An analytic formula for track extrapolation in an inhomogeneous magnetic field

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Extrapolation in Magnetic Field

Differential equation of charged particle motion
in a magnetic field

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = F_l = \kappa \cdot q \cdot \mathbf{v} \times \mathbf{B}$$

Along Z direction :

$$\begin{aligned} x' &\equiv t_x \\ y' &\equiv t_y \\ t'_x &= \kappa \cdot (q/p) \cdot \sqrt{1 + t_x^2 + t_y^2} \cdot \Big(t_x t_y \cdot B_x - (1 + t_x^2) \cdot B_y + t_y \cdot B_z \Big) \\ t'_y &= \kappa \cdot (q/p) \cdot \sqrt{1 + t_x^2 + t_y^2} \cdot \Big(\Big(1 + t_y^2\Big) \cdot B_x - t_x t_y \cdot B_y - t_x \cdot B_z \Big) \\ (q/p)' &= 0 \end{aligned}$$

Analytic Expression

$$
\begin{aligned}
t_x(z_e) &= t_x(z_0) + \sum_{k=1}^{n} \sum_{i_1,\ldots,i_k=x,y,z} t_{x i_1 \ldots i_k}(z_0) \cdot \left(\int_{z_0}^{z_e} B_{i_1}(z_1) \ldots \int_{z_0}^{z_{k-1}} B_{i_k}(z_k) \mathrm{d}z_k \ldots \mathrm{d}z_1 \right) + O(\epsilon) \\
t_y(z_e) &= t_y(z_0) + \sum_{k=1}^{n} \sum_{i_1,\ldots,i_k=x,y,z} t_{y i_1 \ldots i_k}(z_0) \cdot \left(\int_{z_0}^{z_e} B_{i_1}(z_1) \ldots \int_{z_0}^{z_{k-1}} B_{i_k}(z_k) \mathrm{d}z_k \ldots \mathrm{d}z_1 \right) + O(\epsilon) \\
x(z_e) &= x(z_0) + \int_{z_0}^{z_e} t_x(z) \mathrm{d}z + O(\epsilon) \cdot \Delta z \\
y(z_e) &= y(z_0) + \int_{z_0}^{z_e} t_y(z) \mathrm{d}z + O(\epsilon) \cdot \Delta z
\end{aligned}
$$

Coefficients:

$$
\begin{aligned}
t_{x i_1 \ldots i_k}(z) &\equiv \frac{\partial t_{x i_1 \ldots i_{k-1}}}{\partial t_x} a_{i_k}(z) + \frac{\partial t_{x i_1 \ldots i_{k-1}}}{\partial t_y} b_{i_k}(z) \\
t_{y i_1 \ldots i_k}(z) &\equiv \frac{\partial t_{y i_1 \ldots i_{k-1}}}{\partial t_x} a_{i_k}(z) + \frac{\partial t_{y i_1 \ldots i_{k-1}}}{\partial t_y} b_{i_k}(z)
\end{aligned}
$$

