TWISTOR APPROACH TO ONE-LOOP AMPLITUDES

Andreas Brandhuber

QUEEN MARY UNIVERSITY OF LONDON

- AB - SPENCE - TRAVAGLINI

  hep-th/0407214

- BEDFORD - AB - SPENCE - TRAVAGLINI

  hep-th/0410280, 0412108

- AB - McNAMARA - SPENCE - TRAVAGLINI

  to appear

WITTEN 2003:

PERTURBATIVE $\mathcal{N}=4$ SUPER YANG MILLS $\Longleftrightarrow$ TOPOLOGICAL STRING ON TWISTOR SPACE

WEAK/WEAK DUALITY

WHY IS THAT INTERESTING?

- Explains unexpected simplicity of Scattering Amplitudes (YM + Gravity)

  $\Rightarrow$ SIMPLE GEOMETRIC STRUCTURE IN TWISTOR SPACE!

  $\Leftrightarrow$ NEW DIFF. EQNS. FOR AMPLITUDES

- New Tools to calculate amplitudes

  $\Rightarrow$ MHV DIAGRAMS FOR TREES AND LOOPS

  $\Rightarrow$ GENERALIZED UNITARITY

  $\Rightarrow$ NEW RECURSION FORMULAS

MOTIVATION:

- LHC IS COMING!
  - PRECISION PERT. QCD CALCULATIONS
  - LONG WISHLIST OF PROCESSES TO BE COMPUTED
- NEW TECHNIQUES ARE NEEDED!
  * TEXTBOOK METHODS HIDE SIMPLICITY OF SCATTERING AMPLITUDES
  * INTERMEDIATE EXPRESSIONS ARE LARGE
  * FACTORIAL GROWTH OF # DIAGRAMS

$gg \to n\,g$	$n=7$	$n=8$	$n=9$
	559405	10525900	224449225

☺ LUCKILY NOBODY USES TEXTBOOK TECHNIQUES

⇒ COLOR DECOMPOSITION, SPINOR HELICITY, UNITARITY, SUSY, STRING THEORY, ...

AND TWISTOR (INSPIRED) TECHNIQUES

COLOUR DECOMPOSITION

- AT TREE LEVEL, YANG-MILLS INTERACTIONS ARE PLANAR (AND EFF. SUPERSYMMETRIC)

$$A_n^{tree}(\{p_i, \epsilon_i, a_i\}) = \sum_{\sigma \in S_n/\mathbb{Z}_n} \mathrm{Tr}\left(T^{a_{\sigma(1)}} \dots T^{a_{\sigma(n)}}\right) \times \underbrace{A(\sigma(p_1,\epsilon_1), \dots, \sigma(p_n,\epsilon_n))}_{\text{"COLOUR ORDERED PARTIAL AMPL."}}$$

  * ONLY DIAGRAMS WITH A FIXED CYCLIC ORDERING CONTRIBUTE

  * ANALYTIC STRUCTURE IS SIMPLER

- AT LOOP LEVEL, MULTI-TRACE STRUCTURES APPEAR (SUBLEADING IN $1/N$)

  * HOWEVER AT ONE-LOOP A SIMPLE RELATION BETWEEN PLANAR AND NON-PLANAR TERMS EXISTS!

SPINOR HELICITY:

- RESPONSIBLE FOR COMPACT FORMULAS OF TREE AND LOOP AMPLITUDES IN MASSLESS THEORIES

- THE 4D LORENTZ GROUP IS $SL(2,\mathbb{C})$

$$p_\mu \iff p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}} \; ; \; a, \dot{a} = 1, 2$$

$$\text{iff} \quad p^2 = 0 \Rightarrow p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} \, , \; \tilde{\lambda} = \bar{\lambda}$$

λ_a: L, $\tilde{\lambda}_{\dot{a}}$: R SPINOR

- SPINOR PRODUCTS:

$$\langle ij \rangle \equiv \lambda^i_a \lambda^j_b \epsilon^{ab}$$
$$[ij] \equiv \tilde{\lambda}^i_{\dot{a}} \tilde{\lambda}^j_{\dot{b}} \epsilon^{\dot{a}\dot{b}}$$

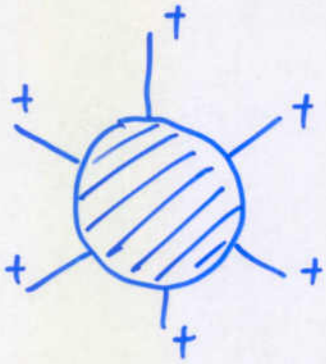
$$\Rightarrow 2 p_i \cdot p_j = - \langle ij \rangle [ij]$$

