Three loop renormalization of QCD in various nonlinear gauges

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Motivation

- QCD has been renormalized to three loops and beyond in various gauges: Landau, Feynman and general covariant
- For various problems these choices of gauge may not be the most appropriate
- One property of the confinement mechanism is that there is abelian dominance and abelian monopole condensation via a Meissner dual superconductor picture
- To study the possibility of abelian dominance the natural choice of gauge is the Maximal Abelian Gauge, MAG
- Currently MAG has only been renormalized to one loop for $SU(2)$

Background

- Main aim is to ascertain whether a dynamical mass is generated for the gluons
- Problem has been examined in a 'laboratory model' called the Curci-Ferrari model ($1 \leq A \leq N_A$)

$$
\begin{aligned}
L \;=\; & -\frac{1}{4} G^A_{\mu\nu} G^{A\,\mu\nu} - \frac{1}{2\alpha} (\partial^\mu A^A_\mu)^2 + \frac{m^2}{2} A^A_\mu A^{A\,\mu} \\
& + \partial_\mu \bar{c}^A \partial^\mu c^A - \alpha m^2 \bar{c}^A c^A - \frac{g}{2} f^{ABC} A^A_\mu \, \bar{c}^B \, \overleftrightarrow{\partial^\mu} \, c^C \\
& + \frac{\alpha g^2}{8} f^{EAB} f^{ECD} \bar{c}^A c^B \bar{c}^C c^D + i \bar{\psi} \not{D} \psi - m_q \bar{\psi} \psi
\end{aligned}
$$

Properties

- Lagrangian is renormalizable; includes quartic ghost interaction
- Gluon mass term is BRST invariant
- Massless Curci-Ferrari (CF) model is known as (nonlinear) Curci-Ferrari gauge
- Corresponds to a particular sector of the MAG

Problems

- Lacks unitarity due to explicit classical mass term - resolved via dynamical mass generation
- Leads to the examination of the condensation of dimension two operator $\frac{1}{2} A^A_\mu A^{A\,\mu} ~-~ \alpha \bar{c}^A c^A$

MAG

- Gauge and ghost fields split $A^A_\mu = A^a_\mu + A^i_\mu$ with $1 \leq a \leq N^o_A$ (off-diagonal), $1 \leq i \leq N^d_A$ (centre or diagonal)
- Gauge fixing is via a modification of covariant gauge fixing procedure to give an interpolating Lagrangian

$$L^{\text{Landau}}_{\text{gf}} = \delta\bar{\delta}\left[\frac{1}{2}A^A_\mu A^{A\,\mu} + \frac{1}{2}\alpha\bar{c}^A c^A\right]$$

$$L^{\text{MAG}}_{\text{gf}} = \delta\bar{\delta}\left[\frac{1}{2}A^a_\mu A^{a\,\mu} + \frac{1}{2}\alpha\bar{c}^a c^a + \frac{1}{2}\zeta A^i_\mu A^{i\,\mu}\right] + (1-\zeta)\delta\left[\bar{c}^i\partial^\mu A^i_\mu\right]$$

- Landau gauge is $\alpha = 0$, $\zeta = 1$
- MAG is $\alpha \neq 0$, $\zeta = 0$ (residual centre gauge symmetry is fixed in Landau gauge $\bar{\alpha} = 0$
- Implement BRST transformations in FORM

