Monte Carlo based studies of polarized positrons source for the International Linear Collider (ILC)

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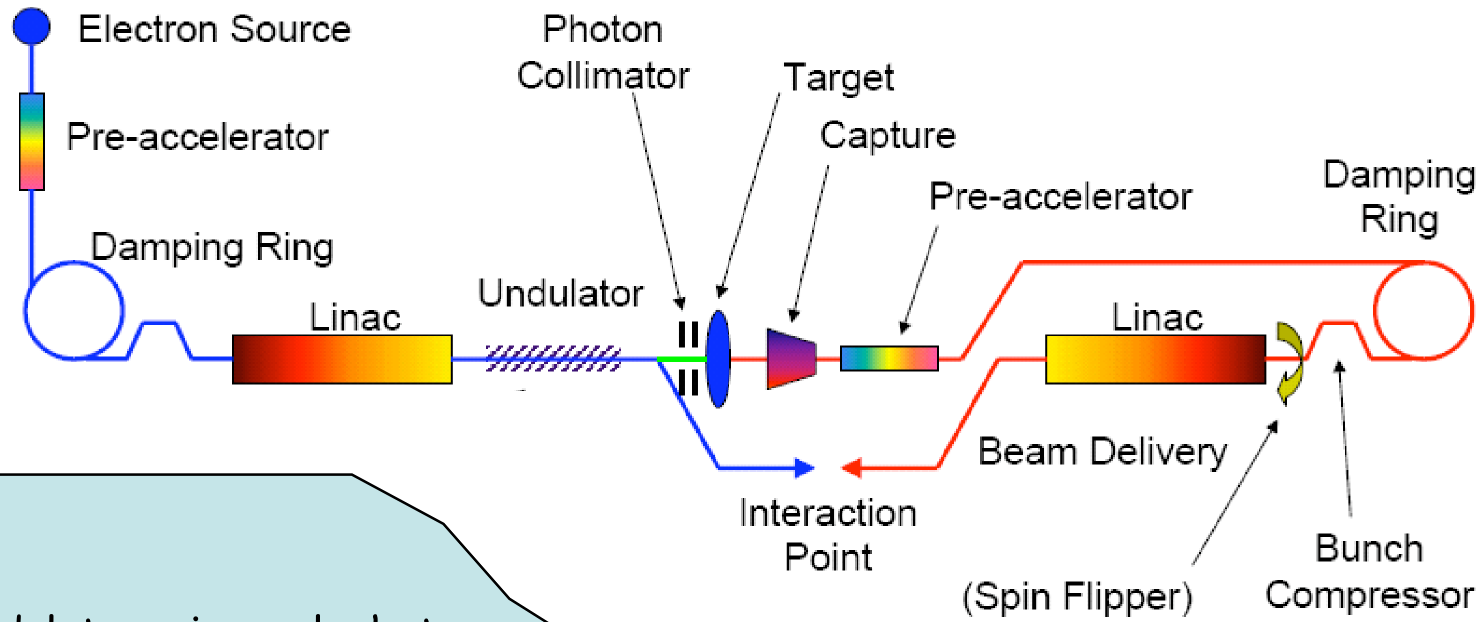
Motivation for „polarized" GEANT4

Simulation of polarization dependent processes for the development of a polarized positron source for the ILC



I. Production:
- Helical undulator: circ. pol. photons
- Conversion Target: via pair production longitudinally polarized e^- and e^+
- Capture of the polarized e^+

II. Polarimetry:
- Transmission polarimetry via Compton scattering (E166)
- other possibilities: Møller / Bhabha scattering ?