- "REDUNDANCY": $p_{a\dot{a}}$ IS INVARIANT UNDER

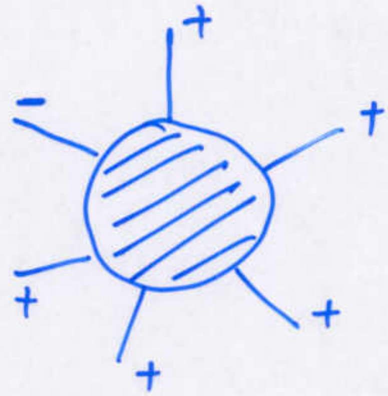
$$(\lambda_a, \tilde{\lambda}_{\dot{a}}) \sim (t\lambda_a, t^{-1}\tilde{\lambda}_{\dot{a}})$$

$\Rightarrow (\lambda, \tilde{\lambda})$ CONTAIN THE CORRECT # D.O.F. TO DESCRIBE MOMENTUM + WAVEFUNCT./POLARIZATION OF MASSLESS PARTICLES WITH ARBITRARY HELICITY h.

n-GLUON TREE AMPLITUDES



$$= 0 =$$



$$= A_n^{tree}$$

PARKE-TAYLOR
(BERENDS-GIELE)

- $A_n^{tree} = i g^{n-2} (2\pi)^4 \delta^{(4)}(\Sigma p_i) \dfrac{\langle \lambda_i \lambda_j \rangle^4}{\prod_{k=1}^{n} \langle \lambda_k \lambda_{k+1} \rangle}$
- VERY SIMPLE!
- HOLOMORPHIC! A_n^{tree} DEPENDS ONLY ON λ's NOT $\tilde{\lambda}$'s (EXCEPT FOR δ-FNCT.)
- TRUE FOR $\mathcal{N}=4,1$ SUPER YANG-MILLS PURE YANG-MILLS, QCD !
- SIMILAR FORMULAS FOR AMPLITUDES WITH TWO GLUON REPLACED BY FERMIONS/SCALARS!

TWISTOR SPACE

- IS A $\frac{1}{2}$ FOURIER TRANSFORM OF SPINOR SPACE $(\lambda_a, \tilde{\lambda}_{\dot{a}})$