Full Lagrangian

$$
\begin{aligned}
L_{\text{gf}} \quad = \quad & - \frac{1}{2\alpha} \left(\partial^\mu A^a_\mu \right)^2 - \frac{1}{2\bar{\alpha}} \left(\partial^\mu A^i_\mu \right)^2 + \bar{c}^a \partial^\mu \partial_\mu c^a + \bar{c}^i \partial^\mu \partial_\mu c^i \\
& + g \Big[(1-\zeta) f^{abk} A^a_\mu \bar{c}^k \partial^\mu c^b - \zeta f^{abk} A^a_\mu \partial^\mu c^b \bar{c}^k - f^{abc} A^a_\mu \bar{c}^b \partial^\mu c^c \\
& \quad - \zeta f^{abk} A^a_\mu \bar{c}^b \partial^\mu c^k - \frac{(1-\zeta)}{\alpha} f^{abk} \partial^\mu A^a_\mu A^b_\nu A^{k\,\nu} - f^{abk} \partial^\mu A^a_\mu c^b \bar{c}^k \\
& \quad - \frac{1}{2} f^{abc} \partial^\mu A^a_\mu \bar{c}^b c^c - (2-\zeta) f^{abk} A^k_\mu \bar{c}^a \partial^\mu \bar{c}^b - f^{abk} \partial^\mu A^k_\mu \bar{c}^b c^c \Big] \\
& + g^2 \Big[(1-\zeta) f^{acbd}_d A^a_\mu A^{b\,\mu} \bar{c}^c c^d - \frac{(1-\zeta)^2}{2\alpha} f^{akbl}_o A^a_\mu A^{b\,\mu} A^k_\nu A^{l\,\nu} \\
& \quad + (1-\zeta) f^{adcj}_o A^a_\mu A^{j\,\mu} \bar{c}^c c^d - \frac{(1-\zeta)}{2} f^{ajcd}_o A^a_\mu A^{j\,\mu} \bar{c}^c c^d \\
& \quad + (1-\zeta) f^{ajcl}_o A^a_\mu A^{j\,\mu} \bar{c}^c c^l + (1-\zeta) f^{alcj}_o A^a_\mu A^{j\,\mu} \bar{c}^c c^l \\
& \quad - (1-\zeta) f^{cjdi}_o A^i_\mu A^{j\,\mu} \bar{c}^c c^d - \frac{\alpha}{4} f^{abcd}_d \bar{c}^a \bar{c}^b c^c c^d \\
& \quad - \frac{\alpha}{8} f^{abcd}_o \bar{c}^a \bar{c}^b c^c c^d + \frac{\alpha}{8} f^{acbd}_o \bar{c}^a \bar{c}^b c^c c^d - \frac{\alpha}{4} f^{abcl}_o \bar{c}^a \bar{c}^b c^c c^l \\
& \quad + \frac{\alpha}{4} f^{acbl}_o \bar{c}^a \bar{c}^b c^c c^l - \frac{\alpha}{4} f^{albc}_o \bar{c}^a \bar{c}^b c^c c^l + \frac{\alpha}{2} f^{akbl}_o \bar{c}^a \bar{c}^b c^k c^l \Big]
\end{aligned}
$$

Dimension two operator

- For condensation of operators in MAG examine the BRST invariant dimension two operator $\mathcal{O} = \frac{1}{2} A^a_\mu A^{a\,\mu} \; - \; \alpha \bar{c}^a c^a$
- Similar to CF operator but restricted to the off-diagonal sector
- For application of local composite operator, LCO, formalism require its anomalous dimension at three loops
- For the CF and Landau gauges the renormalization structure was already available
- Landau gauge LCO completed at two loops with massless quarks (pole mass $m_g = 2.13 \Lambda_{\overline{\mathrm{MS}}}$ using ONSHELL)

Renormalization

- Shortcut available in Landau and CF from Slavnov-Taylor identity $\gamma_{\mathcal{O}}(a) ~=~ -\beta(a)/a ~+~ \gamma_c(a)$
- Similar result in MAG $\gamma_{\mathcal{O}}(a) ~=~ -\beta(a)/a ~+~ \gamma_{c^i}(a)$
- Algebraic renormalization defines structure of renormalization constants

