The quark-gluon vertex

in arbitrary gauge and dimension

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Work done with Andrei Davydychev and Leo Saks

- Introduction
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- Basics
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  - Charge Conjugation
  - Kinematics
- One-Loop Vertex
  - Selected Off-Shell Results
  - Special Limits

Related work:

ggg: J.S. Ball and T.-W. Chiu,
Phys. Rev. D22 (1980) 2550; E: D23 (1981) 3085.

ggg: A. I. Davydychev, P. Osland, and O. V. Tarasov,
Phys. Rev. D 54 (1996) 4087–4113; E: D 59 (1999) 109901

$ee\gamma$: J.S. Ball and T.-W. Chiu,
Phys. Rev. D22 (1980) 2542

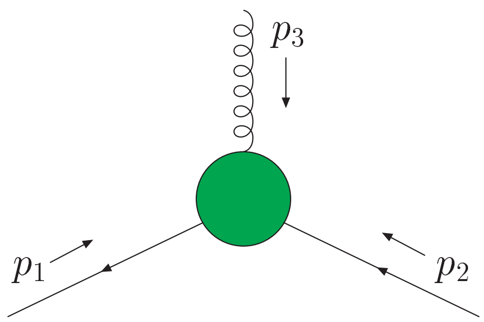
$ee\gamma$: A. Kızılersü, M. Reenders and M.R. Pennington,
Phys. Rev. D52 (1995) 1242

Lowest order:

$$g(T^a)_{ji}\,[\gamma_\mu]_{\beta\alpha}$$

Introduction

Notation:



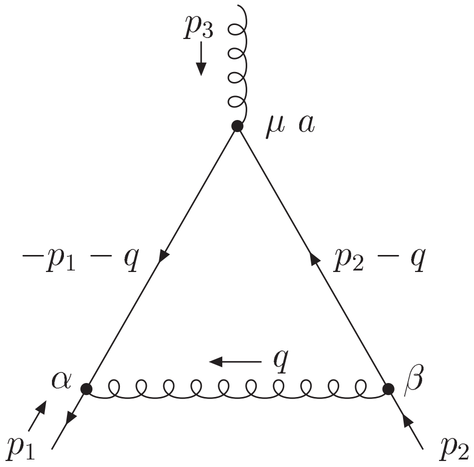
In general:

$$\Gamma^a_\mu(p_1, p_2, p_3) = gT^a\, \Gamma_\mu(p_1, p_2, p_3)$$

with 12 Dirac-matrix structures:

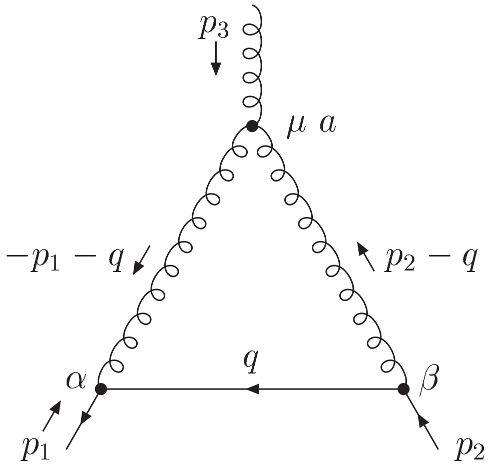
$$\Gamma^\mu(p_1, p_2, p_3) = \sum_{i=1}^{12} V_i^\mu h_i$$

At one loop, two diagrams:



QED-like, 'a'

$C_F - \frac{1}{2}C_A$



Non-Abelian, 'b'

$\frac{1}{2}C_A$

$$C_A = N, \qquad C_F = \frac{N^2 - 1}{2N}$$

'Naive' decomposition:

$$\begin{aligned}
\Gamma^\mu_{\text{naive}} \quad = \quad & \gamma^\mu h_1 + p_2^\mu h_2 + p_1^\mu h_3 + \gamma^\mu \not{p}_2 h_4 + \gamma^\mu \not{p}_1 h_5 \\
& + p_2^\mu \not{p}_2 h_6 + p_2^\mu \not{p}_1 h_7 + p_1^\mu \not{p}_2 h_8 + p_1^\mu \not{p}_1 h_9 \\
& + p_2^\mu \not{p}_1 \not{p}_2 h_{10} + p_1^\mu \not{p}_1 \not{p}_2 h_{11} + \gamma^\mu \not{p}_1 \not{p}_2 h_{12}
\end{aligned}$$