GEANT4 Status

Particle:
Particle definition
Momentum
Polarization

Material/Volume:
Atomic composition
Density
Fields (electric/magnetic)

Processes:

• at rest	particle decay annihilation
• along step	ionization multiple scattering
• post step	Compton scattering Bremsstrahlung Møller-, Bhabha scattering Pair production

- only low energy Compton scattering (linear pol. optical photons on unpol. e^-)
- no polarization of the medium
- placeholder for pol. vector of particles (3-vector) exists

What is needed in GEANT4 for polarization studies

TARGET

Gammas:

- GammaConversion
- ComptonScattering
- PhotoElectricEffect

Electrons and Positrons:

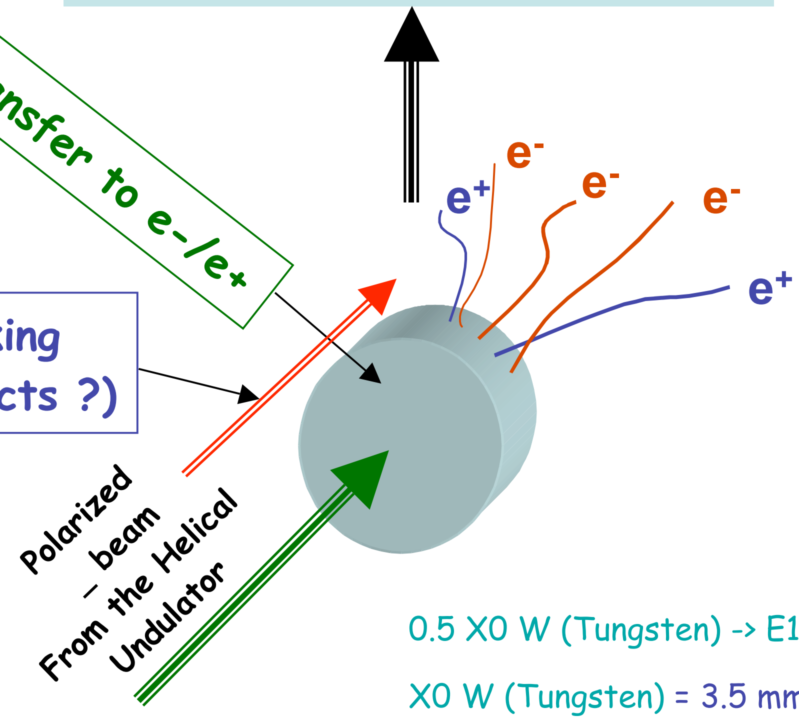
- MultipleScattering
- Ionization
- Bremsstrahlung

Diagnostics (Polarimetry)
Cross sections polarization dependent

- Compton Scattering
- Bhabha Scattering
- Moller Scattering
- Positron annihilation in Flight
- ....

Polarization transfer to e-/e+

Polarization tracking
(depolarization effects ?)



0.5 X0 W (Tungsten) -> E166

X0 W (Tungsten) = 3.5 mm

MAGNETIC FIELD:

Proposal for the implementation

- Use the 3-vector (particlePolarization) for bookkeeping of particle polarization
  but: how to define the polarization ?
- introduce a polarization manager to handle the medium-polarization
- implement polarization dependent cross sections for the desired processes in a universal way (for future extension)

Proposal: use Spin density matrix and Stokes-Parameters

Advantages:

- can handle all polarization states
- provides a unique definition of the polarization

Stokes parameters

G.Stokes, Trans. Cambridge Phil. Soc. 9 (1852) 399

Wave function :

$$\Psi(\boldsymbol{x}, t) = a_1 \Psi_1 + a_2 \Psi_2$$

Jones vector :

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad |a_1|^2 + |a_2|^2 = 1$$

Spin density matrix :

$$\rho = \boldsymbol{a} \otimes \boldsymbol{a}^* = \begin{pmatrix} a_1 a_1^* & a_1 a_2^* \\ a_2 a_1^* & a_2 a_2^* \end{pmatrix} = \frac{1}{2}(1 + \boldsymbol{\xi}\boldsymbol{\sigma})$$

Stokes parameter :

$$\boldsymbol{\xi} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \boldsymbol{a}^\dagger \boldsymbol{\sigma}\, \boldsymbol{a}$$

$$\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Stokes parameters

Stokes parameter	Photon observation	Fermion observation
ξ_1	Plane polarization	Spin in *z* - direction
ξ_2	Plane polarization at an angle of $\pi/4$ to the right	Spin in *x* - direction
ξ_3	Left/Right circular polarization	Spin in *y* - direction