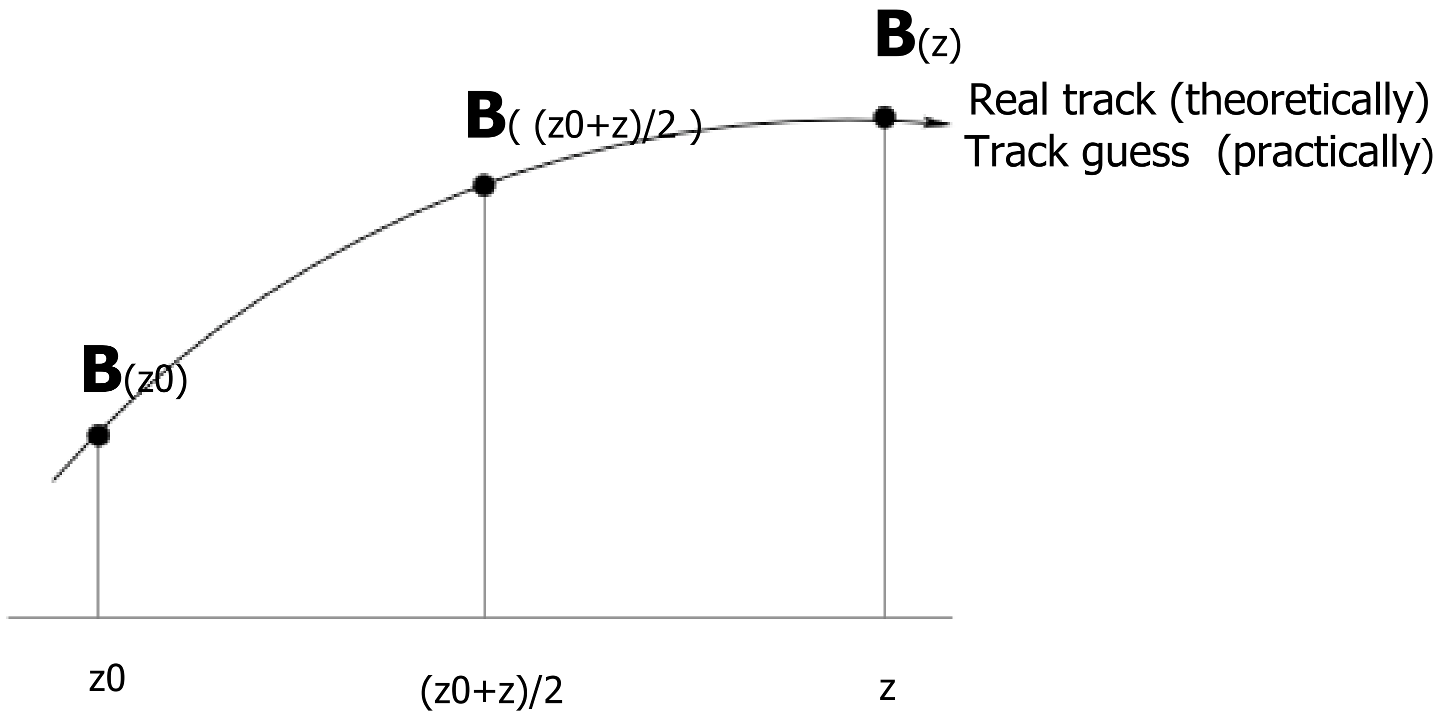
Equation of motion:

$$
\begin{aligned}
\mathbf{a}(z) &\equiv \kappa \cdot (q/p) \cdot \sqrt{1 + t_x^2 + t_y^2} \cdot \left(\quad t_x t_y, \quad -\left(1 + t_x^2\right), \quad t_y \right) \\
\mathbf{b}(z) &\equiv \kappa \cdot (q/p) \cdot \sqrt{1 + t_x^2 + t_y^2} \cdot \left(\left(1 + t_y^2\right), \quad -t_x t_y, \quad -t_x \right)
\end{aligned}
$$

Extrapolation error:

$$
\epsilon = \frac{(\kappa B (q/p) \Delta z)^{n+1}}{(n+1)!}
$$

Analytic Expression Features



Features :

- Field is taken as a function of z coordinate.

Analytic Expression Features

$$
\begin{aligned}
t_x(z_e) &= t_x(z_0) + \sum_{k=1}^{n} \sum_{i_1,\ldots,i_k=x,y,z} t_{x i_1 \ldots i_k}(z_0) \cdot \left(\int_{z_0}^{z_e} B_{i_1}(z_1) \ldots \int_{z_0}^{z_{k-1}} B_{i_k}(z_k) \mathrm{d}z_k \ldots \mathrm{d}z_1 \right) + O(\epsilon) \\
t_y(z_e) &= t_y(z_0) + \sum_{k=1}^{n} \sum_{i_1,\ldots,i_k=x,y,z} t_{y i_1 \ldots i_k}(z_0) \cdot \left(\int_{z_0}^{z_e} B_{i_1}(z_1) \ldots \int_{z_0}^{z_{k-1}} B_{i_k}(z_k) \mathrm{d}z_k \ldots \mathrm{d}z_1 \right) + O(\epsilon) \\
x(z_e) &= x(z_0) + \int_{z_0}^{z_e} t_x(z) \mathrm{d}z + O(\epsilon) \cdot \Delta z \\
y(z_e) &= y(z_0) + \int_{z_0}^{z_e} t_y(z) \mathrm{d}z + O(\epsilon) \cdot \Delta z
\end{aligned}
$$

$$
\epsilon = \frac{(\kappa B(q/p)\Delta z)^{n+1}}{(n+1)!}
$$

Features :

- The precision of extrapolation does not depend on a shape of the magnetic field.
- One can cut off the higher-order terms in the series.