where $h_i \equiv h_i(p_1^2, p_2^2, p_3^2)$

Transverse and longitudinal parts:

$$\Gamma_\mu(p_1,p_2,p_3) = \Gamma_\mu^{(\mathrm{L})}(p_1,p_2,p_3) + \Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3)$$

where

$$p_3^\mu \Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3) = 0$$

Longitudinal part involved in Ward-Slavnov-Taylor identity

$$\begin{aligned}&p_3^\mu\ \Gamma_\mu(p_1,p_2,p_3)\\ &= G(p_3^2)\Big[S^{-1}(-p_1)H(p_1,p_2,p_3) - \overline{H}(p_2,p_1,p_3)S^{-1}(p_2)\Big]\end{aligned}$$

More notation!

Scalar Integrals

Diagram 1 (2 massive lines)

$$J_2(\nu_1, \nu_2, \nu_3) \equiv \int \frac{\mathrm{d}^n q}{[(p_2 - q)^2 - m^2]^{\nu_1} \, [(p_1 + q)^2 - m^2]^{\nu_2} \, [q^2]^{\nu_3}}$$

Diagram 2 (1 massive line)

$$J_1(\nu_1, \nu_2, \nu_3) \equiv \int \frac{\mathrm{d}^n q}{[(p_2 - q)^2]^{\nu_1} \, [(p_1 + q)^2]^{\nu_2} \, [q^2 - m^2]^{\nu_3}}$$

Space-time dimension: $n = 4 - 2\varepsilon$

Extract:

$$\eta \equiv \frac{\Gamma^2(\frac{n}{2} - 1)}{\Gamma(n - 3)} \, \Gamma(3 - \tfrac{n}{2}) = \frac{\Gamma^2(1 - \varepsilon)}{\Gamma(1 - 2\varepsilon)} \, \Gamma(1 + \varepsilon)$$

Three-point function:

$$J_i(1, 1, 1) = \mathrm{i} \, \pi^{n/2} \, \eta \, \varphi_i(p_1^2, p_2^2, p_3^2; m)$$

Two-point functions:

$$\begin{aligned}
J_1(1, 1, 0) &= J_0(1, 1, 0) = \mathrm{i} \, \pi^{n/2} \, \eta \, \kappa_{0,3} \\
J_1(0, 1, 1) &= J_2(0, 1, 1) = \mathrm{i} \, \pi^{n/2} \, \eta \, \kappa_{1,1} \\
J_1(1, 0, 1) &= J_2(1, 0, 1) = \mathrm{i} \, \pi^{n/2} \, \eta \, \kappa_{1,2}
\end{aligned}$$

Tadpole integral:

$$\begin{aligned} J_1(0,0,1) &= J_2(1,0,0) = J_2(0,1,0) \\ &= \mathrm{i}\,\pi^{n/2}\,\frac{\Gamma(1+\varepsilon)}{\varepsilon(1-\varepsilon)}\,(m^2)^{1-\varepsilon} = \mathrm{i}\,\pi^{n/2}\,\eta\,m^2\,\widetilde{\kappa} \\ \widetilde{\kappa} &\equiv \widetilde{\kappa}(m^2) \equiv \frac{\Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\,\frac{1}{\varepsilon(1-\varepsilon)}\,(m^2)^{-\varepsilon} \end{aligned}$$

Integration-by-parts techniques
F.V. Tkachov (1981); K.G. Chetyrkin and F.V. Tkachov (1981)

$\Longrightarrow$ Recurrence relations

$\Longrightarrow$ Gram determinants

Play an important role in 'compactification'