Example linear polarized photon:

$$\boldsymbol{E} = \cos\phi\,\boldsymbol{E}_1 + \sin\phi\,\boldsymbol{E}_2$$

$$\boldsymbol{\xi} = \begin{pmatrix} \cos^2\phi - \sin^2\phi \\ 2\sin\phi\cos\phi \\ 0 \end{pmatrix}$$

Matrix formalism

$$\begin{pmatrix} I \\ \xi \end{pmatrix} = T \begin{pmatrix} I_0 \\ \xi_0 \end{pmatrix}$$

W.H. McMaster, Rev.Mod.Phys. 33 (1961) 8

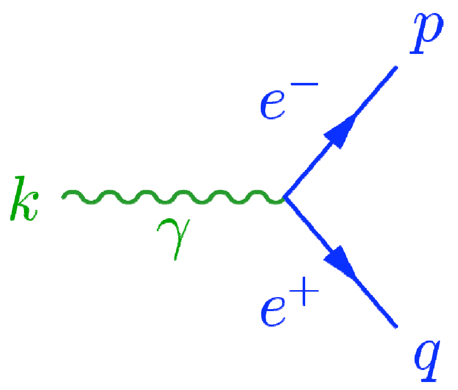
Transformation matrix

$$T = \begin{pmatrix} S & A_1 & A_2 & A_3 \\ P_1 & M_{11} & M_{21} & M_{31} \\ P_2 & M_{12} & M_{22} & M_{32} \\ P_3 & M_{13} & M_{23} & M_{33} \end{pmatrix}$$

- Differential cross section
- Asymmetry
- Polarization
- Depolarization and polarization transfer

Pair production in field of nucleus

$$T = \begin{pmatrix} I & -D & 0 & 0 \\ 0 & 0 & 0 & -L \\ 0 & 0 & 0 & -T \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



$$I = [p^2 + (p-k)^2](3 + F(p,k;Z)) - 2p(p-k)(1 + G(p,k;Z))$$

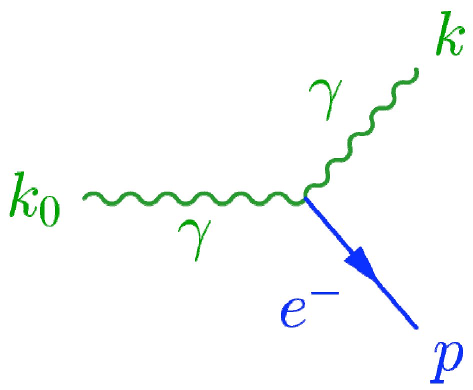
$$D = 8p(p-k)\,G(p,k;Z)$$

$$L = k\{(2p-k)[3 + F(p,k;Z)] + 2(p-k)[1 + G(p,k;Z)]\}$$

$$T = 4k(p-k)\,H(p,k;Z)$$

Compton scattering

$$T = \begin{pmatrix} I & A & 0 & E \\ A & B & 0 & H_1 \\ 0 & 0 & C & H_2 \\ F & G_1 & G_2 & D \end{pmatrix}$$



Independent of electron spin S:
(I, A, B, C, D)

$$I = 1 + \cos^2\theta + (k_0 - k)(1 - \cos\theta)$$

$$A = \sin^2\theta$$

$$D = 2\cos\theta + (k_0 - k)(1 - \cos\theta)\cos\theta$$

Dependent on electron spin S:
(E, F, G_i, H_i)

$$E = -(1 - \cos\theta)(\boldsymbol{k}_0 \cos\theta + \boldsymbol{k}) \cdot \boldsymbol{S}$$

$$F = -(1 - \cos\theta)(\boldsymbol{k} \cos\theta + \boldsymbol{k}_0) \cdot \boldsymbol{S}$$

Proposal for the implementation

Input:

