Higgs-mass predictions and electroweak precision observables in the Standard Model and the MSSM

Georg Weiglein, CERN

Bastei, 04/2000

1. Introduction

2. Precision observables in the SM and the MSSM at present and future colliders

3. Higher-order results for m_{h} in the MSSM

4. Conclusions

1. Introduction

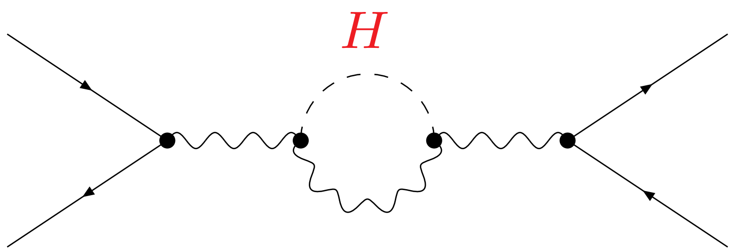
Comparison of ew precision data with theory:

EW precision data:	Theory:
$M_Z, M_W, \sin^2\theta_{\rm eff}^{\rm lept}, \ldots$	SM, MSSM, ...

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



All parameters of the model enter

$\Downarrow$

Improve indirect constraints on unknown parameters: M_H, $m_{\tilde{t}}$, ...

effects of "new physics"?

Indirect determination of m_t from precision data:

$$m_t = 172^{+14}_{-11} \text{ GeV}$$

direct measurement:

$$m_t = 174.3 \pm 5.1 \text{ GeV}$$

One-loop corrections to precision observables:

$$\sim m_t^2$$

$$\sim \ln M_H$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

– unknown higher-order corrections

– exp. error of input parameters: m_t, $\Delta\alpha_{had}$, ...

Assume: SUSY particles accessible at next
generation of colliders

In unconstrained MSSM: 105 new parameters

masses, $\tan\beta$, mixing angles, complex phases

$\Rightarrow$ Parameter determination is difficult task

Is the model consistent with data?

Patterns of SUSY breaking?

(M-SUGRA, GMSB, AMSB, ...)

Indirect information from precision observables
complementary to direct production processes

Stringent direct test of SUSY:

Light Higgs boson h required

m_h calculable from MSSM parameters:

Tree level: $m_\mathsf{h} < M_\mathsf{Z}$

Large radiative corrections: $\Rightarrow$ $m_\mathsf{h} \lesssim 135$ GeV

m_h, $\sigma_{e^+e^-\to Zh}$, Γ_h:

Precise prediction needed for Higgs search:
Discovery/exclusion potential of LEP2 and upgraded Tevatron

High-precision measurement of m_h:

$$\begin{aligned} \text{LHC}: &\quad \Delta m_\mathsf{h} \approx 0.2 \text{ GeV} \\ \text{LC}: &\quad \Delta m_\mathsf{h} \approx 0.05 \text{ GeV} \\ \text{Muon Collider}: &\quad \Delta m_\mathsf{h} \approx 0.1 \text{ MeV} \end{aligned}$$

$\Rightarrow$ Very sensitive test of the theory

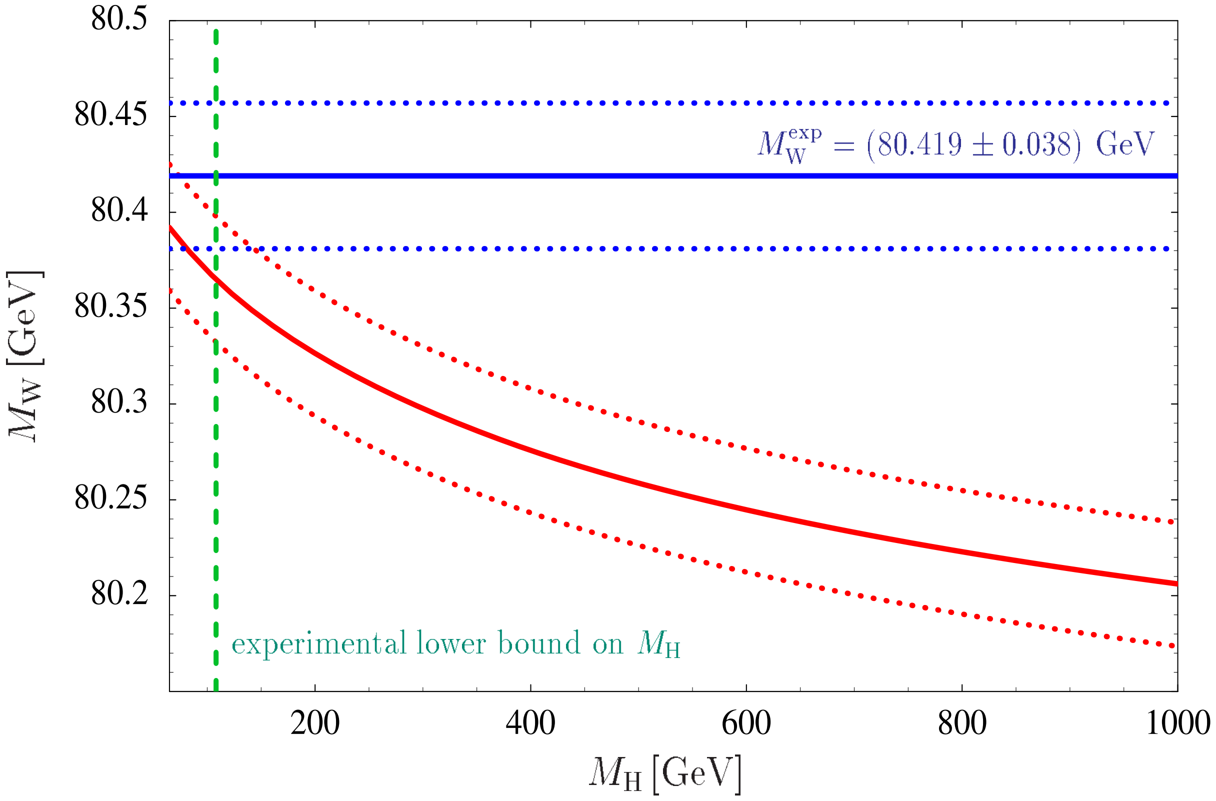
Precise prediction for m_H:

$\Rightarrow$ Resolution of splitting m_H, M_A

2. Precision observables in the SM and the MSSM at present and future colliders

SM prediction for M_{W} vs. experimental result:

[*A. Freitas, S. Heinemeyer, W. Hollik, W. Walter, G.W. '00*]



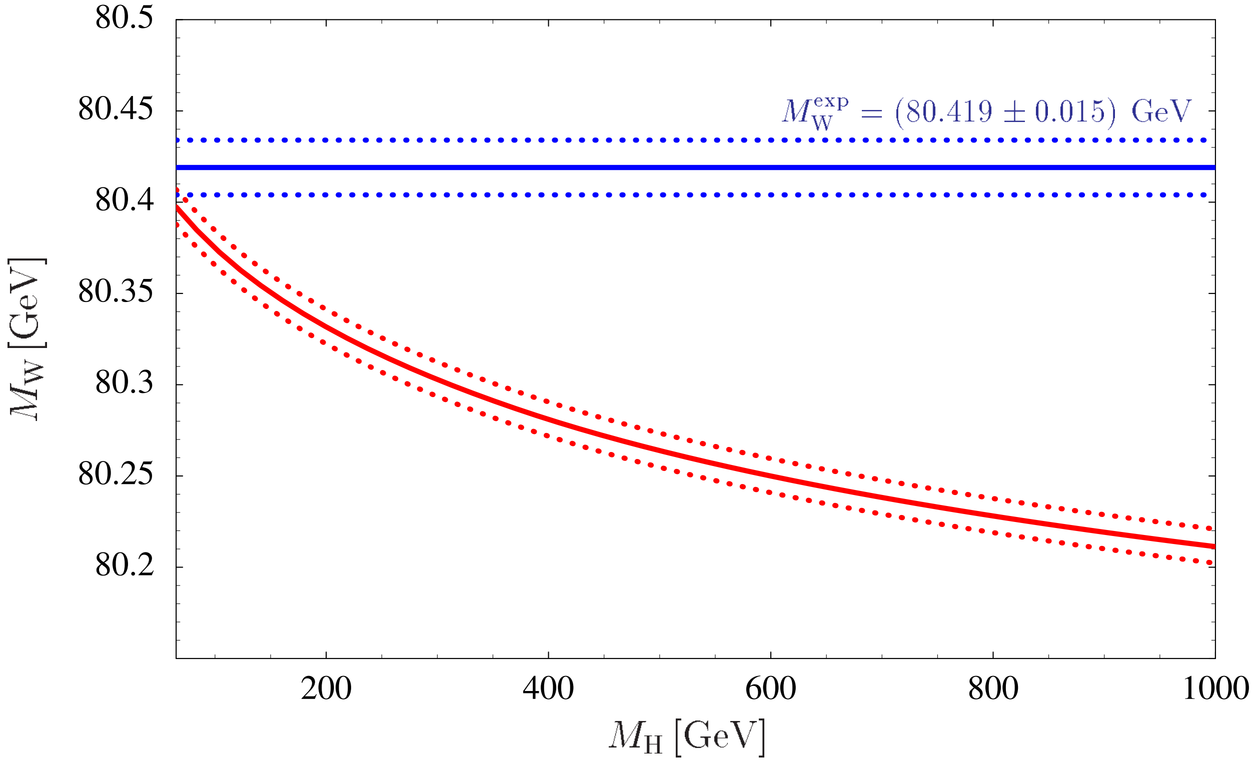
Exact fermionic 2-loop contrib., no expansion in m_{t}

Estimate of theoretical uncertainty from unknown higher-order electroweak corrections:

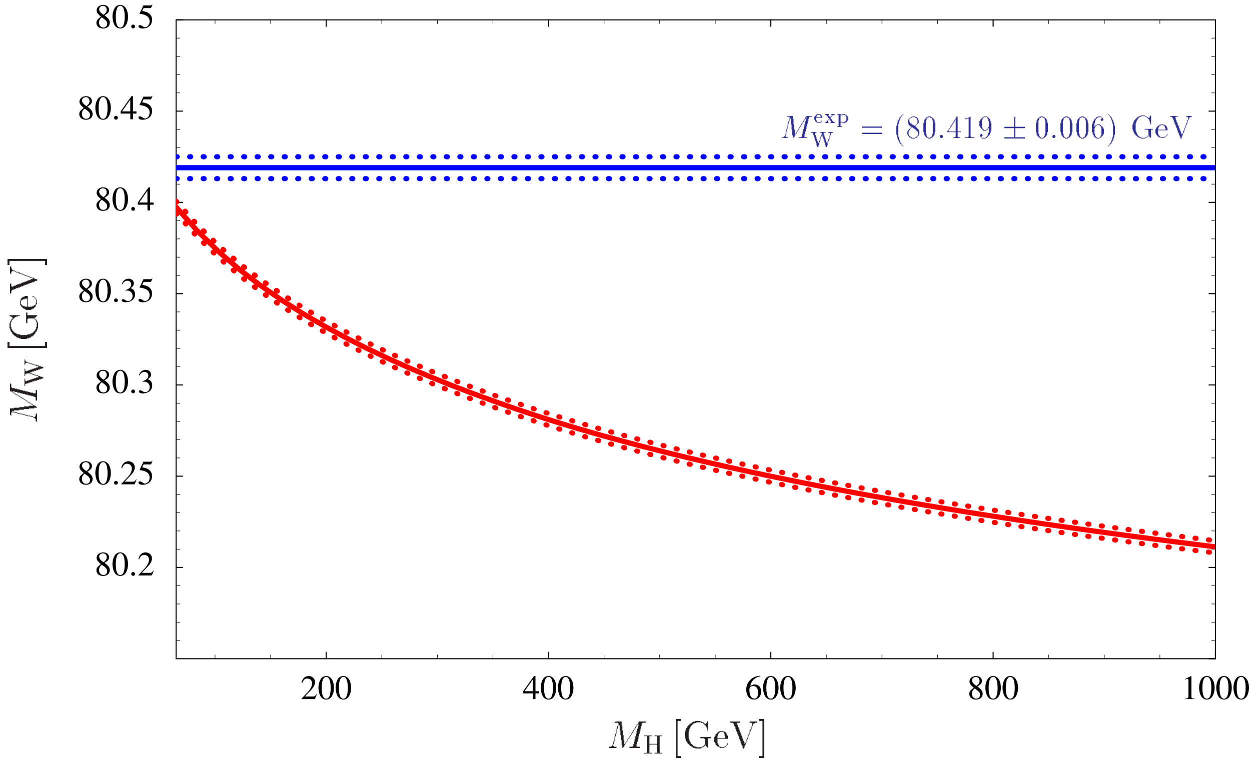
$$\Delta M_{\mathrm{W}}^{\mathrm{theo,ew}} \lesssim 3\,\mathrm{MeV}$$

$M_W(M_H)$ in the SM, prospective future accuracies:

LHC precision:

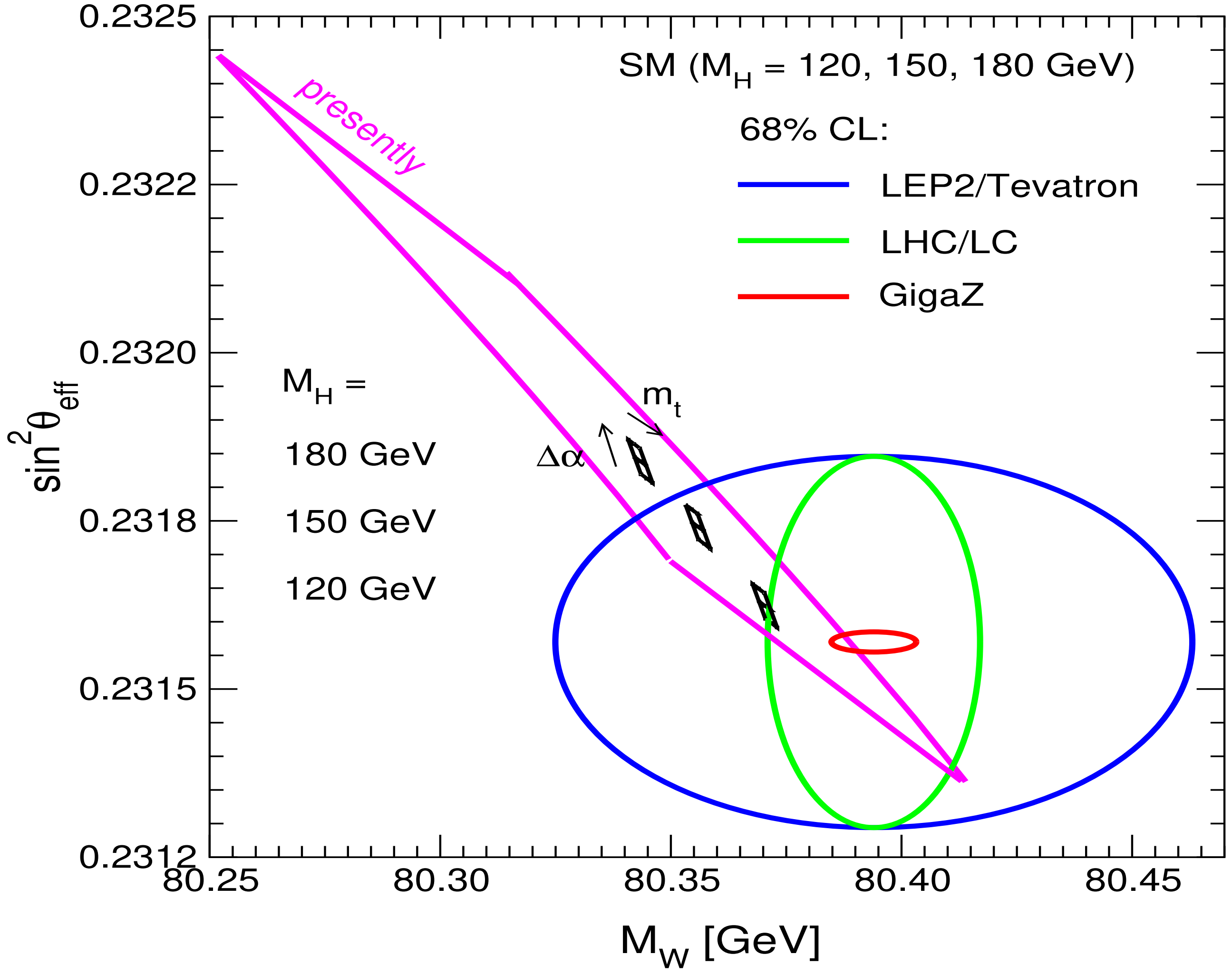


GigaZ precision:



SM prediction for M_W and $\sin^2\theta_{\mathrm{eff}}$ vs. prospective accuracies at LEP2/Tevatron, Tev./LHC/LC, GigaZ:

[*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]



Expected accuracies

	LEP2/Tev.	Tev./LHC	LC	GigaZ
M_W	30 MeV	15 MeV	15 MeV	6 MeV
$\sin^2\theta_{\mathrm{eff}}$	0.00017	0.00017	0.00017	0.00001
m_t	5 GeV	2 GeV	0.2 GeV	0.2 GeV
m_h	?	0.2 GeV	0.05 GeV	0.05 GeV

Expected precisions of indirect Higgs-mass determinations:

[*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]

	M_W	$\sin^2\theta_{eff}$	all
now	229 %	62 %	61 %
Tevatron Run IIA	77 %	46 %	41 %
Tevatron Run IIB	39 %	28 %	26 %
LHC	28 %	24 %	21 %
LC	18 %	20 %	15 %
GigaZ	12 %	7 %	7 %

Precision observables in the MSSM:

In order to treat MSSM at same level of accuracy as SM

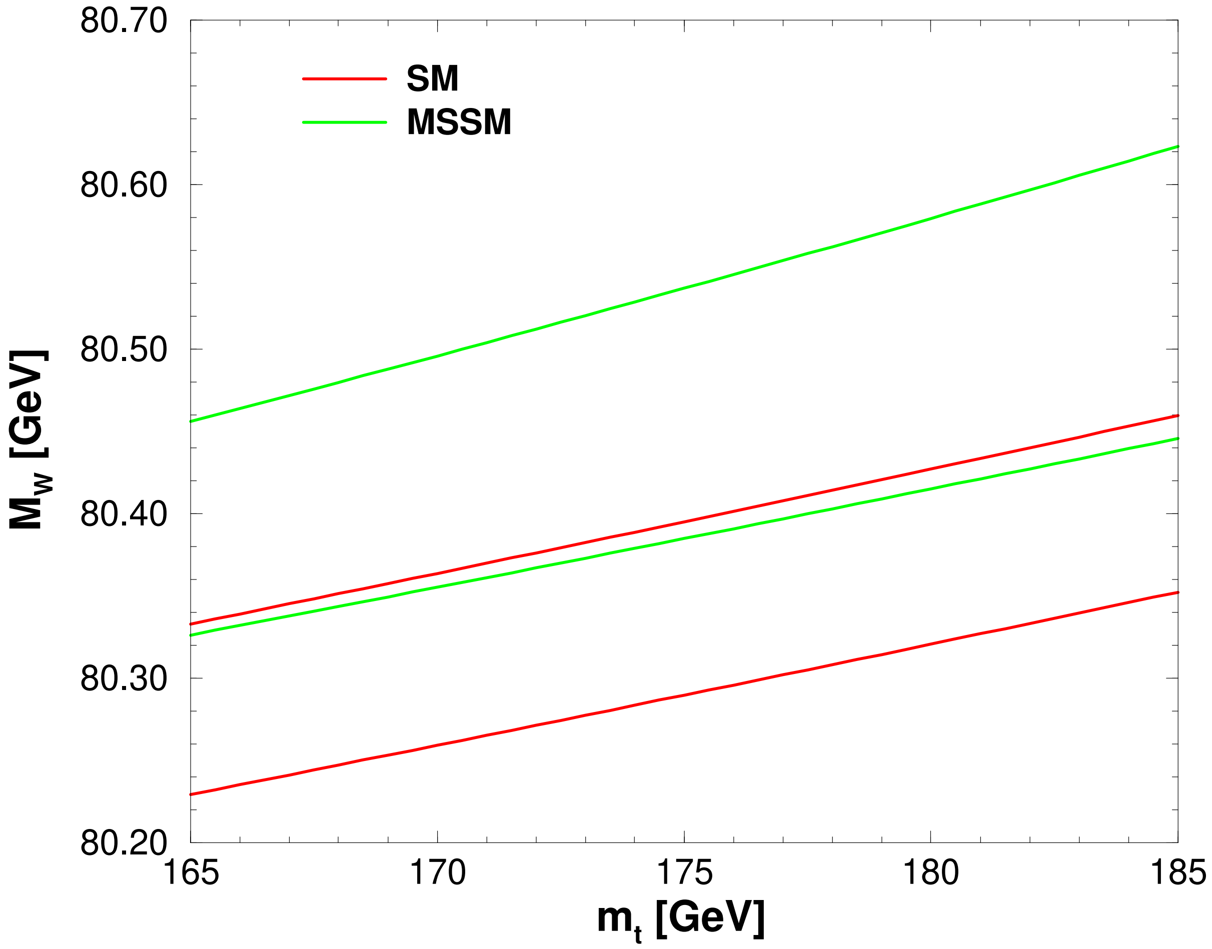
$\Rightarrow$ Include higher-order corrections

Prediction for M_W, $\sin^2\theta_{\rm eff}$ in the MSSM:

- Complete 1-loop results
  [*P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek '94*]
  [*D. Garcia, J. Solà '94*]
  [*D. Garcia, R. Jiménez, J. Solà '95*]
  [*A. Dabelstein, W. Hollik, W. Mösle '95*]
  [*P. Chankowski, S. Pokorski '96*]