Two-point functions

Lowest-order gluon propagator:

$$\delta^{a_1 a_2} \frac{1}{p^2} \left(g_{\mu_1 \mu_2} - \xi \, \frac{p_{\mu_1} p_{\mu_2}}{p^2} \right)$$

gauge parameter: ξ

Fermion propagator:

$$S^{-1}(p) \equiv \alpha(p^2) \not{p} + \beta(p^2) I$$

$$\begin{aligned} \alpha^{(1)}(p^2) &= \frac{g^2 \, \eta}{(4\pi)^{n/2}} \frac{C_F}{2p^2} (n-2)(1-\xi) \\ &\quad \times \left[(p^2 + m^2) \kappa_1(p^2; m) - m^2 \widetilde{\kappa}(m^2) \right] \\ \beta^{(1)}(p^2) &= -\frac{g^2 \, \eta}{(4\pi)^{n/2}} \, C_F \, m(n-\xi) \kappa_1(p^2; m) \end{aligned}$$

Inverse ghost propagator:

$$\widetilde{\Pi}^{a_1 a_2}(p^2) = \delta^{a_1 a_2} \, p^2 \, \left[G(p^2) \right]^{-1}$$

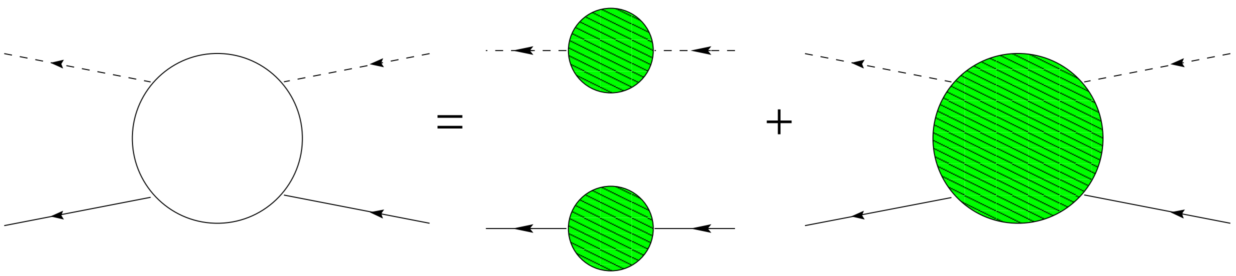
Lowest-order: $G^{(0)} = 1$; one-loop contribution:

$$G^{(1)}(p^2) = -\frac{g^2 \, \eta}{(4\pi)^{n/2}} \frac{C_A}{4} \left[2 + \xi(n-3) \right] \, \kappa_0(p^2)$$

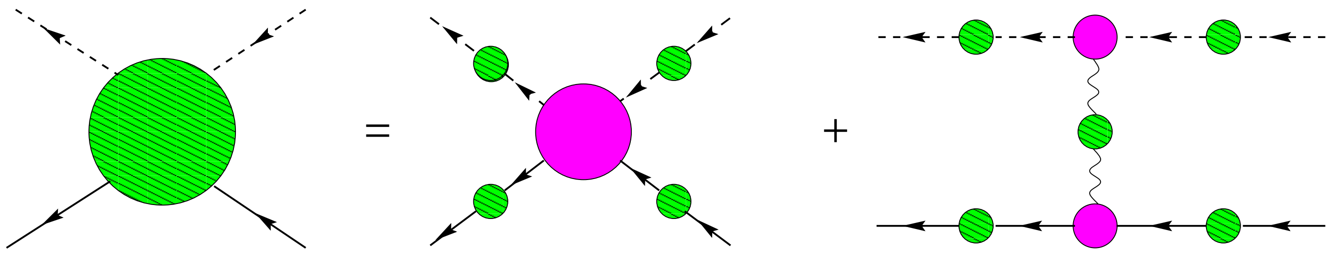
Ward-Slavnov-Taylor

$$p_3^{\mu}\ \Gamma_{\mu}(p_1,p_2,p_3)$$
$$= G(p_3^2)\left[S^{-1}(\text{-}p_1)\ H(p_1,p_2,p_3)\text{-}\overline{H}(p_2,p_1,p_3)\ S^{-1}(p_2)\right]$$