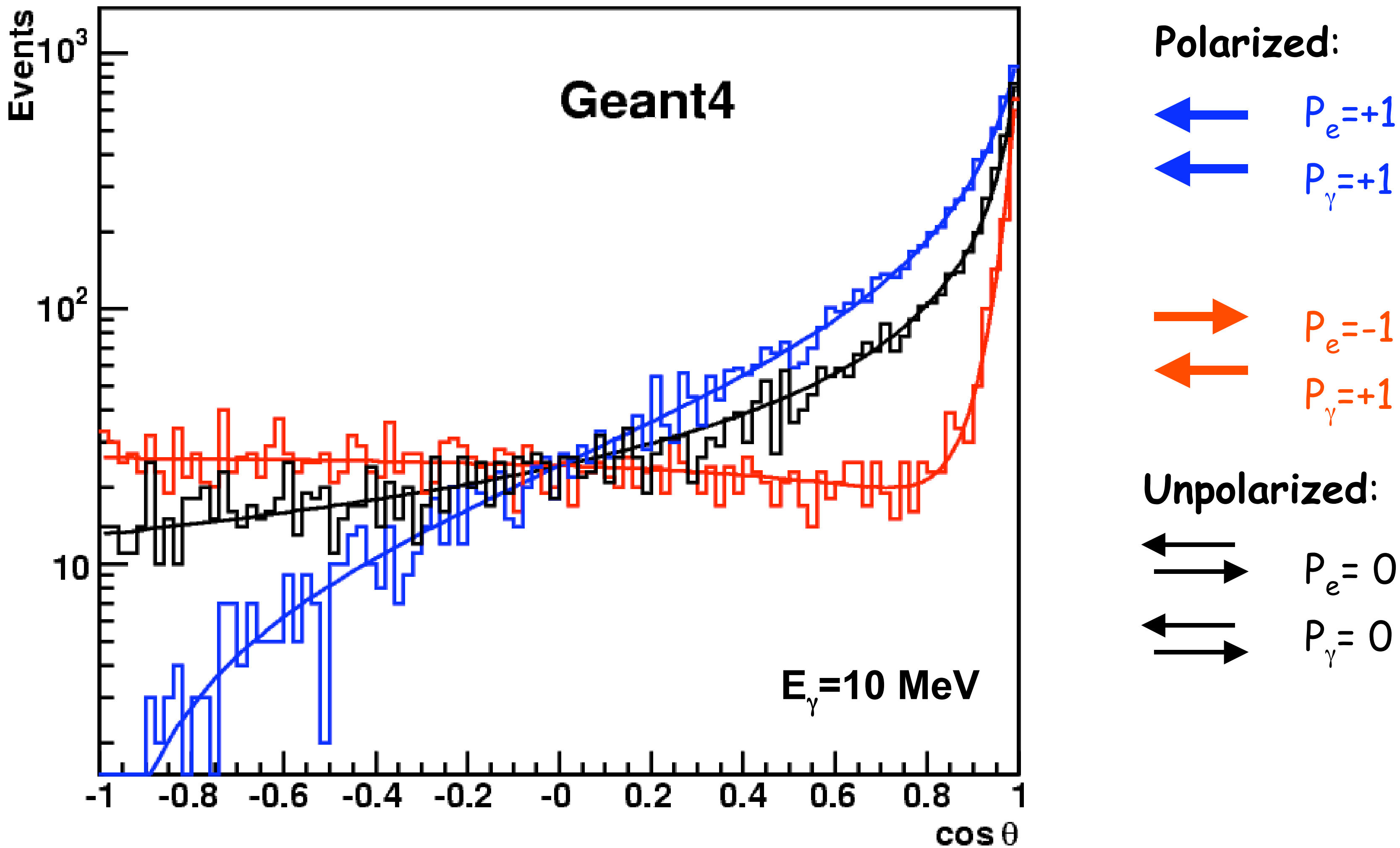
Stokes parameters of the particle (beam) –> G4ThreeVector
(bookkeeping already included in GEANT4)
Polarization of the Volume –> G4ThreeVector (PolarizationManager)

Calculate cross sections (total, differential) –> interaction length
Derive polarization transfer from initial to final state

Output:

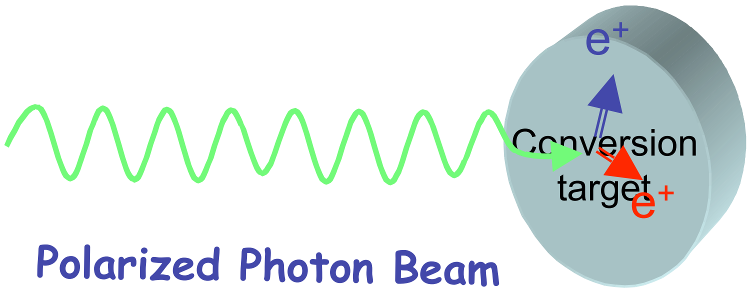
Stokes parameters of the final states

Results Compton scattering

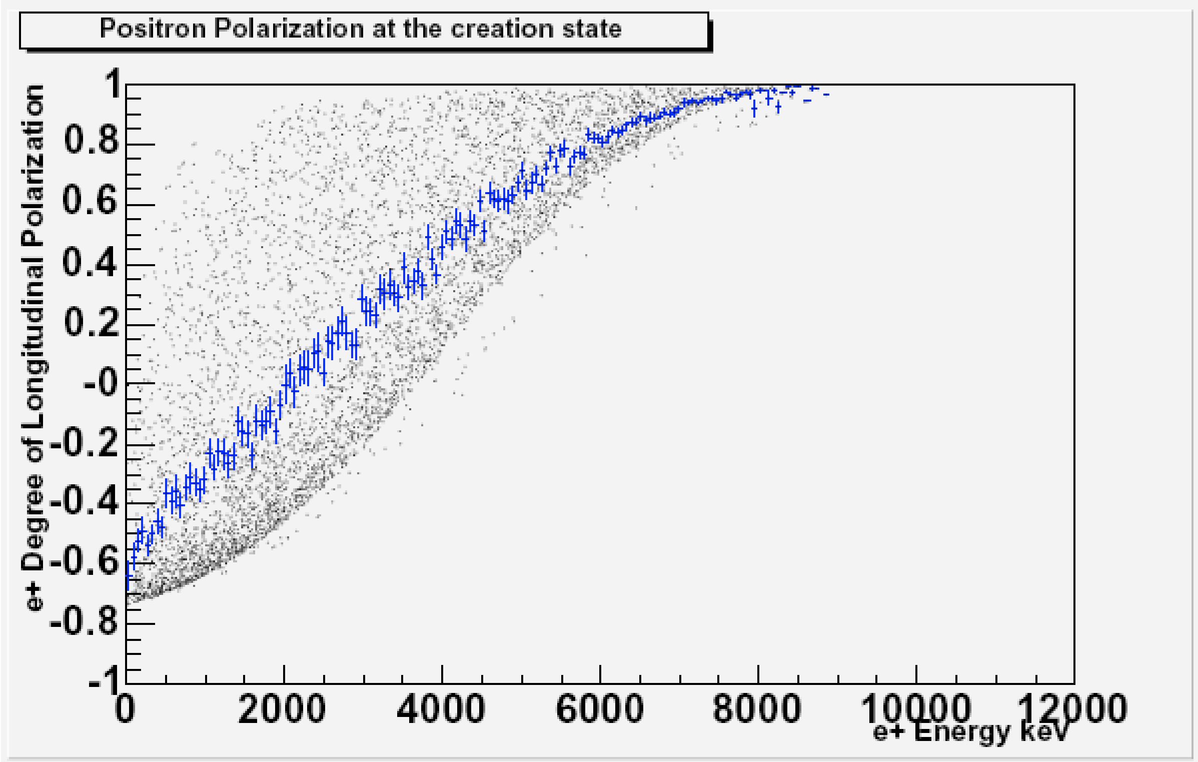


$$\begin{pmatrix} I \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

Stokes Vector



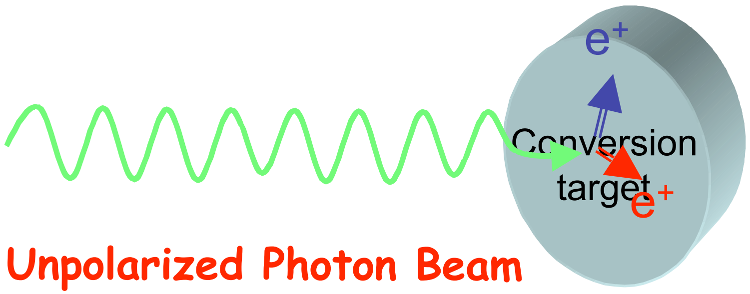
Positron Polarization profile from polarized Undulator photons (creation point)