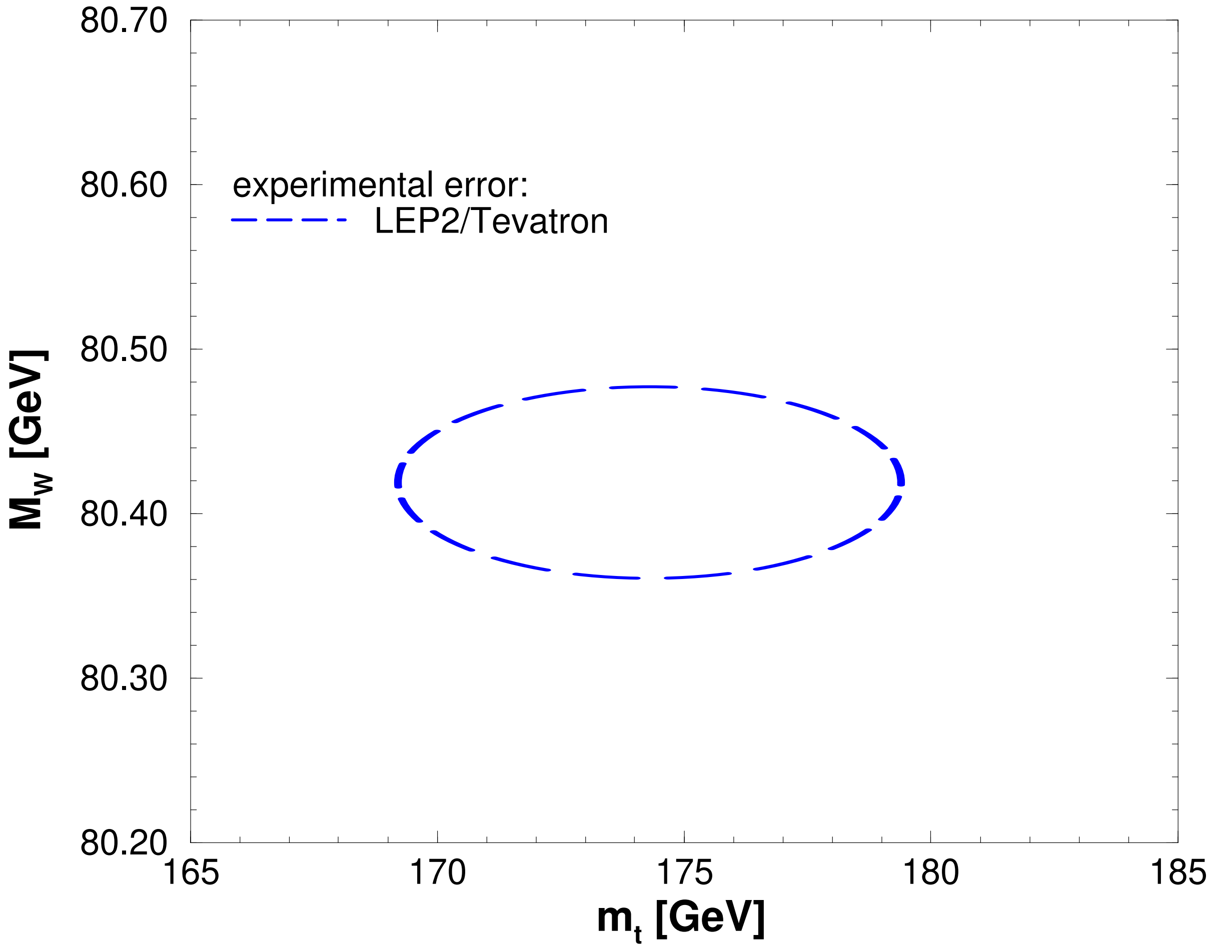
- Leading 2-loop $\mathcal{O}(\alpha\alpha_s)$ corrections
  [*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97*]
  [*S. Heinemeyer, W. Hollik, G. W. '98*]

Prediction for M_W in the SM and the MSSM:

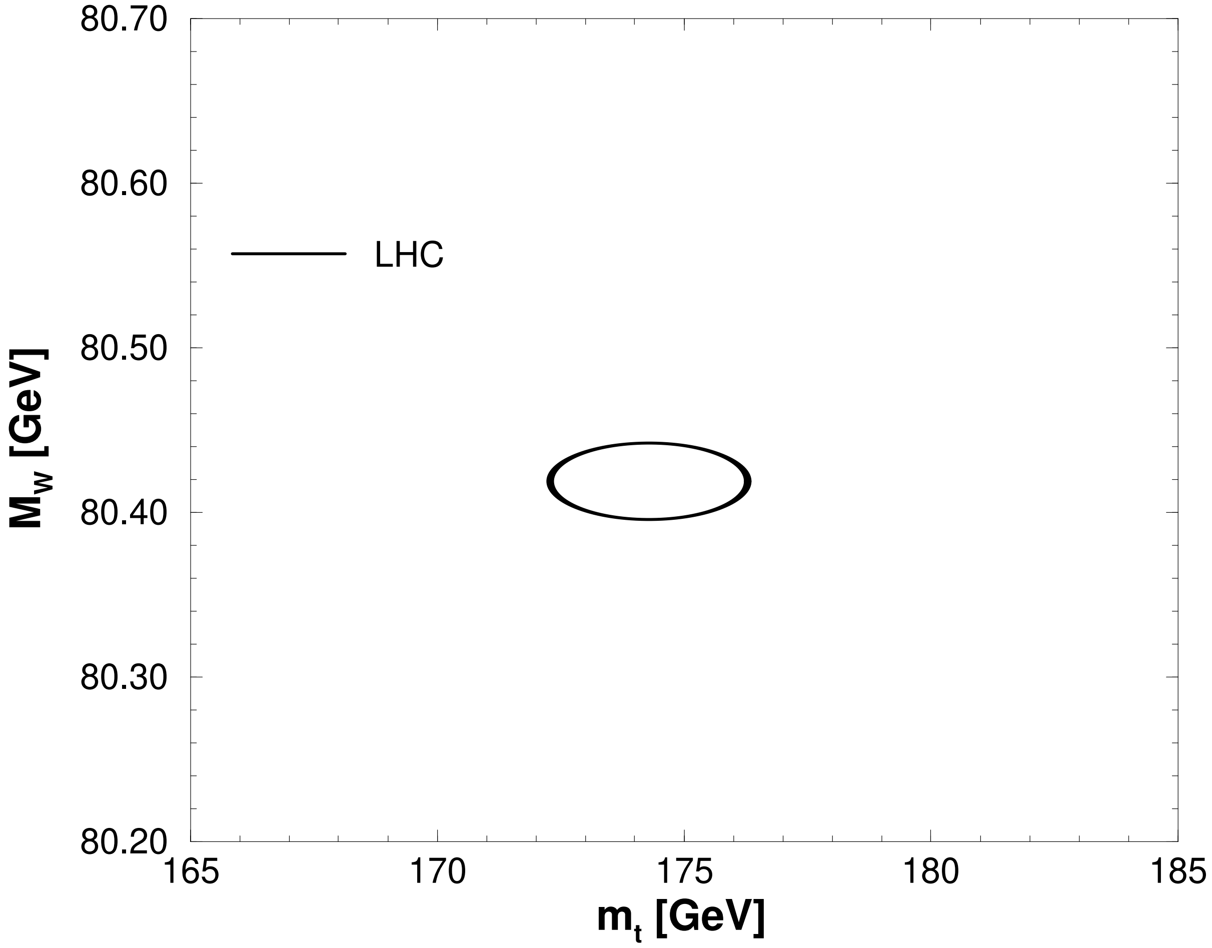


SM: M_H varied

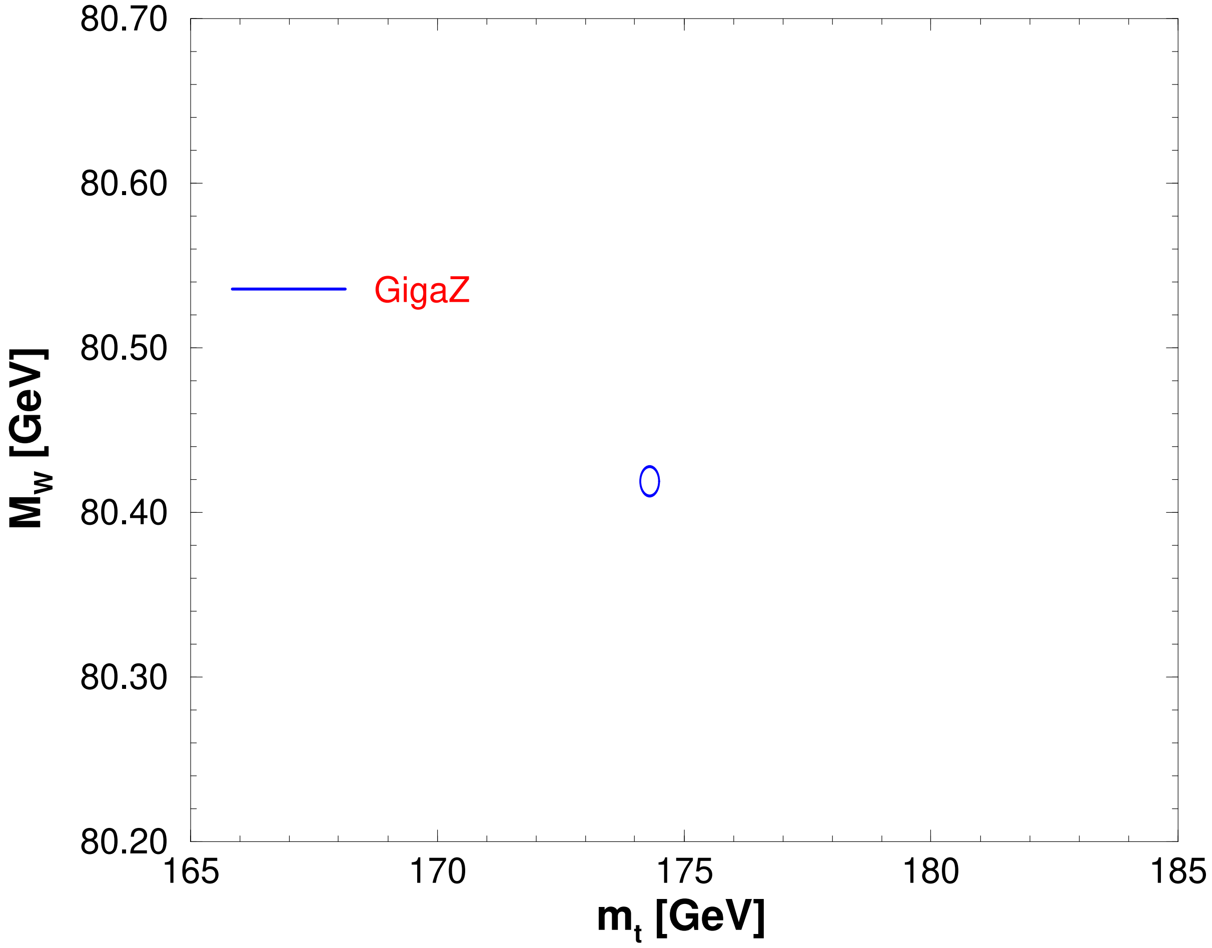
MSSM: SUSY parameters varied



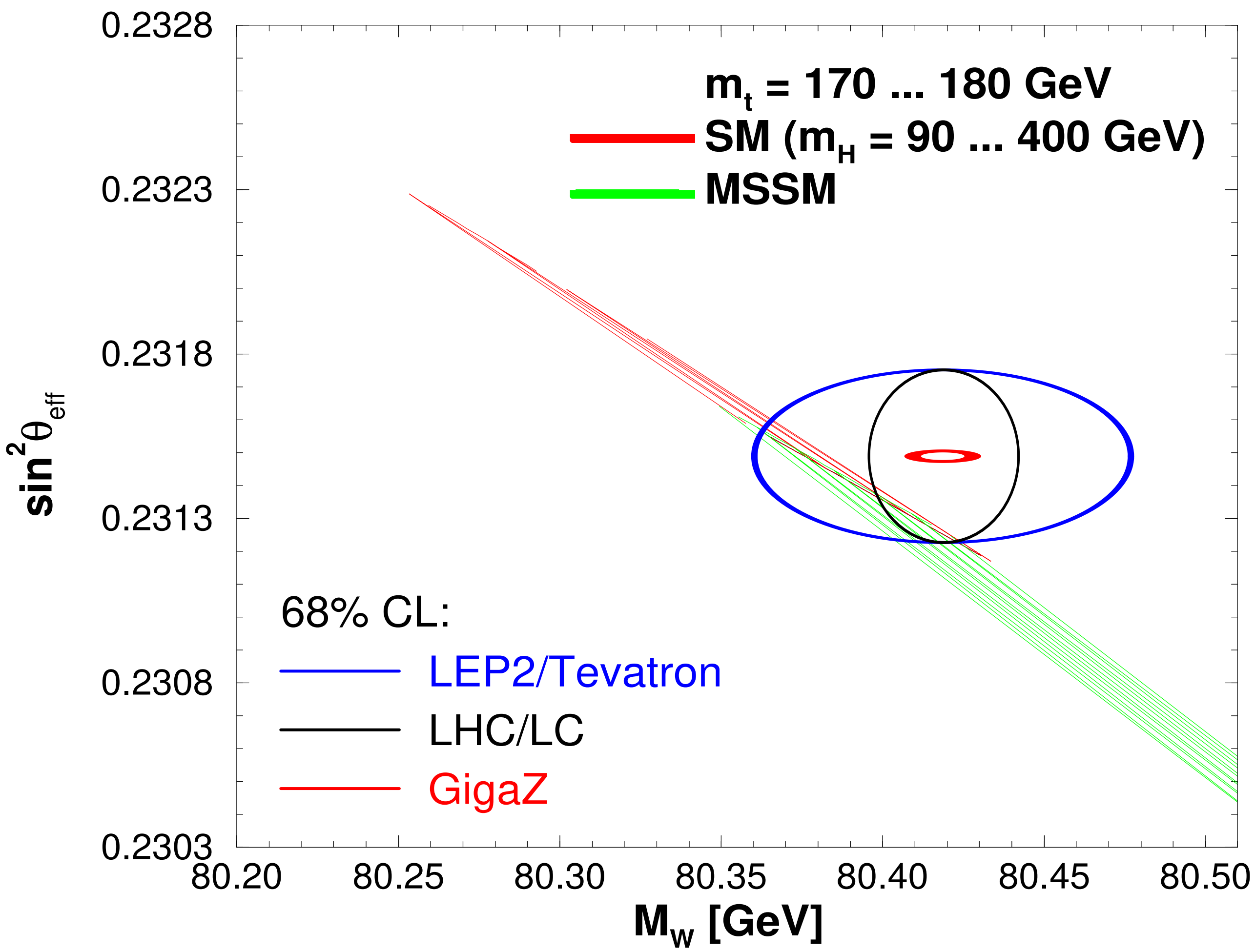
80.70
80.60
80.50
80.40
80.30
80.20
165
170
175
180
185
M_W [GeV]
m_t [GeV]
experimental error:
LEP2/Tevatron



80.70
80.60
80.50
80.40
80.30
80.20
LHC
M_W [GeV]
165
170
175
180
185
m_t [GeV]


GigaZ
80.70
80.60
80.50
80.40
80.30
80.20
165
170
175
180
185
M_W [GeV]
m_t [GeV]

Prediction for M_W, $\sin^2\theta_{\rm eff}$ in SM and MSSM:



⇒ Large improvement in experimental accuracy LEP/TevI → TevII/LHC/LC → GigaZ:

⇒ Very sensitive test of theory

3. Higher-order results for m_h in the MSSM

In SUSY: m_h not a free parameter $\Rightarrow$ calculable

Higgs sector of the MSSM:

Two Higgs doublets

$$\begin{aligned} H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\ H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix} \end{aligned}$$

Higgs potential:

$$\begin{aligned} V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ &\quad + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2 \end{aligned}$$

Physical states: $h^0, H^0, A^0, H^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

Tree-level result for m_{h}, m_{H}:

$$m^2_{\mathrm{H,h}} = \frac{1}{2}\left[M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2 \pm \sqrt{(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2)^2 - 4 M_{\mathrm{Z}}^2 M_{\mathrm{A}}^2 \cos^2 2\beta}\right]$$

$\Rightarrow$ $m_{\mathrm{h}} \leq M_{\mathrm{Z}}$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Large radiative corrections:

Yukawa couplings: $\frac{e\, m_{\mathrm{t}}}{2 M_{\mathrm{W}} s_{\mathrm{W}}}$, $\frac{e\, m_{\mathrm{t}}^2}{M_{\mathrm{W}} s_{\mathrm{W}}}$, ...

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_{\mathrm{t}}^4 \ln\left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_{\mathrm{t}}^2}\right)$

$\tilde{t}$ sector:

$$M^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_{\mathrm{t}}^2 + DT_1 & m_{\mathrm{t}} X_{\mathrm{t}} \\ m_{\mathrm{t}} X_{\mathrm{t}} & M^2_{\tilde{t}_R} + m_{\mathrm{t}}^2 + DT_2 \end{pmatrix}$$

$X_{\mathrm{t}} = A_{\mathrm{t}} - \mu \cot\beta$; large mixing possible

Input parameters: $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$, X_{t}

often used: $M_{\mathrm{SUSY}} \equiv M_{\tilde{t}_L} = M_{\tilde{t}_R}$, $M_{\mathrm{S}}^2 \equiv M_{\mathrm{SUSY}}^2 + m_{\mathrm{t}}^2$

$\Rightarrow$ Physical parameters: $m_{\tilde{\mathrm{t}}_1}$, $m_{\tilde{\mathrm{t}}_2}$, $\theta_{\tilde{\mathrm{t}}}$

Present status of m_h prediction in the MSSM

Main approaches for radiative corrections to m_h:

Effective potential / RG approach:

Corrections to Higgs potential:
effective couplings, $p^2 = 0$

[*H. Haber, R. Hempfling '91*]

[*Y. Okada, M. Yamaguchi, T. Yanagida '91*]

[*J. Ellis, G. Ridolfi, F. Zwirner '91*]

[*R. Barbieri, M. Frigeni '91*]

[*R. Hempfling, A. Hoang '93*]

[*R. Zhang '99*]

[*J. Espinosa, R. Zhang '00*]

Renormalization group improvement of one-loop result

⇒ Leading logs at two-loop level

[*J. Casas, J. Espinosa, M. Quirós, A. Riotto '95*]

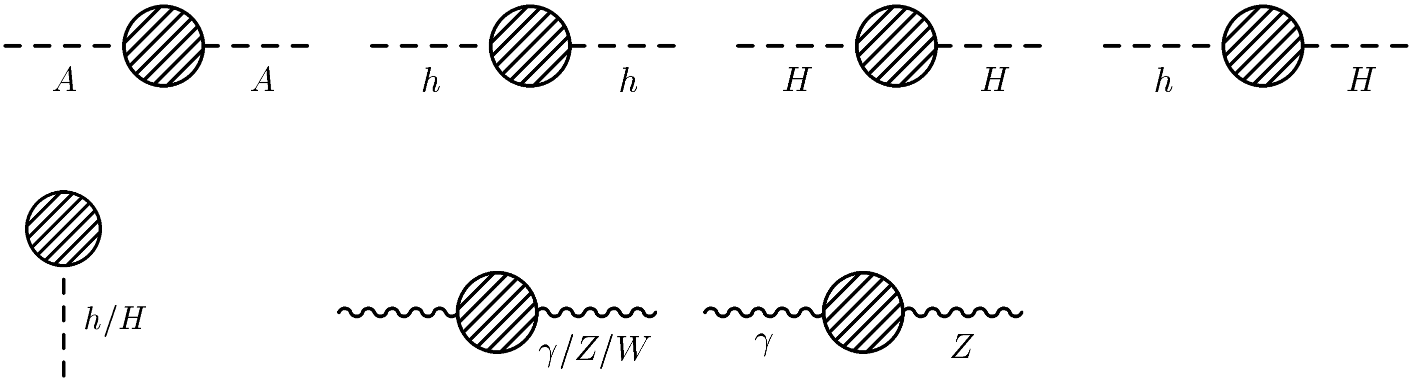
[*M. Carena, M. Quirós, C. Wagner '95*]

[*H. Haber, R. Hempfling, A. Hoang '97*]

Renormalization in $\overline{\text{MS}}$ scheme

Feynman-diagrammatic approach:

Explicit Feynman-diagrammatic calculation,
Higgs mass: pole of propagator



Complete one-loop result

[*P. Chankowski, S. Pokorski, J. Rosiek '94*]

[*A. Dabelstein '95*]

[*D. Pierce, J. Bagger, K. Matchev, R. Zhang '97*]

Dominant two-loop contributions

[*S. Heinemeyer, W. Hollik, G. W. '98, '99*]