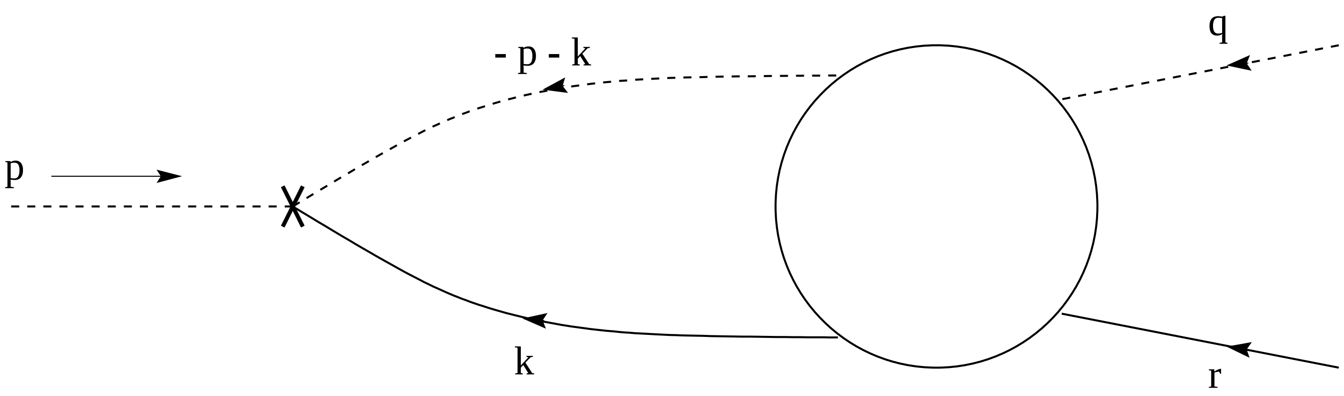
$H(p_1,p_2,p_3)$ is determined by
the quark-quark-ghost-ghost four-point function:



unconnected part　　　connected part



proper 4-point function　　　one-particle reducible contribution



Decomposition into scalar functions:

$$\begin{aligned} H(p_1, p_2, p_3) \quad &= \quad \chi_0(p_1^2, p_2^2, p_3^2) I + \chi_1(p_1^2, p_2^2, p_3^2) \not{p}_1 \\ &\quad + \chi_2(p_1^2, p_2^2, p_3^2) \not{p}_2 + \chi_3(p_1^2, p_2^2, p_3^2) \sigma_{\mu\nu} p_1^\mu p_2^\nu \end{aligned}$$

$$\sigma_{\mu\nu} = \tfrac{1}{2} \left(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu\right)$$

Similarly for $\overline{H}$ (quark line reversed)

Convenient to split RHS into colour coefficients of diagrams 1 and 2

Two WST identities ('QED' and 'Non-Abelian'):

'QED':

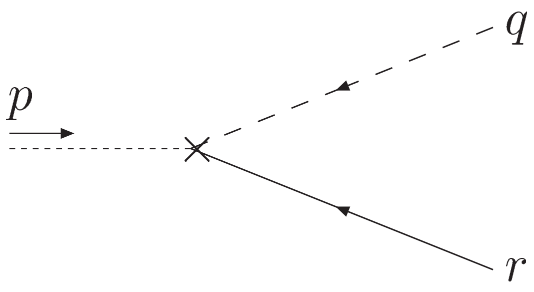
$$
\begin{aligned}
& p_3^{\mu}\Gamma_{\mu}^{(1a)}(p_1,p_2,p_3) \\
&= \Big(C_F - \tfrac{1}{2}C_A\Big)\ C_F^{-1}\ \Big[S^{-1}(-p_1) - S^{-1}(p_2)\Big]^{(1)}
\end{aligned}
$$

'Non-Abelian':

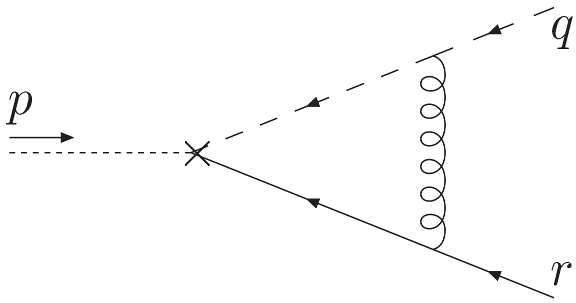
$$
\begin{aligned}
& p_3^{\mu}\Gamma_{\mu}^{(1b)}(p_1,p_2,p_3) \\
&= \Big[S^{-1}(-p_1)\Big]^{(0)}\ H^{(1)}(p_1,p_2,p_3) - \overline{H}^{(1)}(p_2,p_1,p_3)\ \Big[S^{-1}(p_2)\Big]^{(0)} \\
&+ \tfrac{1}{2}C_A\ C_F^{-1}\ \Big[S^{-1}(-p_1) - S^{-1}(p_2)\Big]^{(1)}\ H^{(0)} \\
&+ 2G^{(1)}(p_3^2)\ \Big[S^{-1}(-p_1) - S^{-1}(p_2)\Big]^{(0)}\ H^{(0)}
\end{aligned}
$$

