Calculational techniques (not only) for single top production

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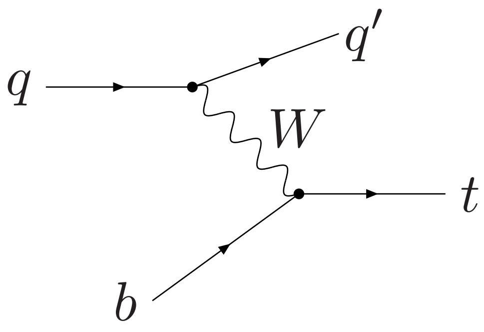
1. **Single top production**
2. **NLO corrections: Traditional and modern techniques**
3. **Spinor methods**
4. **Tensor integral reduction**
5. **Treatment of γ_5**
6. **Conclusions**

Basic facts about top

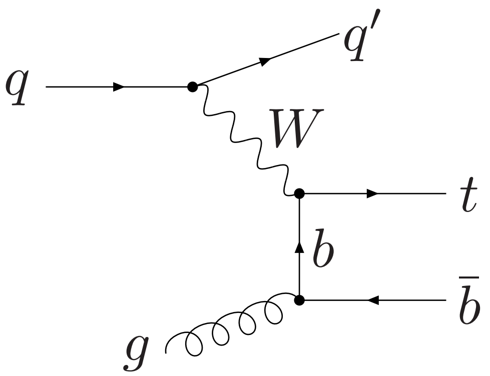
- Discovered in 1995 at Tevatron through $t\bar{t}$-pair production.
- Mass $m_t = 175 \pm 5\ GeV$, width $\Gamma = 1.7\ GeV$.
- Top is special: Lifetime shorter than characterisitic hadronization time scale.
- Can also be produced through an electroweak Wtb-vertex.

Single top production

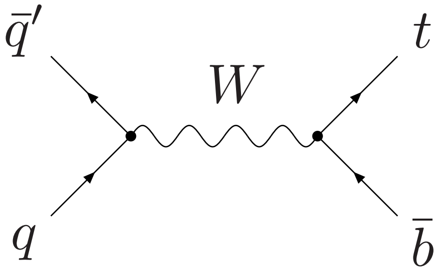
- A single top quark can be produced by weak interaction:



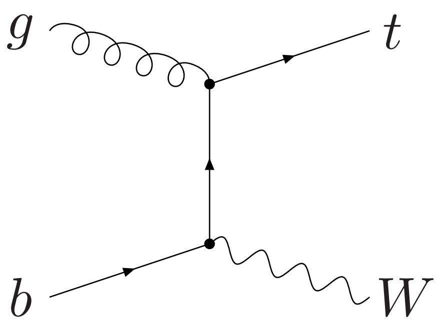
Flavour excitation



W-gluon fusion



s-channel



Associated W-production

Single top production

- Process sensitive to Wtb vertex.
  Non-standard couplings can give a hint on physics beyond SM.

- Direct measurement of CKM matrix element V_{tb}.
  Verification of unitarity of CKM matrix.

- The top quark is produced left-handed.
  Opportunity for polarization studies.

First step:

- Full LO calculation including the production and the decay of the top quark.

- Comparison of full calculation with top narrow width approximation.

Top narrow width approximation

Tevatron	σ_{tot}	± 20 GeV	narrow width
Wg	15.0 ± 0.4 fb	14.3 ± 0.3 fb	14.5 ± 0.1 fb
Wb	87 ± 1 fb	85 ± 2 fb	87 ± 1 fb
$q\bar{q}$	46 ± 1 fb	32.3 ± 0.3 fb	29.0 ± 0.2 fb

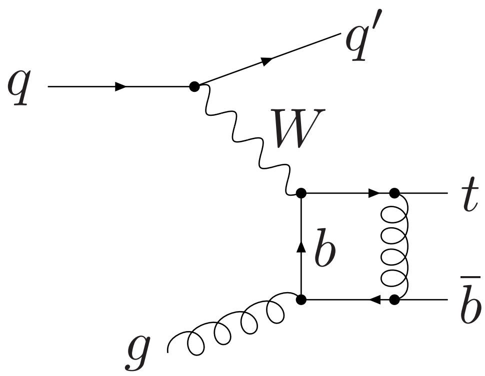
Table 1: Numerical results for Tevatron at 2 TeV.

LHC	σ_{tot}	± 20 GeV	narrow width
Wg	4.6 ± 0.2 pb	4.5 ± 0.4 pb	4.6 ± 0.1 pb
Wb	13.1 ± 0.3 pb	13.0 ± 0.4 pb	13.3 ± 0.1 pb
$q\bar{q}$	685 ± 19 fb	479 ± 16 fb	432 ± 4 fb

Table 2: Numerical results for the LHC at 14 TeV.

NLO calculation

- NLO corrections to W-gluon fusion.
- More reliable prediction on p_T-spectrum.



Complications:

- Cannot neglect the W- and top-mass.
- W-boson has left-handed couplings.

Traditional techniques

- Calculate

$$2 \text{ Re } A^*_{Born} A_{Loop}$$

- Use Pasarino-Veltman reduction for tensor integrals.

- Use 't Hooft-Veltman scheme for γ_5.

Solves the problem in principal, but leads to complicated expressions !