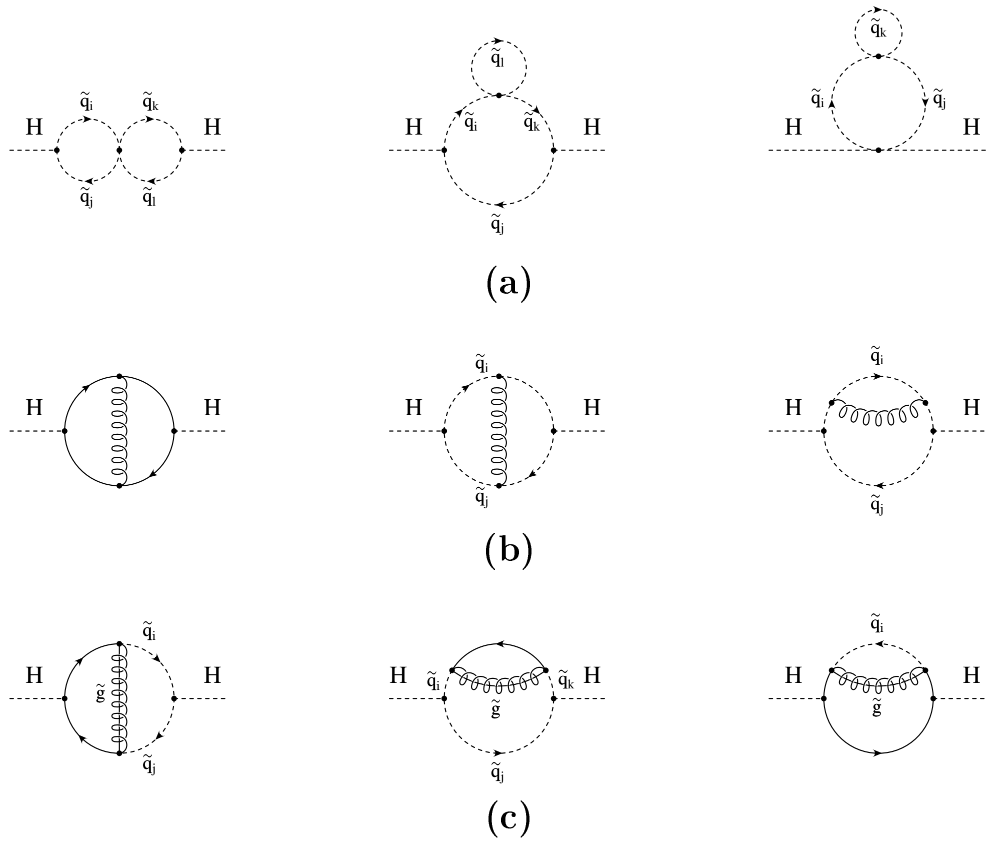
Renormalization: on-shell scheme, physical masses

Physical masses m_{h}^2, m_{H}^2:

poles of h/H propagators $\Rightarrow$ solutions of

$$\left[q^2 - m_{\mathrm{h}}^2 + \hat{\Sigma}_{\mathrm{hh}}(q^2)\right]\left[q^2 - m_{\mathrm{H}}^2 + \hat{\Sigma}_{\mathrm{HH}}(q^2)\right] - \left(\hat{\Sigma}_{\mathrm{hH}}(q^2)\right)^2 = 0.$$

Dominant two-loop corrections:
$\mathcal{O}(\alpha\alpha_s)$ contribution from $t-\tilde{t}$-sector
(Yukawa contribution, $p^2 = 0$)
[*S. Heinemeyer, W. Hollik, G. W. '98*]



$\Rightarrow$ Analytical result in terms of
$\tan\beta, M_A, \mu, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \ldots$

Program: *FeynHiggs*, *FeynHiggsFast*
[*S. Heinemeyer, W. Hollik, G. W. '98, '00*]

`www-itp.physik.uni-karlsruhe.de/feynhiggs`

Comparison of diagrammatic and RG result

in terms of unobservable param. X_t, $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$:

Problem:
From two-loop order on: different meaning in different renormalization schemes!

$\Rightarrow$ Rewrite diagrammatic result in terms of $\overline{\text{MS}}$ param.

$$\overline{m}_t = m_t^{OS}\left(1 - \frac{4}{3}\frac{\alpha_s}{\pi}\right)$$

$$\overline{M}_S^2 = M_S^{2,OS} - \frac{8}{3\pi}\frac{\alpha_s}{} M_S^2 + \mathcal{O}\left(\frac{m_t}{M_S}\right)$$

$$\overline{X}_t = X_t^{OS} + \frac{\alpha_s}{3\pi} M_S\left(8 + 4\frac{X_t}{M_S} - \frac{X_t^2}{M_S^2} - 3\frac{X_t}{M_S}\ln\left(\frac{m_t^2}{M_S^2}\right)\right)$$

Leading m_t^4 contributions:

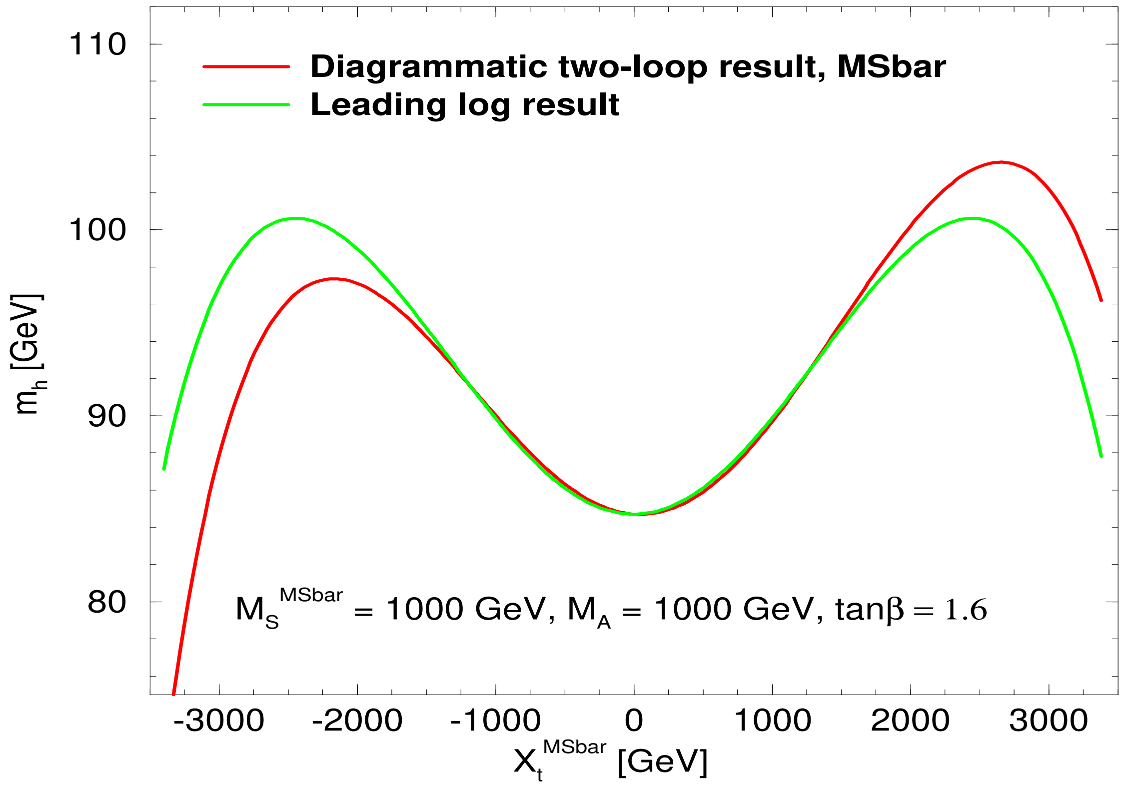
$$m_h^2 = m_h^{2,\text{tree}} + 3\frac{G_\mu\sqrt{2}}{\pi^2}\overline{m}_t^4\Bigg\{ -\frac{1}{2}\ln\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right) + \frac{1}{2}\frac{\overline{X}_t^2}{\overline{M}_S^2}\left(1 - \frac{1}{12}\frac{\overline{X}_t^2}{\overline{M}_S^2}\right)$$
$$- \frac{\alpha_s}{\pi}\Bigg\{\ln^2\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right) + \left[\frac{2}{3} - 2\frac{\overline{X}_t^2}{\overline{M}_S^2}\left(1 - \frac{1}{12}\frac{\overline{X}_t^2}{\overline{M}_S^2}\right)\right]\ln\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right)$$
$$+ \frac{\overline{X}_t}{\overline{M}_S}\left(\frac{2}{3} - \frac{7}{9}\frac{\overline{X}_t^2}{\overline{M}_S^2} + \frac{1}{36}\frac{\overline{X}_t^3}{\overline{M}_S^3} + \frac{1}{18}\frac{\overline{X}_t^4}{\overline{M}_S^4}\right)\Bigg\}\Bigg\} + \ldots$$

$\Rightarrow$ Leading-log terms = RG result
[*M. Carena, H. Haber, S. Heinemeyer, W. Hollik, C. Wagner, G. W. '00*]

Non-logarithmic two-loop terms: **new**

Agreement with 2-loop eff. potential calculation
[*J. Espinosa, R. Zhang '99*]

Effect of new genuine two-loop contributions:



$\Rightarrow$ Sizable numerical effect

result is asymmetric in $\pm X_t$

Increase in m_h^{max} by up to 4 GeV

Non-leading one-loop terms, variation of $m_{\tilde{g}}$, ...
$\Rightarrow$ further deviations

Constraints on $\tan\beta$ from Higgs search at LEP

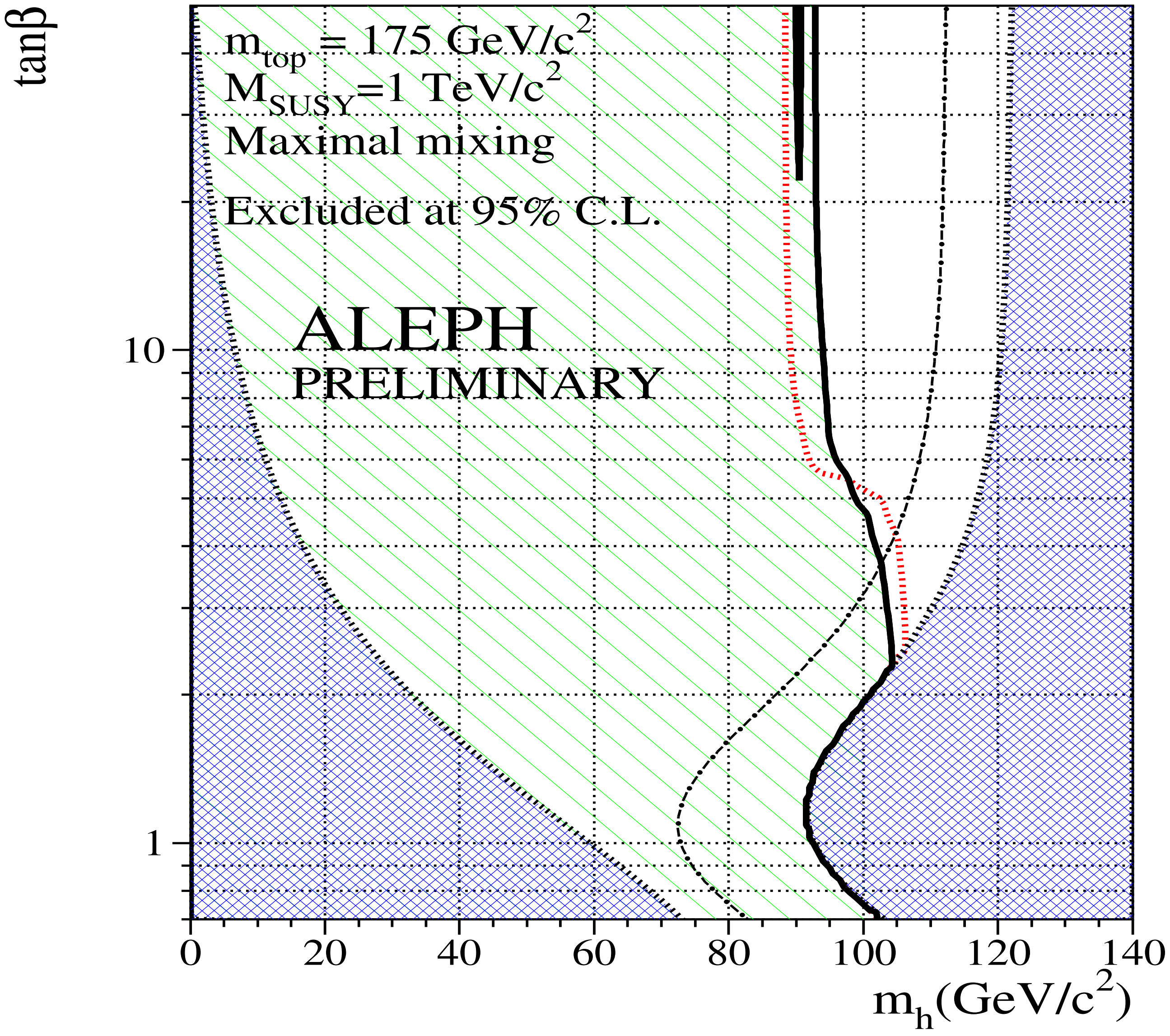
Previous LEP analysis: “benchmark” scenario:

$$m_t = 175 \text{ GeV}, \qquad M_{SUSY} = 1 \text{ TeV}$$

Further parameters:

$\mu = -100$ GeV, $M_2 = 1630$ GeV, $M_A \leq 500$ GeV
$\overline{m}_t$ includes 2-loop QCD corrections
$A_t = \sqrt{6} M_{SUSY}$ (“maximal mixing”)

Experimental bound vs. RG result for m_h:
[*ALEPH Collaboration, LEPC meeting 11 ’99*]

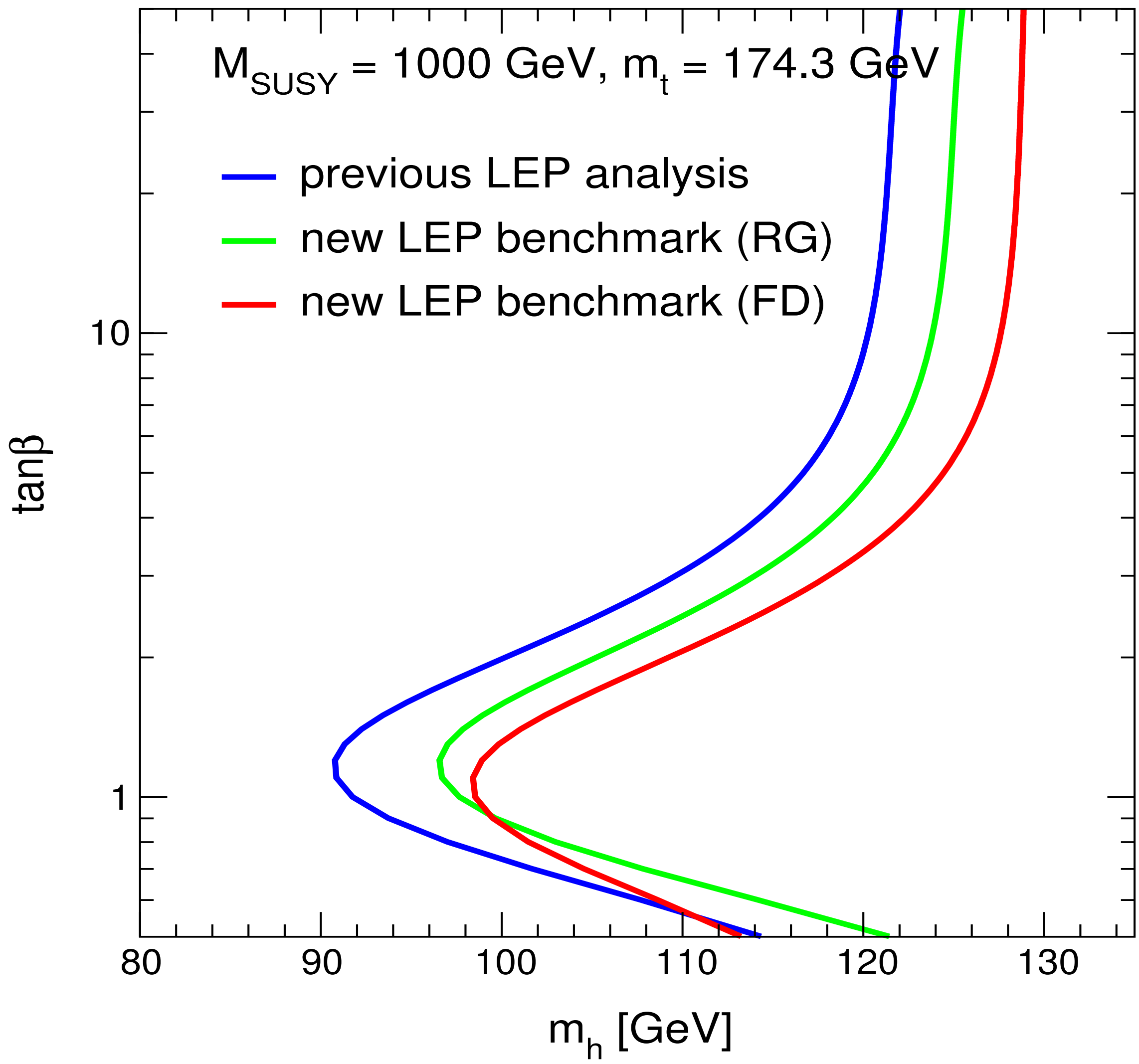


$m_h^{\rm max}$-scenario:

$$m_t = m_t^{\rm exp},\ M_{\rm SUSY} = 1\ {\rm TeV}$$
$$M_2 = 200\ {\rm GeV},\ M_A = 1\ {\rm TeV},\ \overline{m}_t = 1/(1 + \tfrac{4}{3\pi}\alpha_s),$$
$$X_t = \sqrt{6} M_{\rm SUSY}\ ({\rm RG}),\ X_t = 2 M_{\rm SUSY}\ ({\rm FD})$$

$\Rightarrow$ Conservative upper bound

Prev. analysis vs. $m_h^{\rm max}$ scenario, RG vs. FD result:



$m_h^{\rm max}$ increased: up to 5 GeV: prev. an. $\rightarrow$ $m_h^{\rm max}$ scen.

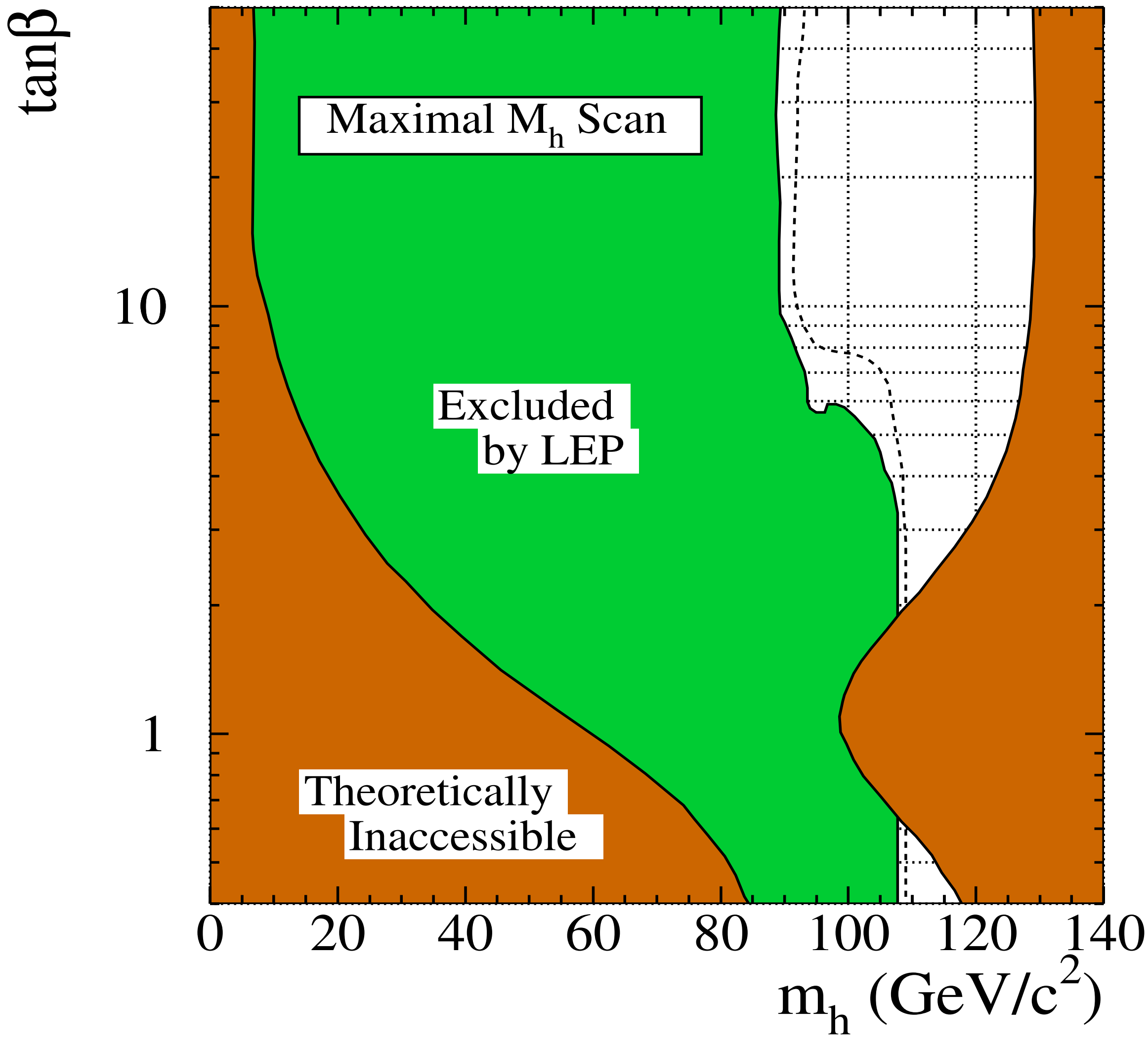
up to 4 GeV: RG $\rightarrow$ FD result

$\Rightarrow$ $m_h^{\rm max}$ higher by up to ≈ 9 GeV

Accessible $\tan\beta$ region decreased

Experimental search vs. upper m_h-bound (*FeynHiggs*)
[*LEP Higgs Working Group '00*]

$m_\mathrm{h}^\mathrm{max}$-scenario:



$m_\mathrm{h} > 88.3\,\mathrm{GeV}, \quad M_\mathrm{A} > 88.7\,\mathrm{GeV}, \quad 95\%$ C.L.

$$m_\mathrm{h}^\mathrm{SM} > 107.7\,\mathrm{GeV}$$

Excluded $\tan\beta$ region:

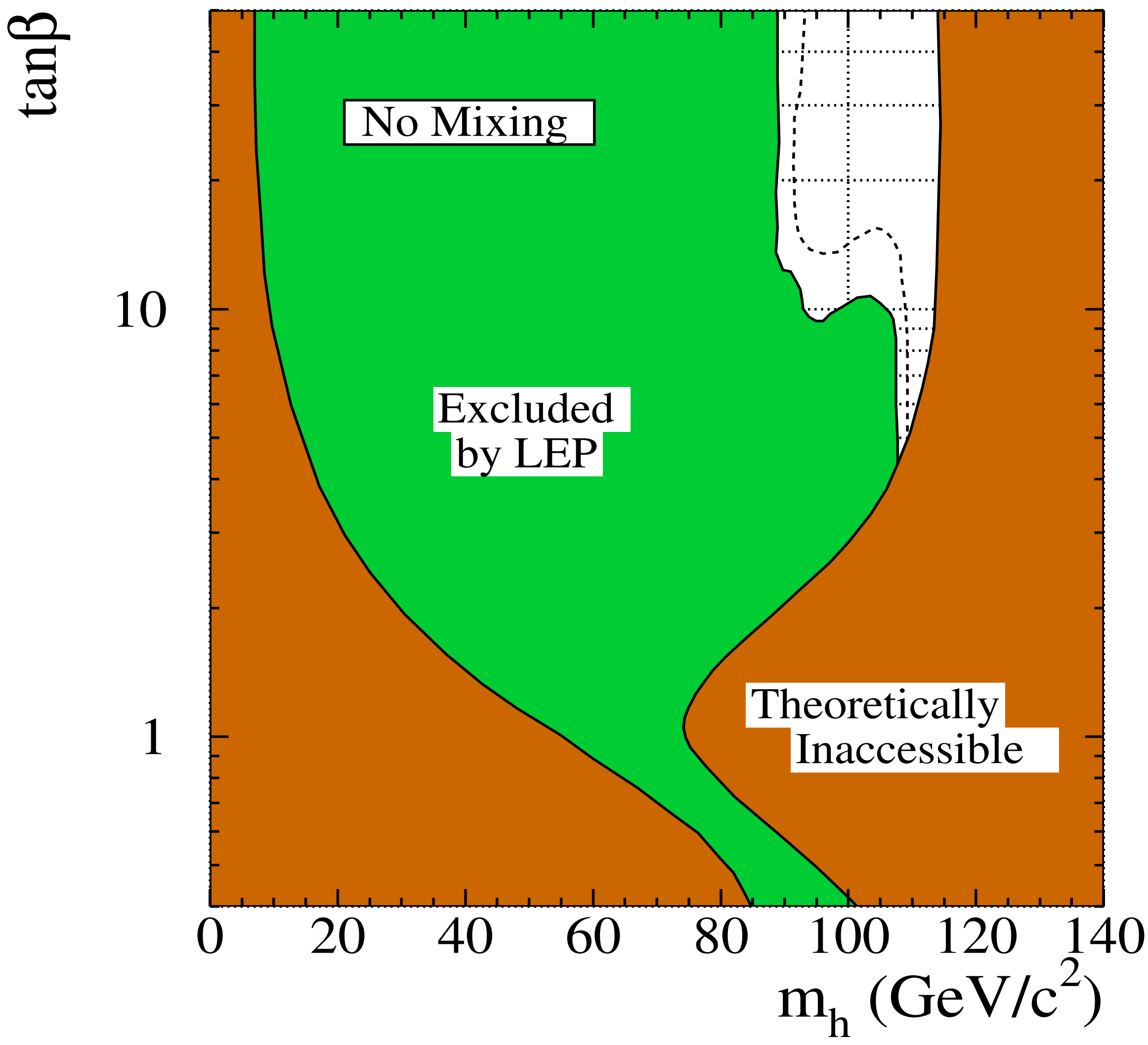
$$0.7 < \tan\beta < 1.8$$

for $m_\mathrm{t} = 174.3$ GeV, $M_\mathrm{SUSY} = 1$ TeV

Experimental search vs. upper m_{h}-bound (*FeynHiggs*)

[*LEP Higgs Working Group '00*]

no-mixing scenario:



Excluded $\tan\beta$ region:

$$0.4 < \tan\beta < 4.1$$

for $m_{\mathrm{t}} = 174.3$ GeV, $M_{\mathrm{SUSY}} = 1$ TeV

Upper bound on m_{h} in the MSSM:

$$m_{\mathrm{h}} \lesssim 130\,\mathrm{GeV}$$

for $m_{\mathrm{t}} = 175\,\mathrm{GeV}$

real parameters ($\mathcal{CP}$-conserving case)

no theoretical uncertainties included

Same bound applies also for case of $\mathcal{CP}$-violating Higgs sector
[*A. Pilaftsis, C. Wagner '99*]

Theoretical uncertainties:

– from unknown higher-order corrections

Analysis not as sophisticated yet as for SM precision observables ("blue band" for M_{H})

personal estimate: $\Delta m_{\mathrm{h}} = 3$ GeV

– from uncertainties in input parameters

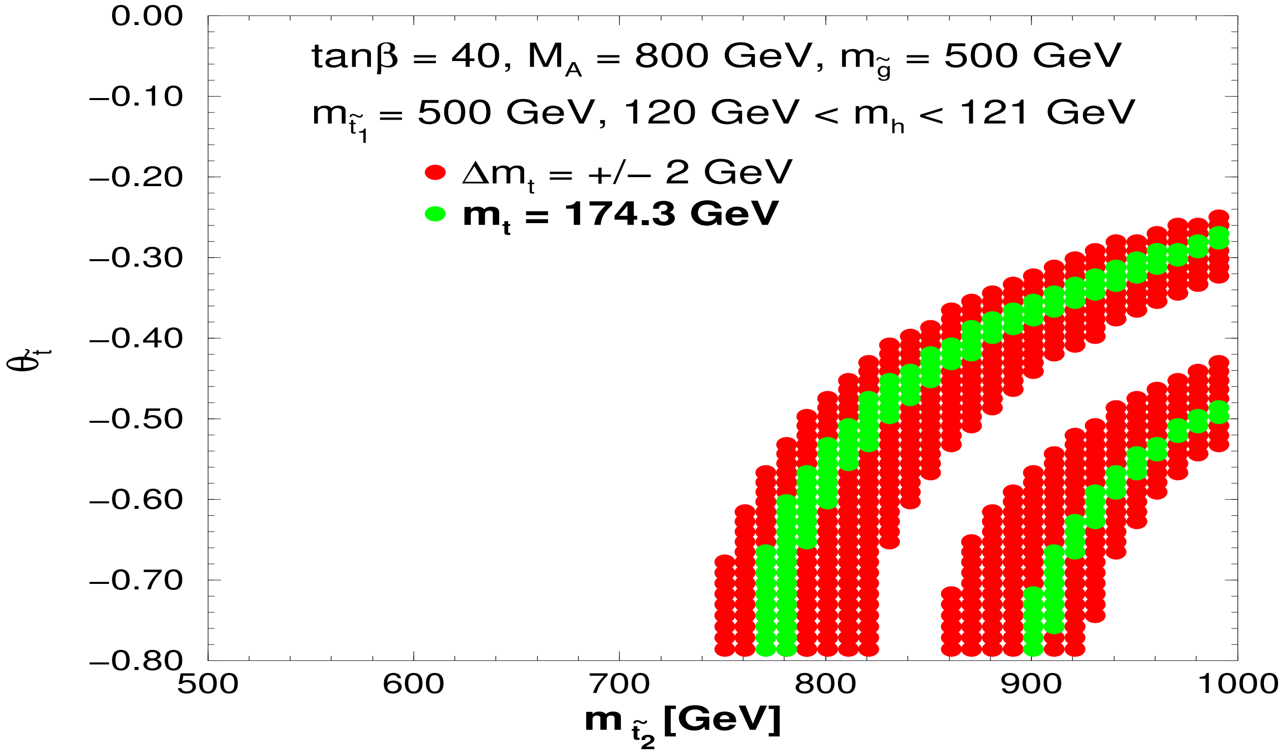
$m_{\mathrm{t}}, \ldots, M_{\mathrm{A}}, \tan\beta, m_{\tilde{\mathrm{t}}_1}, m_{\tilde{\mathrm{t}}_2}, \theta_{\tilde{\mathrm{t}}}, m_{\tilde{\mathrm{g}}}, \ldots$

$\Delta m_{\mathrm{t}} = 1$ GeV $\Rightarrow$ $\Delta m_{\mathrm{h}} = 1$ GeV

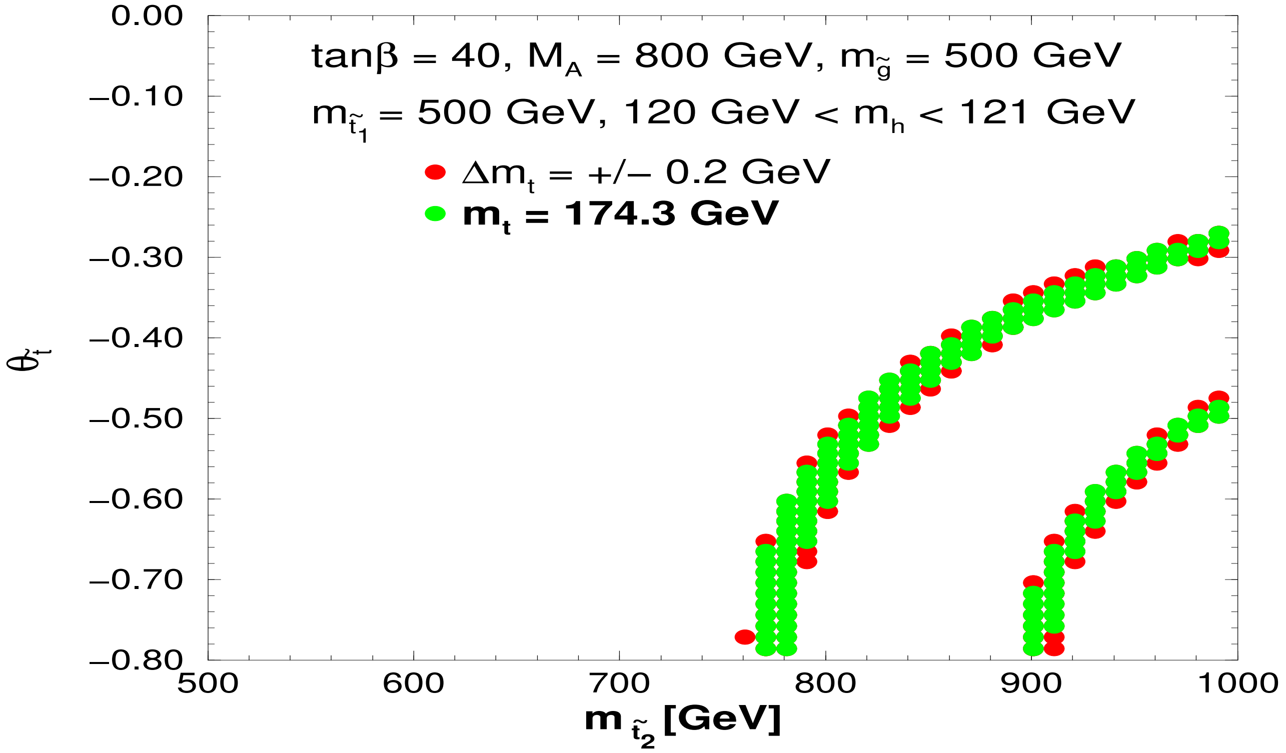
Assume: m_{h} measured at LHC, LC

$\Rightarrow$ constraints for parameters in $\tilde{t}$ sector:

LHC precision, $m_{\tilde{\mathrm{t}}_1} = 500$ GeV, $\tan\beta = 40$:



LC precision, $m_{\tilde{\mathrm{t}}_1} = 500$ GeV, $\tan\beta = 40$:

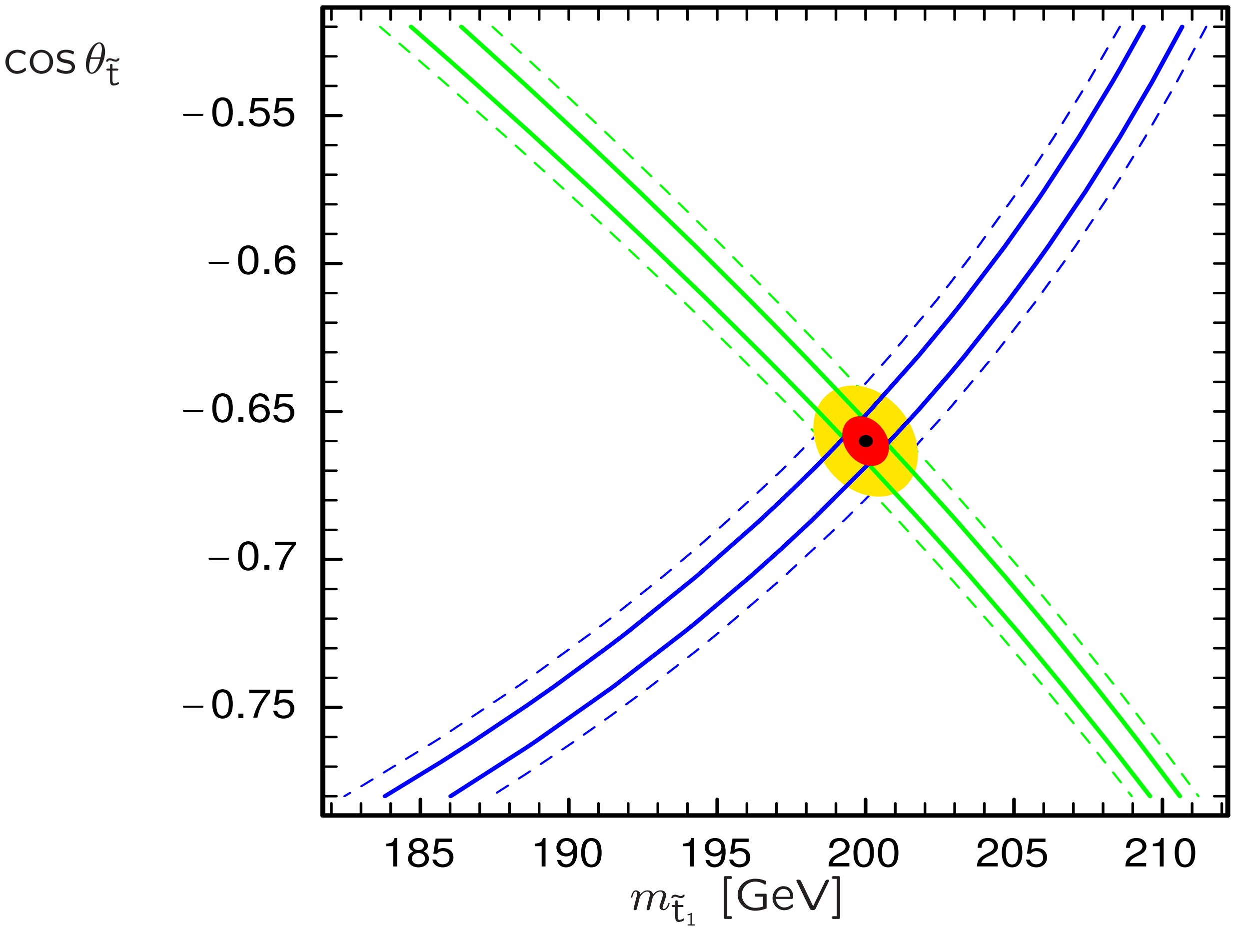


Information on parameters in $\tilde{t}$ sector
from direct production at high luminosity LC:

Determination of $m_{\tilde{t}_1}$ and $\theta_{\tilde{t}}$ in $e^+e^- \to \tilde{t}_1\tilde{t}_1$
with 90% polarized e^- beams, $\sqrt{s} = 500$ GeV
[*A. Bartl, H. Eberl, S. Kraml, W. Majerotto, W. Porod '99*]

Two cases: $\mathcal{L} = 100\ \text{fb}^{-1}$, $\mathcal{L} = 500\ \text{fb}^{-1}$

$\mathcal{P}_L = -0.9$: $\sigma_L = 44.88$ fb, $\Delta\sigma_L = 1.73$, 0.77 fb
$\mathcal{P}_R = +0.9$: $\sigma_R = 26.95$ fb, $\Delta\sigma_R = 1.34$, 0.6 fb



$m_{\tilde{t}_2} = 420$ GeV, $M = 200$ GeV, $\tan\beta = 4$, ...

$\Rightarrow$ direct/indirect information is complementary
very sensitive test of the model

Combined precision observables: m_h, M_W, $\sin^2\theta_{eff}$ (GigaZ accuracy):

[*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]

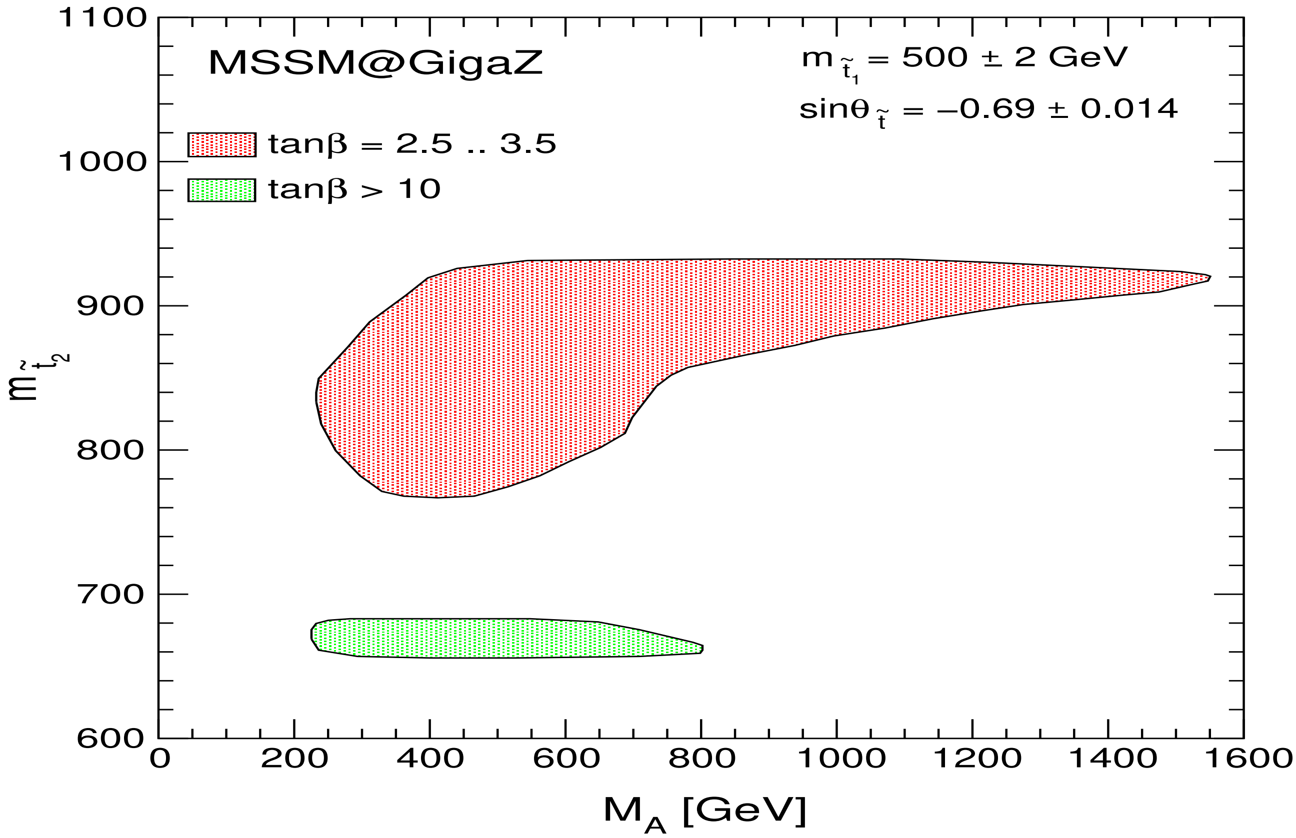
Assume:

$m_h = 115 \pm 0.05$ GeV, $M_W = 80.4 \pm 0.006$ GeV,
$\sin^2\theta_{eff} = 0.23140 \pm 0.00001$

$2.5 < \tan\beta < 3.5$ or $\tan\beta \geq 10$
(from measurements in chargino sector)

$m_{\tilde{t}_1} = 500 \pm 2$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 2\%$, $A_b = A_t \pm 10\%$,
$\mu = -200 \pm 1$ GeV, $M_2 = 400 \pm 2$ GeV $m_{\tilde{g}} = 500 \pm 10$ GeV

Allowed region in M_A–$m_{\tilde{t}_2}$ plane:



$\Rightarrow$ Constraints on $m_{\tilde{t}_2}$
Upper bound on M_A

4. Conclusions

- At TevII/LHC/LC: improved accuracy of precision observables M_{W}, $\sin^2\theta_{\mathrm{eff}}$, m_{h} and input parameters m_{t}, $m_{\tilde{t}}$, ...

  $\Rightarrow$ Highly sensitive test of SM and MSSM

- GigaZ:

  SM: indirect M_{H}-determination: $\delta M_{\mathrm{H}}/M_{\mathrm{H}} \sim 7\%$

  MSSM: Constraints on $\tilde{t}$ sector parameters

  Upper bound on M_{A}

- Diagrammatic calculation of the leading $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM

  Comparison of result for m_{h} with RG result: Agreement in leading logs

  FD result contains new genuine two-loop contributions

  $\Rightarrow$ m_{h} higher by up to ≈ 4 GeV

- Constraints on $\tan\beta$ from Higgs search at LEP:

  New LEP benchmarks, new theoretical input

  $\Rightarrow$ Large increase in $m_{\mathrm{h}}^{\mathrm{max}}$

  Excluded $\tan\beta$ region decreased