Lowest-order H and $\overline{H}$:

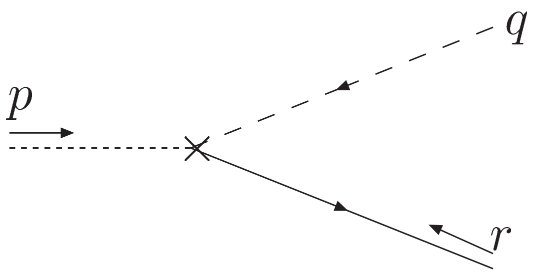
$$H^{(0)}(p,r,q) \quad \longleftrightarrow$$



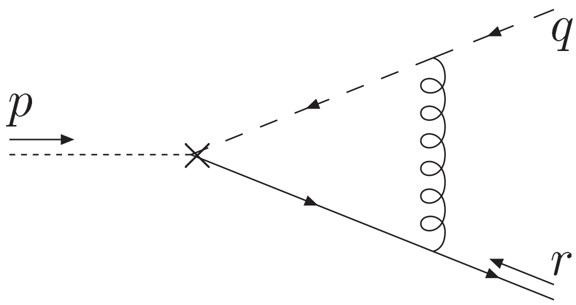
$$H^{(1)}(p,r,q) \quad \longleftrightarrow$$



$$\overline{H}^{(0)}(p,r,q) \quad \longleftrightarrow$$



$$\overline{H}^{(1)}(p,r,q) \quad \longleftrightarrow$$



Decompositions

Longitudinal part [Ball-Chiu]:

$$\Gamma_\mu^{(\mathrm{L})}(p_1, p_2, p_3) = \sum_{i=1}^{4} \lambda_i(p_1^2, p_2^2, p_3^2)\ L_{i,\mu}(p_1, p_2)$$

with

$$\begin{aligned}
L_1^\mu &= \gamma^\mu \\
L_2^\mu &= (\not{p}_1 - \not{p}_2)(p_1 - p_2)^\mu \\
L_3^\mu &= (p_1 - p_2)^\mu \\
L_4^\mu &= \sigma^{\mu\nu}(p_1 - p_2)_\nu
\end{aligned}$$

The last one, L_4^μ, is new (not in QED)

All $\lambda_i(p_1^2, p_2^2, p_3^2)$ are related to S (or α, β), G and H, $\overline{H}$ (or χ_i) by WST identities

Transverse part [Ball-Chiu]:

$$\Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3)=\sum_{i=1}^{8}\tau_i(p_1^2,p_2^2,p_3^2)T_{i,\mu}(p_1,p_2)$$

where the transverse tensors are the following:

$$\begin{aligned}
T_1^\mu &= p_1^\mu(p_2p_3)-p_2^\mu(p_1p_3)\\
T_2^\mu &= \left[-p_1^\mu(p_2p_3)+p_2^\mu(p_1p_3)\right](\not p_1-\not p_2)\\
T_3^\mu &= p_3^2\gamma^\mu-p_3^\mu\not p_3\\
T_4^\mu &= \left[p_1^\mu(p_2p_3)-p_2^\mu(p_1p_3)\right]p_1^\nu p_2^\lambda\sigma_{\nu\lambda}\\
T_5^\mu &= p_{3\nu}\sigma^{\mu\nu}\\
T_6^\mu &= \gamma^\mu(p_1^2-p_2^2)+(p_1-p_2)^\mu\not p_3\\
T_7^\mu &= \frac{1}{2}(p_2^2-p_1^2)\left[\gamma^\mu(\not p_1-\not p_2)-(p_1-p_2)^\mu\right]\\
&\quad -(p_1-p_2)^\mu p_1^\nu p_2^\lambda\sigma_{\nu\lambda}\\
T_8^\mu &= -\gamma^\mu p_1^\nu p_2^\lambda\sigma_{\nu\lambda}+p_1^\mu\not p_2-p_2^\mu\not p_1
\end{aligned}$$