Spinor methods

- Basic objects: Massless two-component Weyl spinors

$$|p\pm\rangle, \qquad \langle p\pm|$$

- Gluon polarization vectors:

$$\varepsilon_\mu^+(k,q) = \frac{\langle k+|\gamma_\mu|q+\rangle}{\sqrt{2}\langle q-|k+\rangle}, \qquad \varepsilon_\mu^-(k,q) = \frac{\langle k-|\gamma_\mu|q-\rangle}{\sqrt{2}\langle k+|q-\rangle}$$

- Massive spinors:

$$\begin{aligned} u(\pm) &= \frac{1}{\sqrt{2pq}}(\not{p}+m)\,|q\mp\rangle \\ \bar{u}(\pm) &= \frac{1}{\sqrt{2pq}}\langle q\mp|\,(\not{p}+m) \end{aligned}$$

Tensor integral reduction

- The Pasarino-Veltman algorithm based on Lorentz invariance.
- The Feynman parameter space technique by Z. Bern, L. Dixon and D.A. Kosower.
- Partial integration
- Dual vectors In four dimensions there can be only four linearly independent vectors (J.A.M. Vermaseren and W. van Neerven).
- Spinor algebra Loop momenta are usually sandwiched between spinors (R. Pittau).

Tensor integral reduction with spinors

Example:

$$\gamma^\mu \gamma^\nu \int \frac{d^D k}{(2\pi)^D} \frac{k_\mu k_\nu}{(k^2 - m_1^2) \ldots ((k - q_n)^2 - m_n^2)}$$

Trick:

$$\gamma^\mu \gamma^\nu k_\mu k_\nu \quad = \quad (k^2 - m_1^2) + m_1^2$$

Basic identities:

- Fierz identity:

$$\langle A + |\gamma_\mu| B + \rangle \langle C - |\gamma^\mu| D - \rangle \quad = \quad 2 \, [AD] \, \langle CB \rangle$$

- Schouten identity for spinors:

$$\langle AB \rangle \langle CD \rangle \quad = \quad \langle AD \rangle \langle CB \rangle + \langle AC \rangle \langle BD \rangle$$

- Schouten identity for four-vectors:

$$\varepsilon(q_1, q_2, q_3, q_4) k_\mu \quad = \quad \sum_{a=1}^{4} (k \cdot q_a) \, v_\mu^a$$

$$v_\mu^a = \frac{1}{6} \varepsilon^{abcd} \varepsilon(\mu, q_b, q_c, q_d)$$

Tensor integral reduction with spinors

Step 1: Rank ≥ 2 (R. Pittau).

$$\langle p - |\not{k}|q-\rangle\langle p + |\not{k}|q+\rangle = \text{rank-1}$$

Step 2: Rank $= 1$ (S.W.).

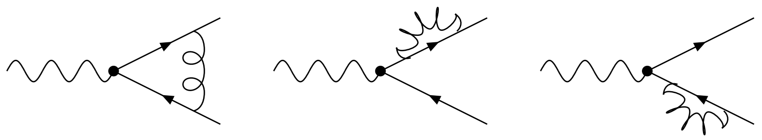
Box : $\text{Tr}\ \not{k}\not{p}_1\not{p}_2\not{p}_3 = \text{scalar}$

Pentagon : $\text{Tr}\ \not{k}\not{p}_1\not{p}_2\not{p}_3\not{p}_4\not{p}_5 = \text{scalar}$

No Gram determinants involved !

γ_5 in D dimensions

- Naive dimensional regularization is not consistent.
- Dimensional reduction is not consistent.
- The 't Hooft-Veltman scheme is consistent.
  - violates the Ward identity for the axial current.



  - this Ward identity has to be restored by hand through a finite renormalization.

Algebraic consistency of dimensional regularization

- Axiomatic approach ('t Hooft, Veltman, Breitenlohner, Maison):
  Take γ_μ, γ_5, $g_{\mu\nu}$ etc. as a set of abstract symbols with a given set of relations. Show that these relations are self-consistent.

- Constructive approach (Wilson, Collins):
  Find a represenation for each object in some (finite or infinit-dimensional) vectorspace.

The constructive approach

Take the natural numbers $\mathbb{N} = \{1, 2, 3, ...\}$ and consider pairs (p, q) with $p, q \in \mathbb{N}$.

Call two pairs (p, q) and (k, l) equivalent if

$$p + l \quad = \quad k + q$$

Define an addition by components

$$(a, b) + (c, d) \quad = \quad (a + c, b + d)$$

The set of equivalence classes yields the integer numbers.

The inverse of (p, q) is (q, p).

The neutral element is (a, a).

K - theory

- Repeat the exercise with pairs (a, b) and (c, d), where $a, b, c, d \in \mathbb{Z}$, and the equivalence relation

$$a \cdot d \quad = \quad c \cdot b$$

- Consider pairs (V_i, V_j) and (V_k, V_l) from a set of vector spaces $\{V_1, V_2, V_3, ...\}$ and the relation

$$V_i \oplus V_l \quad \sim \quad V_k \oplus V_j$$

This construction is quite general and forms the basis of what mathematicians call K - theory.

More contructivism

- Take a pair (V_i, V_j) of vectorspaces. The total number of dimensions is

$$\text{dim } V_i + \text{dim } V_j$$

On the other hand

$$\text{rank} \quad = \quad \text{dim } V_i - \text{dim } V_j$$

is relevant to dimensional regularization.

- Can construct n-tuples of vectorspace such that their ranks form a dense subset of $\mathbb{C}$.

- The integration is independent of the chosen representative.

- Can work with vectorspace of even dimension only.

Regularization scheme

This yields a regularization scheme:

- The scheme is consistent.
- The Dirac algebra can effectively be performed in four dimensions.
- The scheme violates Ward identities, which have to be restored by finite renormalizations.

Summary

- Single top production.
- NLO corrections to W-gluon fusion.
- Spinor methods.
- Reduction of tensor integrals
- Construction of regularization schemes.