$$
\begin{aligned}
A^{a\,\mu}_{\mathrm{o}} &= \sqrt{Z_A}\,A^{a\,\mu} ~,~ A^{i\,\mu}_{\mathrm{o}} ~=~ \sqrt{Z_{A^i}}\,A^{i\,\mu} \\
c^a_{\mathrm{o}} &= \sqrt{Z_c}\,c^a ~,~ \bar{c}^a_{\mathrm{o}} ~=~ \sqrt{Z_c}\,\bar{c}^a ~,~ \zeta_{\mathrm{o}} ~=~ Z_\zeta \zeta \\
c^i_{\mathrm{o}} &= \sqrt{Z_{c^i}}\,c^i ~,~ \bar{c}^i_{\mathrm{o}} ~=~ \frac{\bar{c}^i}{\sqrt{Z_{c^i}}} ~,~ \psi_{\mathrm{o}} ~=~ \sqrt{Z_\psi}\psi ~, \\
g_{\mathrm{o}} &= \mu^\epsilon Z_g\,g ~,~ \alpha_{\mathrm{o}} ~=~ Z^{-1}_\alpha Z_A\,\alpha ~,~ \bar{\alpha}_{\mathrm{o}} ~=~ Z^{-1}_{\alpha^i} Z_{A^i}\,\bar{\alpha}
\end{aligned}
$$

Strategy

- Renormalize 2-point functions to deduce wave function renormalizations (except for c^i); β-function emerges from A^i (cf background field gauge)
- All the anomalous dimensions are required for checking purposes
- Renormalize $A^a_\mu \bar{c}^i c^b$ vertex for Z_{c^i}
- Deduce $\gamma_{\mathcal{O}}(a)$ from these anomalous dimensions (explicit two loop check)
- Generate graphs with QGRAF
- Convert output into FORM input
- Partially substitute Feynman rules

Strategy continued

- Apply group theory rules (zero diagrams)
- Substitute remaining Feynman rules
- Apply MINCER algorithm
- For one and two loop calculate with full interpolating Lagrangian
- For three loop calculate with Landau gauge first ($\alpha = 0$, $\zeta = 1$)
- Repeat for MAG ($\alpha \neq 0$, $\zeta = 0$)
- Extra parameters slows the MINCER algorithm
- Infinities absorbed minimally ($\overline{\text{MS}}$) automatically

Triple gluon vertex

```
id once vtxggg(AA(A1?,m1?,p1?),AA(A2?,m2?,p2?),phot(E3?,m3?,p3?))
= - i_*gg*( - d_(m1,m2)*p1(m3) + d_(m1,m2)*p2(m3)
            + d_(m1,m3)*p1(m2)
            - d_(m1,m3)*p2(m2)*al^-1*zeal
            + d_(m1,m3)*p2(m2)*al^-1
            - d_(m1,m3)*p3(m2)
            + d_(m2,m3)*p1(m1)*al^-1*zeal
            - d_(m2,m3)*p1(m1)*al^-1
            - d_(m2,m3)*p2(m1) + d_(m2,m3)*p3(m1) );

id once vtxggg(AA(A1?,m1?,p1?),AA(A2?,m2?,p2?),phot(E3?,m3?,p3?))
= - i_*gg*( - d_(m1,m2)*p1(m3)
            + d_(m1,m2)*p2(m3)
            + 2*d_(m1,m3)*p1(m2)
            + d_(m1,m3)*p2(m2)*[(1-zeal)/al+1]
            - d_(m2,m3)*p1(m1)*[(1-zeal)/al+1]
            - 2*d_(m2,m3)*p2(m1) );
```

Diagram count

Green's function	One loop	Two loop	Three loop	Total
$A^a_\mu \, A^b_\nu$	6	131	6590	6727
$A^i_\mu \, A^j_\nu$	3	54	2527	2584
$c^a \, \bar{c}^b$	3	81	4006	4090
$\psi \, \bar{\psi}$	2	27	979	1008
$A^a_\mu \, \bar{c}^i \, c^b$	5	287	22621	22913
Total	19	580	36723	37322

Number of Feynman diagrams for each Green's function for the MAG renormalization.