Analytic 3

Example: Third order extrapolation in a homogeneous B_y field

$$
\begin{aligned}
x(z_e) &= x + \Delta z \cdot t_x - (t_x^2+1)\cdot\frac{(hB_y\Delta z)\Delta z}{2!} + 3t_x(t_x^2+1)\cdot\frac{(hB_y\Delta z)^2\Delta z}{3!} - 3(5t_x^4+6t_x^2+1)\cdot\frac{(hB_y\Delta z)^3\Delta z}{4!}\\
y(z_e) &= y + \Delta z \cdot t_y - t_x t_y\cdot\frac{(hB_y\Delta z)\Delta z}{2!} + t_y(3t_x^2+1)\cdot\frac{(hB_y\Delta z)^2\Delta z}{3!} - t_x t_y(15t_x^2+9)\cdot\frac{(hB_y\Delta z)^3\Delta z}{4!}\\
t_x(z_e) &= t_x - (t_x^2+1)\cdot\frac{(hB_y\Delta z)}{1!} + 3t_x(t_x^2+1)\cdot\frac{(hB_y\Delta z)^2}{2!} - 3(5t_x^4+6t_x^2+1)\cdot\frac{(hB_y\Delta z)^3}{3!}\\
t_y(z_e) &= t_y - t_x t_y\cdot\frac{(hB_y\Delta z)}{1!} + t_y(3t_x^2+1)\cdot\frac{(hB_y\Delta z)^2}{2!} - t_x t_y(15t_x^2+9)\cdot\frac{(hB_y\Delta z)^3}{3!}
\end{aligned}
$$

Small parameter: $h = \kappa(q/p)\sqrt{1+t_x^2+t_y^2}$

Coefficients in the Analytic Formula (CBM)

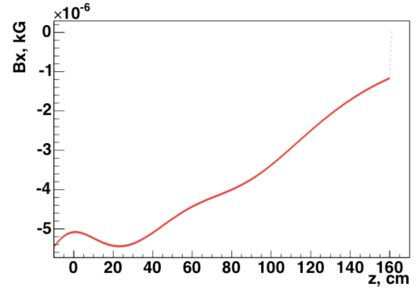
Analytic Light

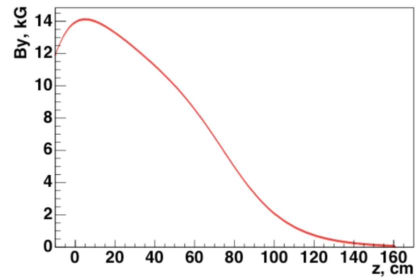
Analytic 1

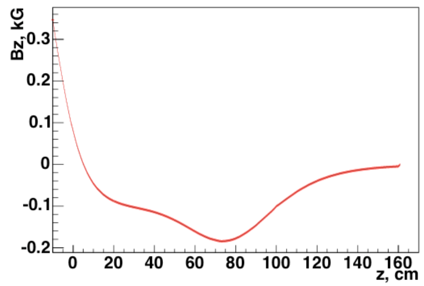
Analytic 2

Analytic 3

c_x	= **1.13e-04**	c_y	= **5.64e-04**	c_z	= **3.09e-04**
c_{xx}	= 3.72e-07	c_{xy}	= 2.57e-06	c_{xz}	= 1.25e-06
c_{yx}	= 3.95e-06	c_{yy}	= **2.39e-04**	c_{yz}	= **3.04e-05**
c_{zx}	= 5.21e-07	c_{zy}	= 4.64e-06	c_{zz}	= 3.74e-06
c_{xxx}	= 5.15e-09	c_{xxy}	= 2.47e-08	c_{xxz}	= 8.73e-09
c_{xyx}	= 3.36e-08	c_{xyy}	= 3.83e-07	c_{xyz}	= 6.57e-08
c_{xzx}	= 5.28e-09	c_{xzy}	= 3.20e-08	c_{xzz}	= 1.78e-08
c_{yxx}	= 4.81e-08	c_{yxy}	= 5.46e-07	C_{yxz}	= 9.34e-08
c_{yyx}	= 7.67e-07	c_{yyy}	= **5.85e-05**	C_{yyz}	= 3.75e-06
c_{yzx}	= 1.30e-07	c_{yzy}	= 1.35e-06	C_{yzz}	= 4.03e-07
c_{zxx}	= 8.33e-09	c_{zxy}	= 3.35e-08	C_{zxz}	= 1.47e-08
c_{zyx}	= 4.76e-08	c_{zyy}	= 8.63e-07	C_{zyz}	= 1.33e-07
c_{zzx}	= 1.59e-08	c_{zzy}	= 9.91e-08	C_{zzz}	= 5.49e-08