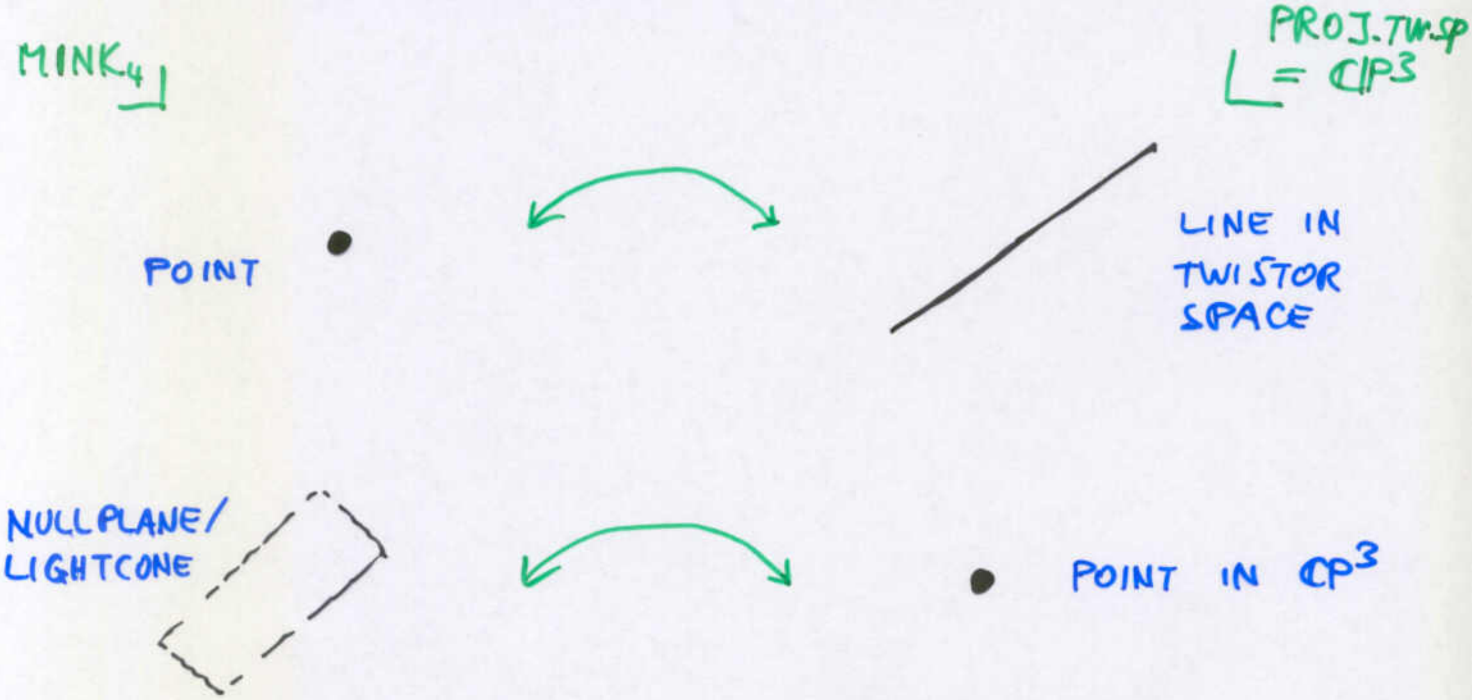
$$\downarrow$$

$$(\lambda_a, \mu^{\dot{a}})$$

- TWISTOR SPACE IS 4 DIM'L $(\lambda_1, \lambda_2, \mu^{\dot{1}}, \mu^{\dot{2}})$
- AMPLITUDES ARE HOMOGENEOUS FUNCTIONS ON TW. SP.

$\Rightarrow$ PROJECTIVE TWISTOR SPACE $(\lambda, \mu) \sim (A\lambda, A\mu)$

- RELATIONS BETWEEN $MINK_4$ AND PROJ. TW. SP.



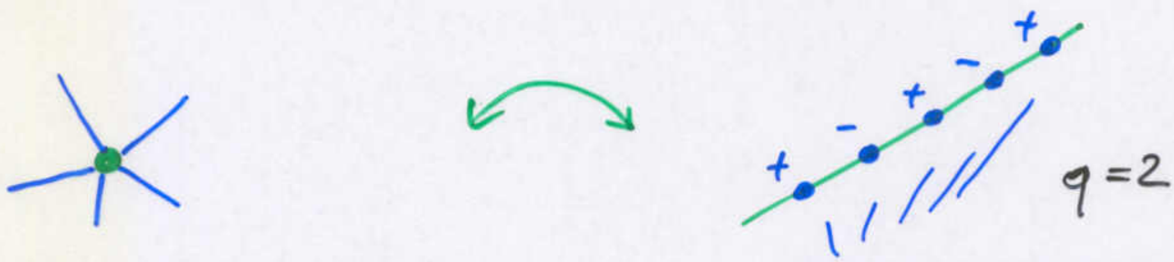
AMPLITUDES IN TWISTOR SPACE

- MHV AMPLITUDES ARE HOLOMORPHIC, EXCEPT FOR MOMENTUM CONSERVATION

PERFORM $\frac{1}{2}$ F.T.

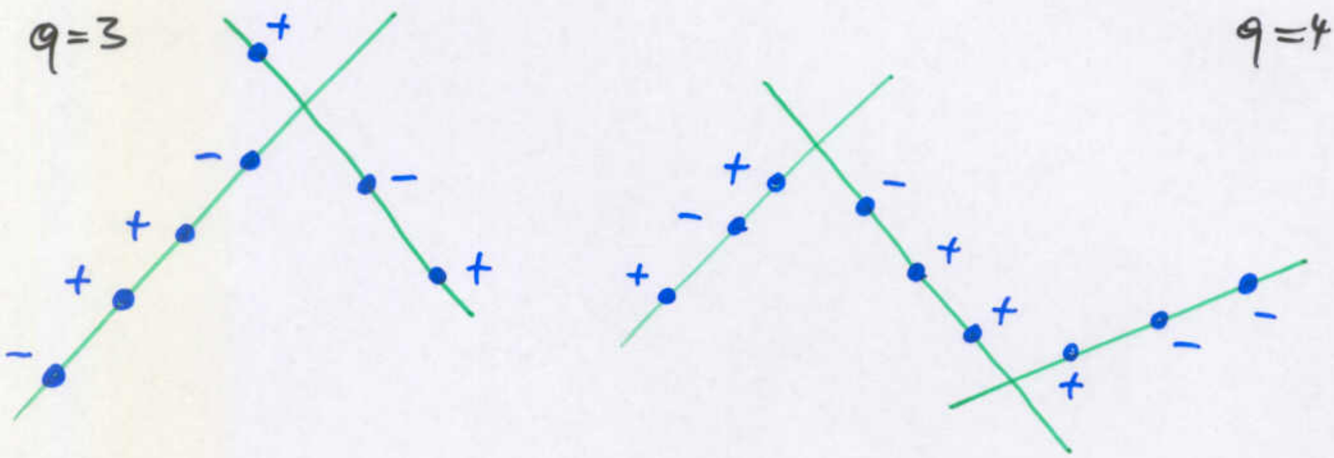
$$\Rightarrow A_{MHV}(\lambda_i)\int \prod_i d\tilde{\lambda}_i \, e^{i\mu_i\tilde{\lambda}_i} e^{ix\lambda_i\tilde{\lambda}_i} \approx \prod_i \delta(\mu_i + x\lambda_i)$$

