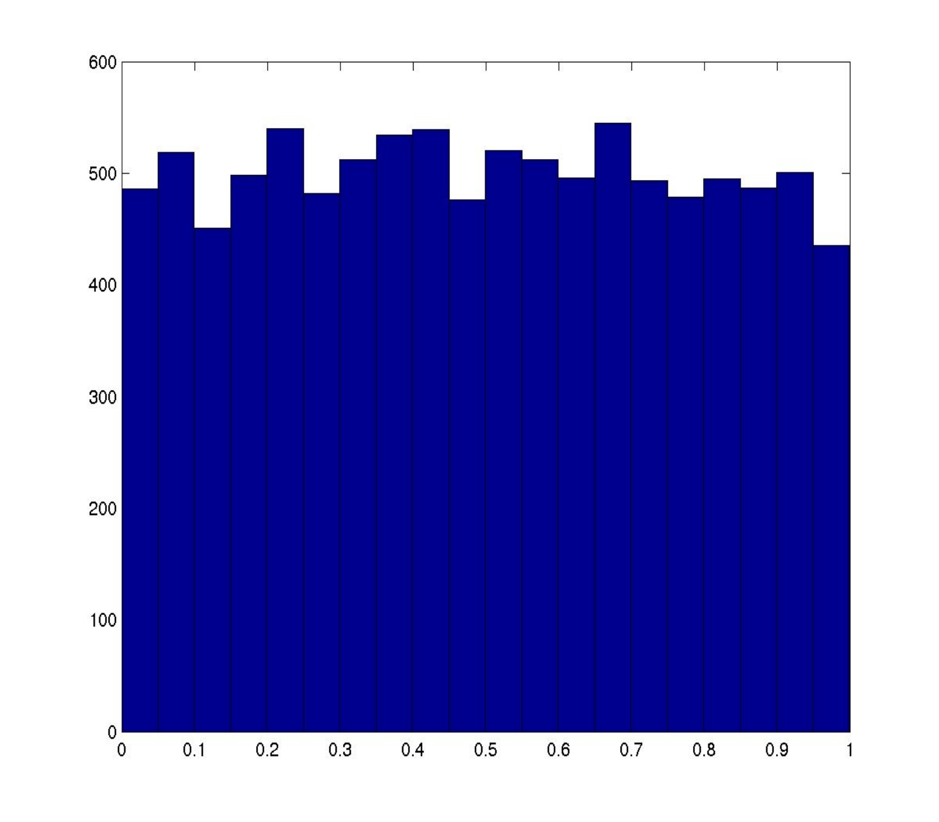


p=5 GeV

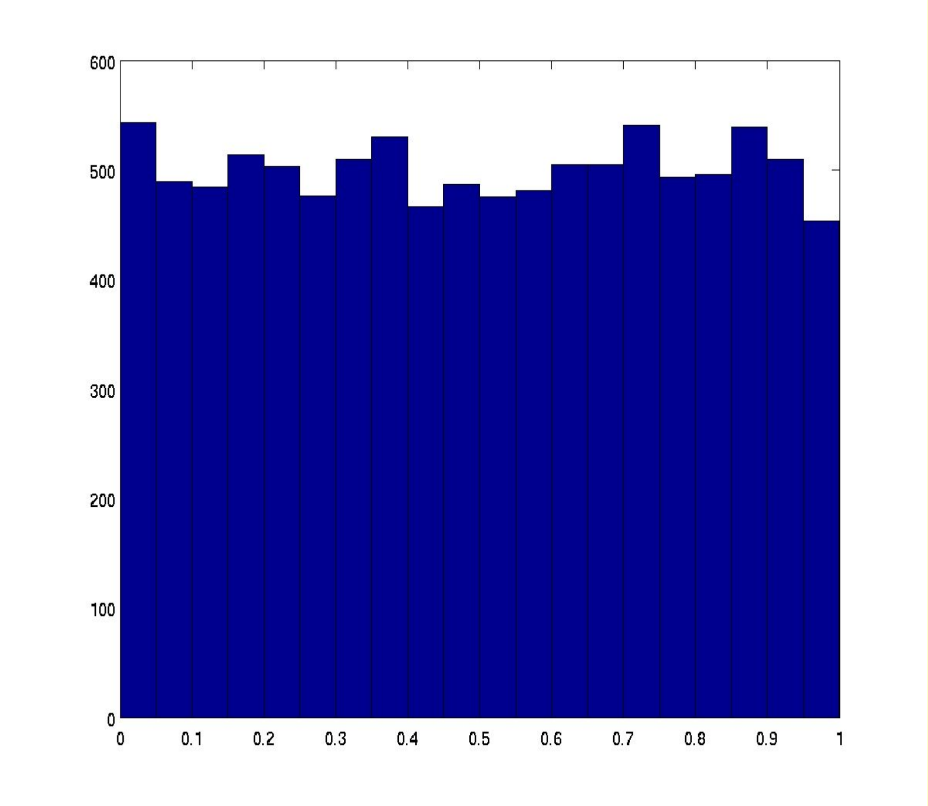


Simulation study

p=10 GeV



p=100 GeV



Summary/outlook

- A GSF for treatment of non-Gaussian tails of multiple scattering has been implemented in a simple detector geometry and for a simple track model
- The GSF is slightly more precise than the KF, particularly at intermediate momenta
- The estimated errors of the GSF closely follows the actual spread of the parameters for all investigated momenta
- The corresponding errors of the KF are OK above 5 GeV

Summary/outlook

- Future work:
  - extending study to full, helical track model and thereby five-dimensional parameter vector
  - more complex detector geometry, for instance ATLAS Inner Detector
  - evaluating impact of quality of estimated impact parameter on e.g. b-tagging performance
    - directly by using mean value and covariance matrix of GSF mixture
    - submitting GSF tracks to GSF secondary vertex fit