Also useful [KRP, dealing with singular limits]

$$\begin{aligned}
\widetilde{T}_4^\mu &= \frac{2}{p_2^2-p_1^2}\left[2T_4^\mu-p_3^2T_7^\mu\right]\\
\widetilde{\tau}_4 &= \tfrac{1}{4}(p_2^2-p_1^2)\tau_4, \qquad \widetilde{\tau}_7=\tau_7+\tfrac{1}{2}p_3^2\tau_4
\end{aligned}$$

$$\widetilde{\lambda}_2\equiv\lambda_2+\tfrac{1}{2}p_3^2\tau_2, \qquad \widetilde{\lambda}_3\equiv\lambda_3-\tfrac{1}{2}p_3^2\tau_1$$

Charge Conjugation

$$\mathrm{C}\ \Gamma^\mu(p_1,p_2,p_3)\ \mathrm{C}^{-1} = -\Gamma^{\mu\ \mathrm{T}}(p_2,p_1,p_3)$$

$$\mathrm{C}\ \gamma^\mu\ \mathrm{C}^{-1} = -\gamma^{\mu\ \mathrm{T}}$$

One finds:

$$\begin{aligned}
L_i^\mu(p_1,p_2) &= -L_i^{\mu\ \mathrm{T}}(p_2,p_1), \qquad && i=1,2,3 \\
L_4^\mu(p_1,p_2) &= L_4^{\mu\ \mathrm{T}}(p_2,p_1), && \\
T_i^\mu(p_1,p_2) &= -T_i^{\mu\ \mathrm{T}}(p_2,p_1), && i=1,2,3,4,5,7,8 \\
T_6^\mu(p_1,p_2) &= T_6^{\mu\ \mathrm{T}}(p_2,p_1) &&
\end{aligned}$$

and correspondingly (opposite signs) for the $\lambda_i(p_1^2,p_2^2,p_3^2)$ and $\tau_i(p_1^2,p_2^2,p_3^2)$

‘Kinematics’

Gram determinants:

$$\begin{aligned} \mathcal{M}_1 &\equiv p_1^2 p_2^2 + 2(p_1 p_2) m^2 + m^4 \\ &= \mathcal{K} + \Big((p_1 p_2) + m^2 \Big)^2 \\ \mathcal{M}_2 &\equiv (p_1^2 - m^2)(p_2^2 - m^2) p_3^2 + m^2 (p_1^2 - p_2^2)^2 \end{aligned}$$

Källén invariant:

$$\mathcal{K} \equiv p_1^2 p_2^2 - (p_1 p_2)^2$$

These functions are related to our choice of basic scalar integrals
Reduce everything to $J_i(1,1,1)$ and ‘lower’

‘Better’ basis
[more compact expressions for transverse functions τ_i]:

$$\varphi_2$$

$$\kappa_{1,1} + \kappa_{1,2}$$

$$\kappa_{1,1} - \kappa_{1,2}$$

$$\kappa_{1,1} + \kappa_{1,2} - 2\kappa_{2,3}$$

$$\kappa_{1,1} + \kappa_{1,2} - 2\widetilde{\kappa}$$

$$(p_1 p_3)\kappa_{1,1} + (p_2 p_3)\kappa_{1,2} + p_3^2 \kappa_{2,3}$$

and similarly for diagram 2

Selected Results for Off-Shell Case

Results from individual diagrams labelled 'a' and 'b',

$$
\begin{aligned}
&\lambda_1^{(1a)}(p_1^2, p_2^2, p_3^2) \\
&= \frac{g^2\eta\left(C_F - \frac{1}{2}C_A\right)}{(4\pi)^{n/2}} \frac{(n-2)(1-\xi)}{4p_1^2p_2^2} \\
&\quad \times \left[p_1^2(p_2^2+m^2)\kappa_{1,2} + p_2^2(p_1^2+m^2)\kappa_{1,1} - (p_1^2+p_2^2)m^2\widetilde{\kappa}\right]
\end{aligned}
$$