$\Rightarrow$ FOR MHV AMPLITUDES ALL POINTS LIE ON A LINE IN TWISTOR SPACE



POINTS CORRESPOND TO EXT. GLUONS

- FOR NON-MHV AMPLITUDES "EXPERIMENTS" REVEAL:



MHV VERTICES

CACHAZO - SVRCEK - WITTEN

- IN A CERTAIN SENSE MHV AMPLITUDES ARE LOCAL INTERACTIONS IN MINK_4

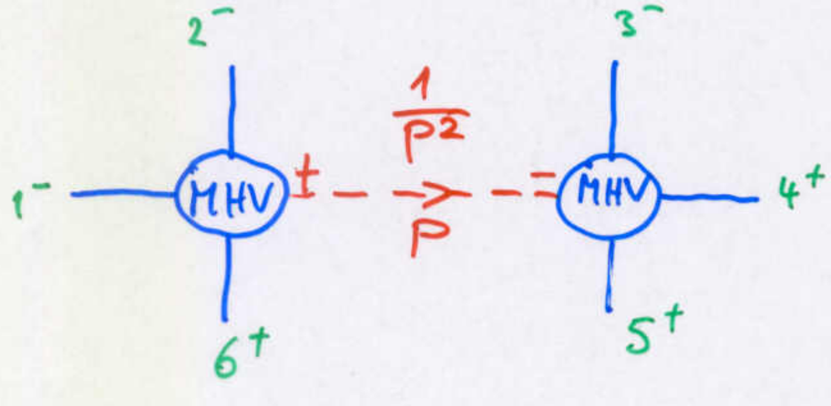
- CSW RULES

  * USE MHV AMPLITUDES CONTINUED OFF-SHELL AS LOCAL VERTICES IN DIAGRAMS

  * CONNECT MHV VERTICES W/ $\frac{1}{P^2}$ PROPAGATORS

  * SUM DIAGRAMS WITH FIXED CYCLIC ORDERING OF EXTERNAL LINES

Ex $\langle 1^- 2^- 3^- 4^+ 5^+ 6^+ \rangle \Rightarrow$ "FIELD THEORY PICTURE"



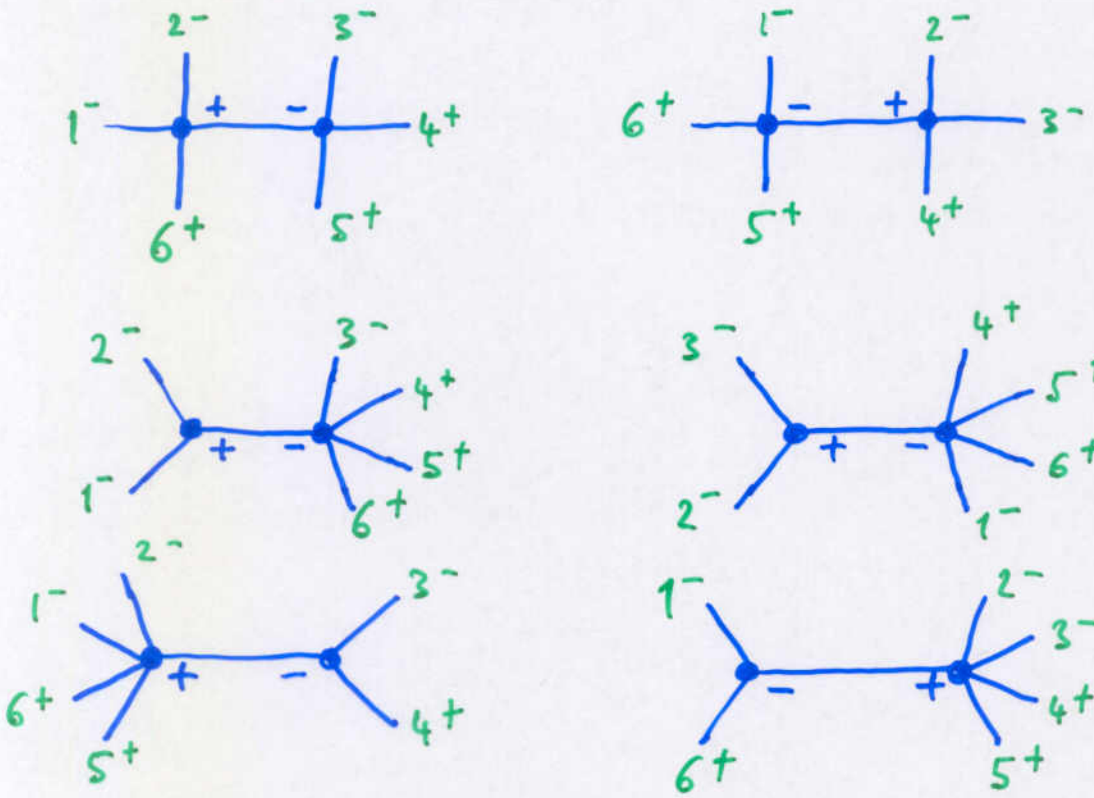
  * OFF-SHELL CONTINUATION: $P^2 \neq 0$