Green's function	One loop	Two loop	Three loop	Total
$A^A_\mu \, A^B_\nu$	3	19	282	304
$c^A \, \bar{c}^B$	1	9	124	134
$\psi \, \bar{\psi}$	1	6	79	86
$A^A_\mu \bar{\psi} \, \psi$	2	33	697	732
Total	7	67	1182	1256

Number of Feynman diagrams for each Green's function for the CF gauge renormalization.

Group theory

- Due to splitting of fields need to develop new group theory results noting $f^{ijk} = 0$ and $f^{ijc} = 0$

$$
\begin{aligned}
f_d^{ABCD} &= f^{iAB} f^{iCD} \quad , \quad f_o^{ABCD} = f^{eAB} f^{eCD} \\
C_A \delta^{ab} &= f^{acd} f^{bcd} + 2 f^{acj} f^{bcj} \quad , \quad C_A \delta^{ij} = f^{icd} f^{jcd}
\end{aligned}
$$

$$
f^{iab} f^{iab} = N_A^d C_A \quad , \quad f^{abc} f^{abc} = \left[N_A^o - 2 N_A^d \right] C_A
$$

$$
f^{icd} f^{jcd} = C_A \delta^{ij} \quad , \quad f^{acj} f^{bcj} = \frac{N_A^d}{N_A^o} C_A \delta^{ab} \quad , \quad f^{acd} f^{bcd} = \frac{[N_A^o - 2 N_A^d]}{N_A^o} C_A \delta^{ab} \, .
$$

$$
\begin{aligned}
f^{apq} f^{bpr} f^{cqr} &= \frac{[N_A^o - 3 N_A^d]}{2 N_A^o} C_A f^{abc} \quad , \quad f^{apq} f^{bpi} f^{cqi} = \frac{N_A^d}{2 N_A^o} C_A f^{abc} \\
f^{ipq} f^{bpr} f^{cqr} &= \frac{[N_A^o - 2 N_A^d]}{2 N_A^o} C_A f^{ibc} \quad , \quad f^{ipq} f^{bpj} f^{cqj} = \frac{N_A^d}{N_A^o} C_A f^{ibc}
\end{aligned}
$$

Quark sector

$$\mathrm{Tr}\left(T^aT^b\right) = T_F\delta^{ab} \;,\; \mathrm{Tr}\left(T^aT^i\right) = 0 \;,\; \mathrm{Tr}\left(T^iT^j\right) = T_F\delta^{ij}$$

$$T^iT^i = \frac{T_F}{N_F}N^d_A I \;,\; T^aT^a = \left[C_F - \frac{T_F}{N_F}N^d_A\right] I$$

$$T^bT^aT^b = \left[C_F - \frac{C_A}{2} - \frac{T_F}{N_F}N^d_A + \frac{C_A N^d_A}{2N^o_A}\right] T^a$$

$$T^iT^aT^i = \left[\frac{T_F}{N_F}N^d_A - \frac{C_A N^d_A}{2N^o_A}\right] T^a$$

$$T^aT^iT^a = \left[\frac{T_F}{N_F}N^o_A - \frac{C_A}{2}\right] T^i \;,\; T^jT^iT^j = \frac{T_F}{N_F}N^d_A T^i$$

$$f^{abc}T^bT^c = \frac{i[N^o_A - 2N^d_A]}{2N^o_A}C_A T^a \;,\; f^{abj}T^bT^j = \frac{iN^d_A}{2N^o_A}C_A T^a$$