Polarization transfer in the pair production process

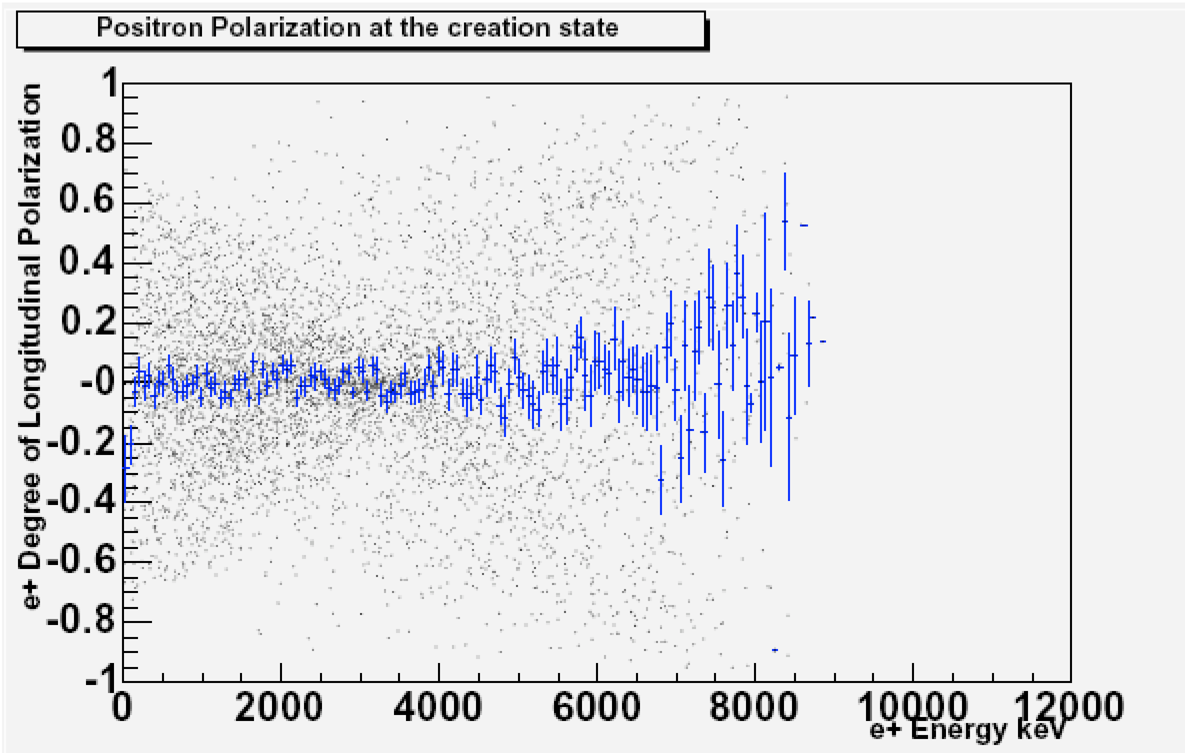
$\begin{pmatrix} I \\ 0 \\ 0 \\ \text{Random} \end{pmatrix}$

Stokes Vector

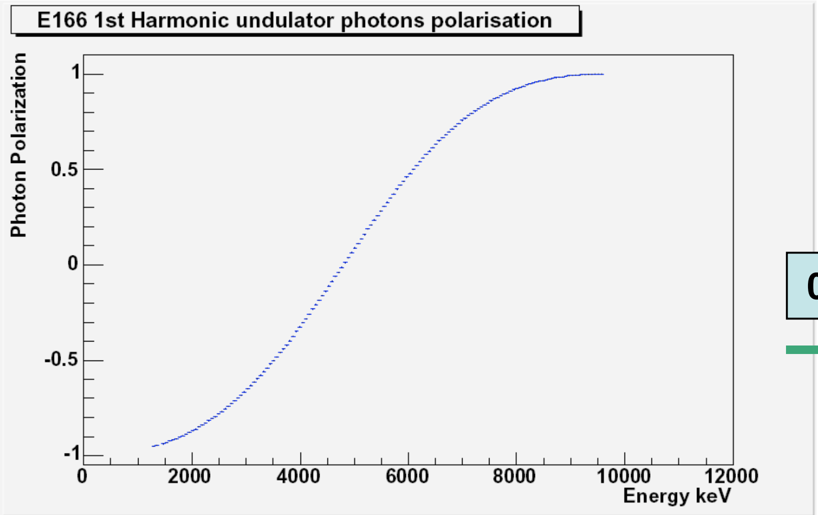


Unpolarized Photon Beam

Positron Polarization profile from polarized Undulator photons (creation point)