$$\lambda_{Pa} = P_{a\dot{a}} \eta^{\dot{a}}$$

↳ REFERENCE SPINOR

MHV VERTICES CONTINUED

$\langle 1^- 2^- 3^- 4^+ 5^+ 6^+ \rangle$



- REPRODUCE KNOWN AND OBTAIN NEW SCATTERING AMPLITUDES IN ANY MASSLESS GAUGE THEORY

  DRAMATIC SIMPLIFICATIONS

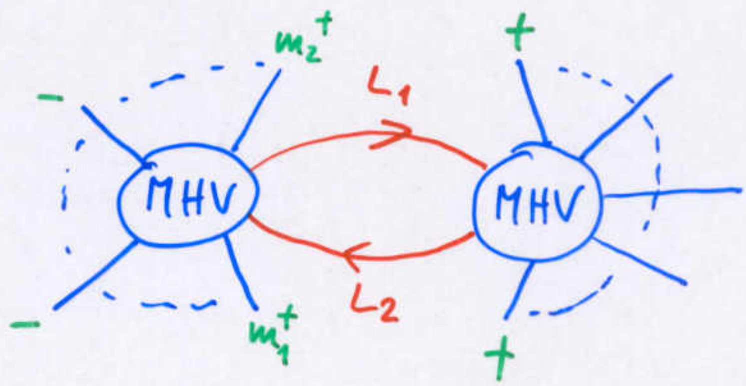
- CORRECT FACTORIZATION
  - MULTIPARTICLE SINGULARITIES ✓
  - COLLINEAR/SOFT LIMITS ✓

FROM TREES TO LOOPS

AB-SPENCE-TRAVAGLINI

- PROGNOSIS FROM TWISTOR STRING THEORY NEGATIVE (BERKOVITS-WITTEN) $\Rightarrow$ "POLLUTION" WITH CONF. SUPER GRAVITY MODES SPOILS DUALITY WITH $\mathcal{N}=4$ SYM
- TRY ANYWAY!
  - CONNECT $V = q - 1 + \ell$ VERTICES
  - USE MHV VERTICES AS FOR TREES
  - PERFORM LOOP INTEGRATION!
- SIMPLEST EXAMPLE: MHV 1-LOOP AMPLITUDES IN $\mathcal{N}=4$ SYM

$$\int \sum_{m_1, m_2, h}$$

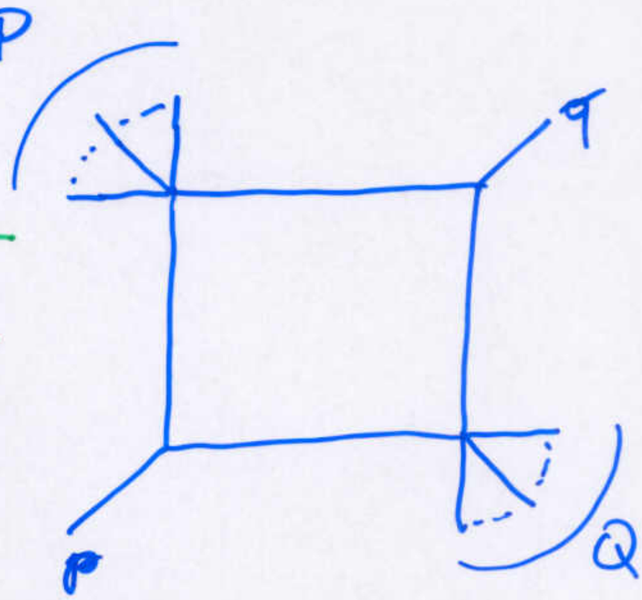


MHV 1-LOOP AMPLITUDES IN $\mathcal{N}=4$ SYM

- COMPUTED BY BERN-DIXON-DUNBAR-KOSOWER (1994) USING "4D CUT-CONSTRUCTIBILITY" (= UNITARITY)