$$f^{ibc}T^bT^c = \frac{i}{2}C_A T^i$$

Identities for three loop

$$
\begin{aligned}
f^{apq} f^{brs} f^{qms} f^{cmt} f^{prt} &= 0 \ , \quad f^{apq} f^{bjs} f^{qms} f^{cmt} f^{pjt} = 0 \\
f^{apq} f^{brs} f^{qjs} f^{cjt} f^{prt} &= 0 \ , \quad f^{apq} f^{brs} f^{qms} f^{imt} f^{prt} = 0 \\
f^{apq} f^{brj} f^{qmj} f^{cmk} f^{prk} &= \frac{N_A^{d\,2} C_A^2}{4 N_A^{o\,2}} f^{abc} \\
f^{apj} f^{brs} f^{jms} f^{imt} f^{prt} &= \frac{N_A^d [N_A^o - 2 N_A^d] C_A^2}{4 N_A^{o\,2}} f^{abi} \\
f^{apj} f^{bks} f^{jms} f^{imt} f^{pkt} &= \frac{N_A^{d\,2} C_A^2}{N_A^{o\,2}} f^{abi} \\
f^{apj} f^{bks} f^{jms} f^{cmt} f^{pkt} &= \frac{N_A^{d\,2} C_A^2}{4 N_A^{o\,2}} f^{abc}
\end{aligned}
$$

- Implemented in group module in FORM via the use of the set notation
- Various input assumptions on isotropy of basic results checked in $SU(2)$ and $SU(3)$

Three point function

- Define matrices $(S^a)_{bc} = f^{abc}$, $(S^i)_{bc} = f^{ibc}$

$$
\begin{aligned}
\mathrm{tr}\left(S^iS^jS^kS^l\right)\mathrm{tr}\left(S^iS^jS^kS^l\right) &= -\mathrm{tr}\left(S^iS^jS^kS^d\right)\mathrm{tr}\left(S^iS^jS^kS^d\right) \\
&\quad + \left[6N_A^d + N_A^o\right]\frac{N_A^{d\,3}C_A^4}{4N_A^{o\,3}} \\
\mathrm{tr}\left(S^iS^jS^kS^d\right)\mathrm{tr}\left(S^iS^jS^kS^d\right) &= -\mathrm{tr}\left(S^iS^jS^cS^d\right)\mathrm{tr}\left(S^iS^jS^cS^d\right) \\
&\quad - \left[4N_A^{d\,2} - N_A^{o\,2}\right]\frac{N_A^{d\,2}C_A^4}{8N_A^{o\,3}} \\
\mathrm{tr}\left(S^iS^jS^cS^d\right)\mathrm{tr}\left(S^iS^jS^cS^d\right) &= -\mathrm{tr}\left(S^iS^bS^cS^d\right)\mathrm{tr}\left(S^iS^bS^cS^d\right) \\
&\quad + \mathrm{tr}\left(S^iS^bS^cS^dS^iS^bS^cS^d\right) \\
&\quad + \mathrm{tr}\left(S^iS^jS^cS^dS^iS^jS^cS^d\right) \\
&\quad - \left[5N_A^{d\,2} - 4N_A^dN_A^o + N_A^{o\,2}\right]\frac{N_A^dC_A^4}{8N_A^{o\,3}}
\end{aligned}
$$

- $\mathrm{tr}\left(S^iS^bS^cS^d\right)\mathrm{tr}\left(S^iS^bS^cS^d\right)$ and $\mathrm{tr}\left(S^iS^bS^cS^dS^iS^bS^cS^d\right)$ cancel