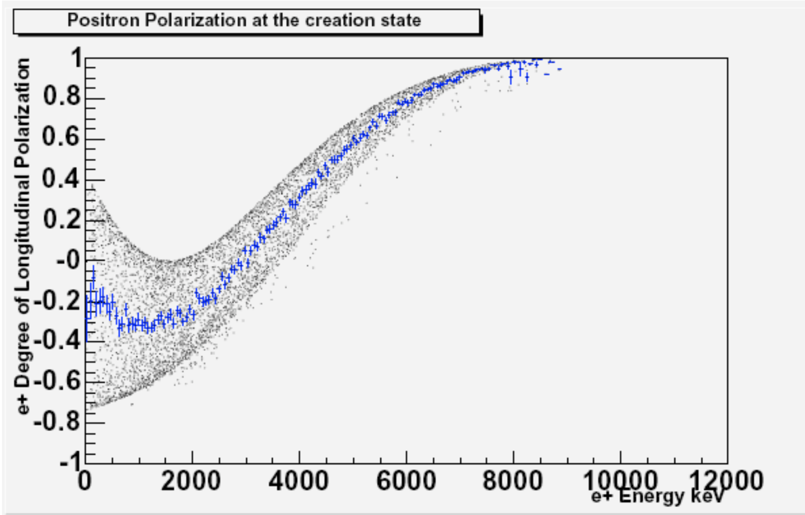


Target studies - results



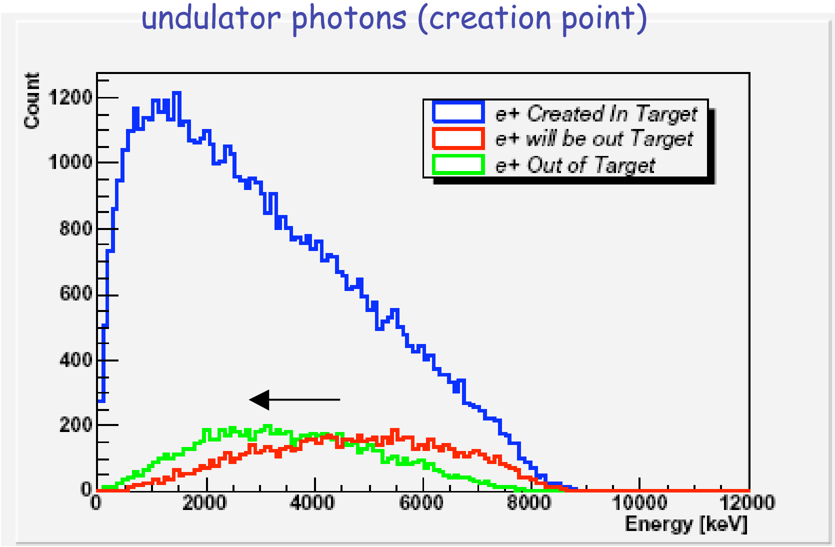
Polarization profile of the first harmonic undulator photons

0.5 X_0 W



Positron Polarization profile created by the undulator photons (creation point)

The target may filter positrons with high degree of polarization



e^+ Energy distribution (inside/outside target)

Outlook

- **Continue the implementation of Polarization in G4 processes**
  - Focus on stokes formalism *(describes all polarization states for γ, e^+, e^-)*
  - First priority to processes needed for the for polarized positron source
  - Other processes will not be neglected
- **Cross check with other existing simulation packages (EGS4…)**
- **Possibility to simulate and cross check with experimental results (E166)**
- **Contact and collaborate with other groups (developing polarized Geant4)**
  - Coordinate the work and the approach on the implementation.
- **Propose the polarized processes to the G4 collaboration (official release)**