- RESULT EXPRESSED IN TERMS OF "2-MASS EASY BOX FUNCTIONS" $I^{2me}(s, t, P^2, Q^2)$

$$A_{MHV}^{1\text{-}LOOP} = A_{MHV}^{TREE} \times \sum_{p,q}$$



MHV DIAGRAMMATIC CALCULATION AGREES WITH BDDK'S RESULT!

- CRUCIAL DECOMPOSITION OF LOOP MOMENTA

$$L_i = \underbrace{l_i}_{\text{ON-SHELL}} + \underbrace{z}_{\text{REAL NR.}}\, \underbrace{\eta}_{\text{FIXED, NULL}}$$

- NATURALLY LEADS TO "DISPERSION INTEGRALS" THAT CAN BE PERFORMED EXPLICITLY ($\approx$ z-INTEGRATION)

- NON-TRIVIAL CHECK OF MHV VERTICES

$\Rightarrow$ CANCELLATION OF η DEPENDENCE IS HIGHLY NON-TRIVIAL

- INCORPORATES LARGE NUMBERS OF CONVENTIONAL FEYNMAN DIAGRAMS

GENERALISATIONS

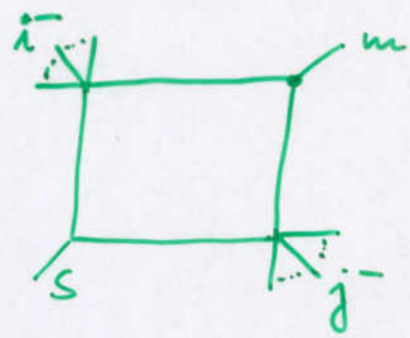
- OUR APPROACH READILY APPLIES TO NON-MHV AMPLITUDES AND THEORIES WITH LESS SUPERSYMMETRY

$\Rightarrow$ MHV, 1-LOOP AMPLITUDES IN $\mathcal{N}=1$ SYM   Bedford, AB, Spence, Travaglini

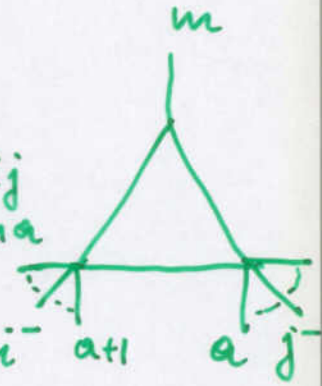
* RESULT EXPRESSED IN TERMS OF SCALAR BOX AND TRIANGLE FUNCTIONS

* $\mathcal{A}_{\text{chiral}}^{\mathcal{N}=1,\, 1\text{-loop},\, \text{MHV}} = \mathcal{A}_{\text{MHV}}^{\text{tree}} \times I$

$$I = \sum_{m,s} b_{m,s}^{i,j}$$



$$+ \sum_{m,a} c_{ma}^{ij}$$



$$+ \; (i \Leftrightarrow j)$$

* AGREES WITH BDDK ✓

* $\nexists$ TWISTOR STRING DUAL FOR $\mathcal{N}=1$ SUSY

MHV AMPLITUDES IN PURE YANG-MILLS

- CONTAIN A "CUT-CONSTRUCTIBLE" PART BUT ALSO RATIONAL TERMS
- FROM MHV-VERTICES WE OBTAIN THE CUT-CONSTRUCTIBLE PART
- SUSY DECOMPOSITION !

$$A^{g} = \underbrace{(A^{g}+4A^{f}+3A^{s})}_{\mathcal{N}=4} - 4\underbrace{(A^{f}+A^{s})}_{\mathcal{N}=1} + \underbrace{A^{s}}_{\text{TO BE COMPUTED}}$$

- RESULT IS EXPRESSED IN TERMS OF

$$F_{\text{finite}} = B(s,t,P^2,Q^2) \quad \text{and} \quad T^{(r)}(p,P,Q) = \frac{\log Q^2/P^2}{(Q^2-P^2)^r}$$

  * COEFFICIENT OF B IS $\left[b^{ij}_{m_1 m_2} \right]^2$
  * AGREES WITH 5-POINT RESULT (BDK)
  * Adjacent neg. Helicity gluons: (BDDK) ✓
  * NEW RESULT FOR NEGATIVE HELICITY GLUONS IN ARBITRARY POSITIONS → A STEP TOWARDS QCD !