Results

$$
\begin{aligned}
\gamma_{c^i}(a) \quad = \quad & \frac{1}{4N_A^o}\left[N_A^o\left((-\alpha-3)C_A\right)+N_A^d\left((-2\alpha-6)C_A\right)\right] a \\
& + \frac{1}{96N_A^{o\,2}}\left[N_A^{o\,2}\left((-6\alpha^2-66\alpha-190)C_A^2+80C_AT_FN_f\right)\right. \\
& + N_A^oN_A^d\left((-54\alpha^2-354\alpha-323)C_A^2+160C_AT_FN_f\right) \\
& \left. + N_A^{d\,2}\left((-60\alpha^2-372\alpha+510)C_A^2\right)\right] a^2 \\
& + \frac{1}{6912N_A^{o\,3}}\left[N_A^{o\,3}((-162\alpha^3-2727\alpha^2-2592\zeta_3\alpha-18036\alpha\right. \\
& - 1944\zeta_3-63268)C_A^3+(6912\alpha+62208\zeta_3+6208)C_A^2T_FN_f \\
& + (-82944\zeta_3+77760)C_AC_FT_FN_f+8960C_AT_F^2N_f^2) \\
& + N_A^{o\,2}N_A^d((-2754\alpha^3+648\zeta_3\alpha^2-28917\alpha^2-4212\zeta_3\alpha-69309\alpha \\
& + 37260\zeta_3-64544)C_A^3+(25488\alpha+103680\zeta_3-13072)C_A^2T_FN_f \\
& + (-165888\zeta_3+155520)C_AC_FT_FN_f+17920C_AT_F^2N_f^2) \\
& + N_A^oN_A^{d\,2}((-7884\alpha^3+22680\zeta_3\alpha^2-84564\alpha^2+97524\zeta_3\alpha-47142\alpha \\
& + 433836\zeta_3-56430)C_A^3+(25056\alpha-124416\zeta_3-18144)C_A^2T_FN_f) \\
& + N_A^{d\,3}((-6480\alpha^3+34992\zeta_3\alpha^2-70092\alpha^2+8424\zeta_3\alpha \\
& \left. + 114912\alpha+77112\zeta_3-161028)C_A^3)\right] a^3 \; + \; O(a^4)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{\mathcal{O}}(a) \quad = \quad & \frac{1}{12 N_A^o} \Big[N_A^o \left((-\,3\alpha + 35) C_A - 16 T_f N_f \right) + N_A^d \left((-\,6\alpha - 18) C_A \right) \Big] \, a \\
& + \frac{1}{96 {N_A^o}^2} \Big[{N_A^o}^2 \left((-\,6\alpha^2 - 66\alpha + 898) C_A^2 - 560 C_A T_f N_f \right. \\
& \left. -\,384 C_F T_f N_f \right) + \; N_A^o N_A^d \left((-\,54\alpha^2 - 354\alpha - 323) C_A^2 + 160 C_A T_f N_f \right) \\
& + {N_A^d}^2 \left((-\,60\alpha^2 - 372\alpha + 510) C_A^2 \right) \Big] \, a^2 \\
& + \frac{1}{6912 {N_A^o}^3} \Big[{N_A^o}^3 ((-\,162\alpha^3 - 2727\alpha^2 - 2592\zeta_3\alpha - 18036\alpha - 1944\zeta_3 \\
& + 302428) C_A^3 + (6912\alpha + 62208\zeta_3 - 356032) C_A^2 T_F N_f + (-\,82944\zeta_3 \\
& - 79680) C_A C_F T_F N_f + 49408 C_A T_F^2 N_f^2 + 13824 C_F^2 T_F N_f \\
& + 33792 C_F T_F^2 N_f^2) + {N_A^o}^2 N_A^d ((-\,2754\alpha^3 + 648\alpha^2\zeta_3 - 28917\alpha^2 \\
& - 4212\alpha\zeta_3 - 69309\alpha + 37260\zeta_3 - 64544) C_A^3 + (25488\alpha + 103680\zeta_3 \\
& - 13072) C_A^2 T_F N_f + (-\,165888\zeta_3 + 155520) C_A C_F T_F N_f \\
& + 17920 C_A T_F^2 N_f^2) + N_A^o {N_A^d}^2 ((-\,7884\alpha^3 + 22680\alpha^2\zeta_3 - 84564\alpha^2 \\
& + 97524\alpha\zeta_3 - 47142\alpha + 433836\zeta_3 - 56430) C_A^3 + (25056\alpha - 124416\zeta_3 \\
& - 18144) C_A^2 T_F N_f) + {N_A^d}^3 ((-\,6480\alpha^3 + 34992\alpha^2\zeta_3 - 70092\alpha^2 \\
& + 8424\alpha\zeta_3 + 114912\alpha + 77112\zeta_3 - 161028) C_A^3) \Big] \, a^3 \; + \; O(a^4)
\end{aligned}
$$