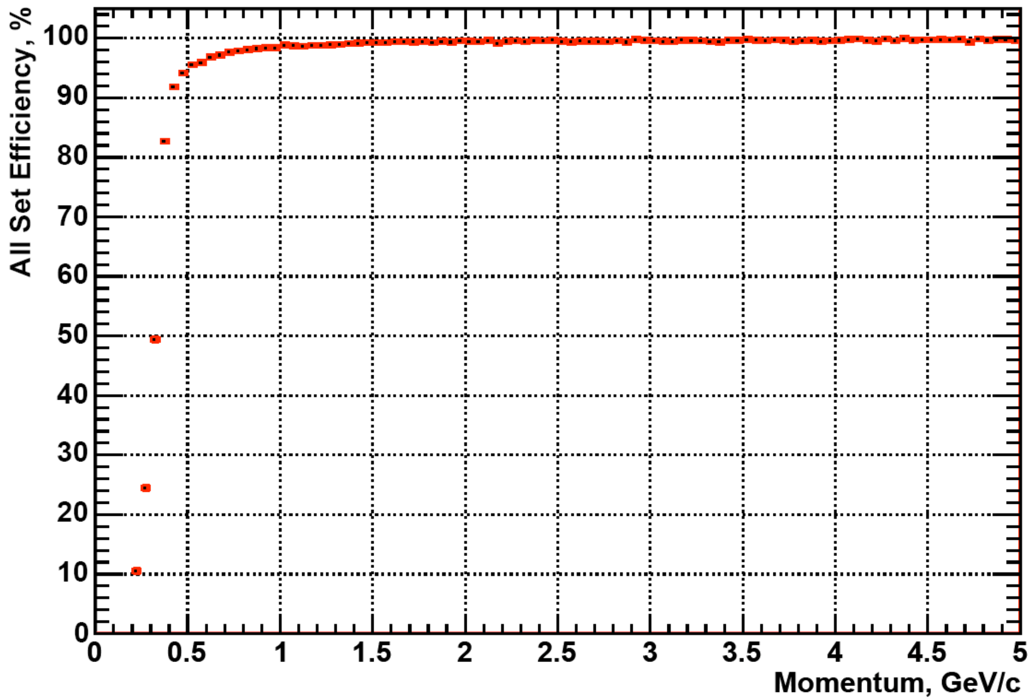


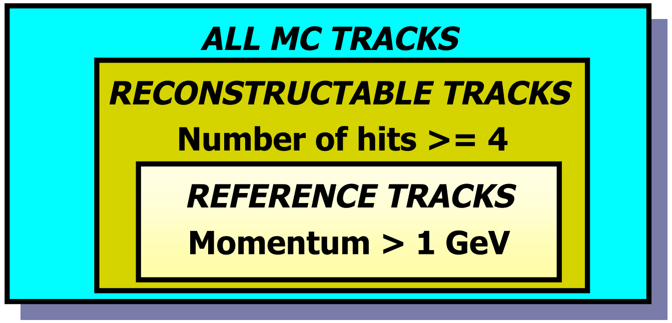
Performance of Extrapolation (CBM)

Method	Residuals					Pulls				
	$\delta p/p$	x	y	t_x	t_y	q/p	x	y	t_x	t_y
Runge-Kutta 4	**0.64**	**27**	**24**	**1.5**	**1.5**	**1.17**	**1.05**	**1.01**	**1.02**	**1.00**
Analytic 3	**0.64**	**27**	**24**	**1.5**	**1.5**	**1.18**	**1.05**	**1.00**	**1.02**	**1.00**
Analytic 2	0.68	27	24	1.5	1.5	1.30	1.08	1.01	1.03	1.00
Analytic 1	0.94	30	25	1.5	1.5	1.90	1.37	1.03	1.10	1.02
Analytic Light	**0.64**	**27**	**24**	**1.5**	**1.5**	**1.19**	**1.05**	**1.00**	**1.02**	**1.00**
Analytic Central	2.49	38	25	1.7	1.5	3.77	2.23	1.03	1.33	1.00

Residuals ($\delta p/p_{[\%]}$, $(x,\ y)_{[\mu\mathrm{m}]}$, $(t_x,\ t_y)_{[\cdot 10^{-3}]}$) and normalized residuals (pulls) of the track parameters at the track vertex z-position after the Kalman filter fitting routine using different extrapolators

Implementation in CBM CA Track Finder





Track category	Efficiency, %
Reference set	99.45
All set	96.98
Extra set	89.46
Clone	0.01
Ghost	0.61

Conclusion

- An analytic formula of extrapolation in magnetic field has been derived
- Precision of extrapolation does not depend on a shape of the field.
- The implementation in the CBM track fitting procedure has the same accuracy as the fourth-order Runge-Kutta method.