GENERALISED UNITARITY

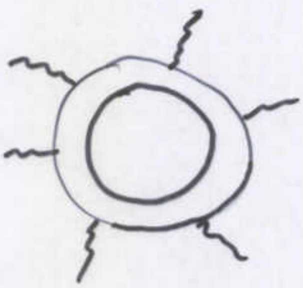
BRITTO
CACHAZO
FENG

BERN
DIXON
KOSOWER

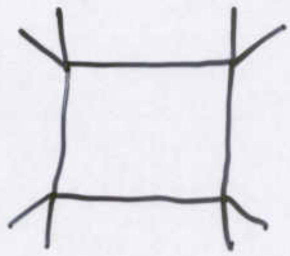
- VERY OLD IDEA, E.G. "ANALYTIC S MATRIX" 1966
  EDEN-LANDSHOFF-OLIVE-POLKINGHORNE

- ONE-LOOP AMPLITUDES IN SUSY GAUGE THEORIES
  ARE 4D CUT-CONSTRUCTIBLE

- 1-LOOP AMPLITUDES IN $\mathcal{N}=4$ SYM ARE OF THE FORM



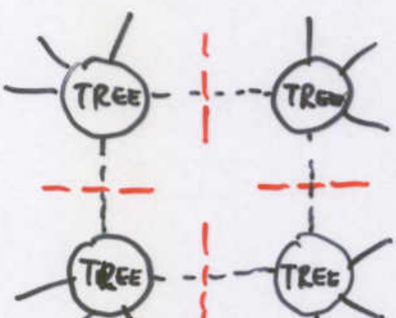
$$\equiv \sum c$$



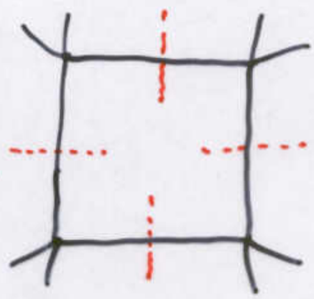
RATIONAL COEFFICIENTS

- CAN WE FIND THE c's WITHOUT PERFORMING INTEGRALS?

YES $\Rightarrow$ USE QUADRUPLE CUTS $\equiv$
REPLACE 4 PROPAGATORS BY $\delta^{+}(L_i^2)$
$i=1,2,3,4$



$$= C$$



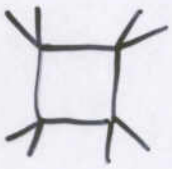
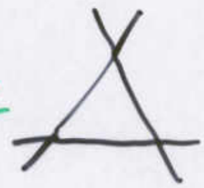
UNIQUE BOX FOR EVERY QUAD. CUT

- LOOP INTEGRATION LOCALISES COMPLETELY!

- C = PRODUCT OF 4 ON-SHELL TREE AMPLITUDES

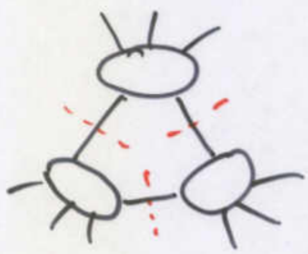
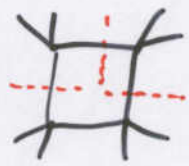
$\Rightarrow$ $\mathcal{N}=4$ SYM AT 1-LOOP IS REDUCED TO AN ALGEBRAIC EXERCISE!

- SIMILAR, BUT MORE WORK FOR 1-LOOP AMPLITUDES IN $\mathcal{N}=1$ SYM:

$\sum b$  $+ \sum c$  $+ \sum d$ 

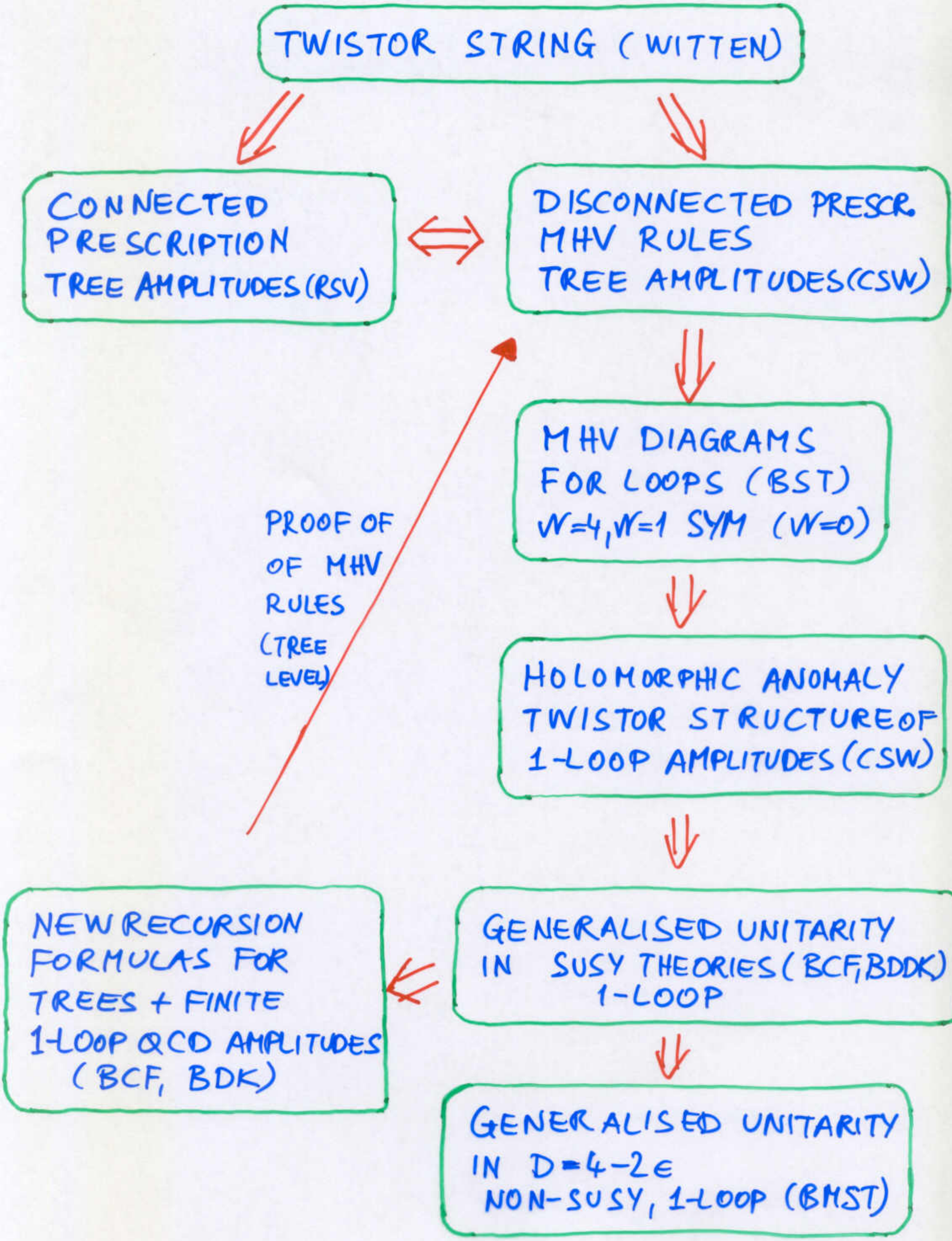
* FIX b's WITH QUAD. CUTS

* FIX c's WITH TRIPLE CUTS

 $= c$  $+ \sum' b$ 

* FIX d's WITH CONVENTIONAL UNITARITY CUTS

$\Rightarrow$ ALL $\mathcal{N}=1$, 1-LOOP, 6 POINT AMPLITUDES AND AN INFINITE SERIES $\langle - - - + \underbrace{\ldots +}_{n-3} \rangle$


TWISTOR STRING (WITTEN)
CONNECTED PRESCRIPTION TREE AMPLITUDES (RSV)
DISCONNECTED PRESCR. MHV RULES TREE AMPLITUDES (CSW)
MHV DIAGRAMS FOR LOOPS (BST) $\mathcal{N}=4, \mathcal{N}=1$ SYM ($\mathcal{N}=0$)
PROOF OF OF MHV RULES (TREE LEVEL)
HOLOMORPHIC ANOMALY TWISTOR STRUCTURE OF 1-LOOP AMPLITUDES (CSW)
NEW RECURSION FORMULAS FOR TREES + FINITE 1-LOOP QCD AMPLITUDES (BCF, BDK)
GENERALISED UNITARITY IN SUSY THEORIES (BCF, BDDK) 1-LOOP
GENERALISED UNITARITY IN $D=4-2\epsilon$ NON-SUSY, 1-LOOP (BMST)