Checks

- Structure of renormalization constants not inconsistent with renormalization group
- Landau gauge results emerge in the limit $\alpha \to 0$, $\zeta \to 1$
- Correct $\overline{\text{MS}}$ β-function emerges at three loops from $\gamma_{A^i}(a)$ - independent of α, N_A^d and N_A^o
- Curci-Ferrari anomalous dimensions correctly emerge for the off-diagonal sector in the limit $N_A^d / N_A^o \to 0$; difficult to see in $SU(N_c)$
- Correct abelian limit emerges $\forall$ N_A^d and N_A^o
- MAG anomalous dimensions cannot be deduced from Landau or CF gauge results

Four loop

$$
\begin{aligned}
\gamma_{A^i}(a) \quad = \quad & \frac{1}{3}\left[4T_F N_f - 11C_A\right] a + \frac{1}{3}\left[-34C_A^2 + 20C_A T_F N_f + 12C_F T_F N_f\right] a^2 \\
& + \frac{1}{54}\left[-2857C_A^3 + 2830C_A^2 T_F N_f + 1230C_A C_F T_F N_f \right. \\
& \left. - 316C_A T_F^2 N_f^2 - 108C_F^2 T_F N_f - 264C_F T_F^2 N_f^2\right] a^3 \\
& + \left[\left(\frac{44}{9}\zeta_3 - \frac{150653}{486}\right) C_A^4 + \left(\frac{39143}{81} - \frac{136}{3}\zeta_3\right) C_A^3 T_F N_f \right. \\
& + \left(\frac{656}{9}\zeta_3 - \frac{7073}{243}\right) C_A^2 C_F T_F N_f + \left(\frac{4204}{27} - \frac{352}{9}\zeta_3\right) C_A C_F^2 T_F N_f \\
& - \left(\frac{7930}{81} + \frac{224}{9}\zeta_3\right) C_A^2 T_F^2 N_f^2 + \left(\frac{704}{9}\zeta_3 - \frac{1352}{27}\right) C_F^2 T_F^2 N_f^2 \\
& - \left(\frac{17152}{243} + \frac{448}{9}\zeta_3\right) C_A C_F T_F^2 N_f^2 - \frac{424}{243} C_A T_F^3 N_f^3 - \frac{1232}{243} C_F T_F^3 N_f^3 \\
& + \left(\frac{80}{9} - \frac{704}{3}\zeta_3\right) \frac{d_A^{ABCD} d_A^{ABCD}}{N_F} \\
& + \left(\frac{1664}{3}\zeta_3 - \frac{512}{9}\right) \frac{d_F^{ABCD} d_A^{ABCD}}{N_F} N_f \\
& \left. - 46C_F^3 T_F N_f + \left(\frac{704}{9} - \frac{512}{3}\zeta_3\right) \frac{d_F^{ABCD} d_F^{ABCD}}{N_F} N_f^2\right] a^5 \; + \; O(a^6)
\end{aligned}
$$

$SU(3)$

$$\gamma_A(a) = \left[3\alpha - 15 + 2N_f\right]\frac{a}{3} + \left[39\alpha^2 + 609\alpha - 1113 + 224N_f\right]\frac{a^2}{24}$$
$$+ \left[-1836\alpha^3\zeta_3 + 9495\alpha^3 - 12258\alpha^2\zeta_3 + 115056\alpha^2 - 14832\alpha N_f\right.$$
$$+ 28620\alpha\zeta_3 + 334701\alpha - 9920N_f^2 - 43776N_f\zeta_3 + 229704N_f$$
$$\left. - 182088\zeta_3 - 839337\right]\frac{a^3}{1152} + O(a^4)$$

$$\gamma_\alpha(a) = \left[-15\alpha^2 + 42\alpha - 36 - 8\alpha N_f\right]\frac{a}{12\alpha}$$
$$+ \left[-138\alpha^3 - 1842\alpha^2 + 1539\alpha - 768 - (408\alpha - 256)N_f\right]\frac{a^2}{48\alpha}$$
$$+ \left[-59292\alpha^4 - 1034424\alpha^3 + 91800\alpha^2N_f - 259200\alpha^2\zeta_3 - 2424195\alpha^2\right.$$
$$+ 64000\alpha N_f^2 + 193536\alpha N_f\zeta_3 - 1301712\alpha N_f + 914976\alpha\zeta_3$$
$$+ 6029820\alpha + 15360N_f^2 - 442368N_f\zeta_3 + 268128N_f$$
$$\left. + 5868288\zeta_3 - 5046732\right]\frac{a^3}{6912\alpha} + O(a^4)$$

$$\gamma_{A^i}(a) = \left[2N_f - 33\right]\frac{a}{3} + 2\left[19N_f - 153\right]\frac{a^2}{3}$$
$$+ \left[-325N_f^2 + 15099N_f - 77139\right]\frac{a^3}{54} + O(a^4)$$

$$\gamma_c(a) = [\alpha - 15]\,\frac{a}{4} + \left[-\,60\alpha^2 - 1020\alpha - 2241 + 200N_f\right]\frac{a^2}{96}$$
$$+ \left[-\,22032\alpha^3\zeta_3 + 10908\alpha^3 - 36936\alpha^2\zeta_3 - 161298\alpha^2 - 17928\alpha N_f\right.$$
$$+ 238464\alpha\zeta_3 - 234657\alpha + 11200N_f^2 + 96768N_f\zeta_3 + 206616N_f$$
$$\left.+ 2301696\zeta_3 - 2791386\right]\frac{a^3}{6912} + O(a^4)$$

$$\gamma_{c^i}(a) = -\,5\,[\alpha + 3]\,\frac{a}{4} + \left[-\,276\alpha^2 - 2028\alpha - 2169 + 200N_f\right]\frac{a^2}{96}$$
$$+ \left[-\,59292\alpha^3 + 108864\alpha^2\zeta_3 - 657666\alpha^2 + 81864\alpha N_f + 193104\alpha\zeta_3\right.$$
$$- 1137267\alpha + 11200N_f^2 + 96768N_f\zeta_3 + 258456N_f$$
$$\left.+ 1661472\zeta_3 - 2619450\right]\frac{a^3}{6912} + O(a^4)$$

$$\gamma_\psi(a) = \alpha\, a + \left[-\,3\alpha^2 + 138\alpha + 262 - 16N_f\right]\frac{a^2}{12} + O(a^3)$$
$$+ \left[8532\alpha^3 + 5832\alpha^2\zeta_3 + 71496\alpha^2 - 19224\alpha N_f + 117936\alpha\zeta_3\right.$$
$$\left.+ 210195\alpha + 1280N_f^2 - 114240N_f + 43848\zeta_3 + 948012\right]\frac{a^3}{1728} + O(a^4)$$

$$\gamma_{\mathcal{O}}(a) = \left[-\,15\alpha + 87 - 8N_f\right]\frac{a}{12} + \left[-\,276\alpha^2 - 2028\alpha + 7623 - 1016N_f\right]\frac{a^2}{96}$$
$$+ \left[-\,19764\alpha^3 + 36288\alpha^2\zeta_3 - 219222\alpha^2 + 27288\alpha N_f + 64368\alpha\zeta_3\right.$$
$$- 379089\alpha + 17600N_f^2 + 32256N_f\zeta_3 - 558072N_f + 553824\zeta_3$$
$$\left.+ 2418114\right]\frac{a^3}{2304} + O(a^4)$$

Conclusions

- Full three loop $\overline{\text{MS}}$ renormalization of QCD in the MAG for arbitrary colour group (hep-th/0504051)
- Interpolating gauge has been extended to a renormalizable six parameter Lagrangian (hep-th/0505037)
- Effective potential of $\mathcal{O}$ for the MAG has been constructed at one loop (hep-th/0406132)
- All the renormalization aspects for a two loop effective potential are in place