$$
\begin{aligned}
&\lambda_1^{(1b)}(p_1^2, p_2^2, p_3^2) \\
&= -\frac{g^2\eta C_A}{(4\pi)^{n/2}} \frac{1}{16\mathcal{K}} \\
&\quad \times \Big\{(2-\xi)\mathcal{K}\left[2(p_1^2+p_2^2-2m^2)\varphi_1 - 4\kappa_{0,3} - n\kappa_{1,2} - n\kappa_{1,1}\right] \\
&\quad + [2+(n-3)\xi]\,(p_1^2-p_2^2)\Big[(p_1^2-p_2^2)\left(((p_1p_2)+m^2)\varphi_1 + \kappa_{0,3}\right) \\
&\qquad\qquad + (p_2^2-(p_1p_2))\kappa_{1,2} - (p_1^2-(p_1p_2))\kappa_{1,1}\Big] \\
&\quad - (n-2)(2-\xi)\mathcal{K}\left[\frac{m^2}{p_2^2}\kappa_{1,2} + \frac{m^2}{p_1^2}\kappa_{1,1} - \frac{p_1^2+p_2^2}{p_1^2p_2^2}m^2\widetilde{\kappa}\right]\Big\}
\end{aligned}
$$

...

$$\lambda_4^{(1a)}(p_1^2, p_2^2, p_3^2) = 0$$

$$\lambda_4^{(1b)}(p_1^2, p_2^2, p_3^2)$$

$$= \frac{g^2\eta C_A}{(4\pi)^{n/2}}\,\frac{\xi m(p_1^2 - p_2^2)}{16\mathcal{K}\mathcal{M}_1}\Big\{\mathcal{K}p_3^2\varphi_1 + (n-3)\mathcal{K}(\kappa_{1,2} + \kappa_{1,1})$$

$$+(n-3)((p_1p_2)+m^2)\big[p_3^2((p_1p_2)+m^2)\varphi_1 + p_3^2\kappa_{0,3}$$

$$+(p_2p_3)\kappa_{1,2} + (p_1p_3)\kappa_{1,1}\big]$$

$$+(n-2)\mathcal{K}\left[\frac{2(p_1p_2)+m^2}{p_1^2p_2^2}m^2\widetilde{\kappa} - \frac{\mathcal{M}_1}{p_1^2 - p_2^2}\left(\frac{\kappa_{1,2}}{p_2^2} - \frac{\kappa_{1,1}}{p_1^2}\right)\right]\Big\}$$

$$
\begin{aligned}
\tau_2^{(1a)}(p_1^2,p_2^2,p_3^2) &= -\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{1}{8\mathcal{K}\mathcal{M}_2}\\
&\times\Bigg[2[(p_1p_2)-m^2]\Bigg((n-1)\frac{\mathcal{M}_2}{\mathcal{K}}p_3^2[(1+\xi)(p_1p_2)\\
&-(1-\xi)m^2]+2(n-2)\xi\mathcal{M}_2-2(n-3)\xi m^2(p_1^2-p_2^2)^2-8\xi\mathcal{K}m^2\\
&+2m^2p_3^2[\xi(p_1^2+p_2^2)-2(p_1p_2)+2(1-\xi)m^2]\Bigg)\varphi_2\\
&+2[(1+\xi)\mathcal{M}_2p_3^2-4(1-\xi)\mathcal{K}m^2(p_3^2-4m^2)]\varphi_2\\
&+(n-2)m^2\left((1-\xi)\frac{(p_1p_2)\mathcal{M}_2}{p_1^2p_2^2}+2\xi p_3^2[(p_1p_2)-m^2]+4\xi\mathcal{K}\right)\\
&\times(\kappa_{1,1}+\kappa_{1,2}-2\widetilde{\kappa})\\
&+2(n-1)\frac{\mathcal{M}_2}{\mathcal{K}}[(1+\xi)(p_1p_2)-(1-\xi)m^2]\\
&\times[(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{2,3}]\\
&-2\xi[(n-2)\mathcal{M}_2-(n-3)(p_1^2-p_2^2)^2m^2](\kappa_{1,1}+\kappa_{1,2}-2\kappa_{2,3})\\
&+(n-4)\{2\xi p_3^2[p_1^2p_2^2-(p_1p_2)m^2]-4\xi\mathcal{K}m^2-(\xi-1)\mathcal{M}_2\}\\
&\times(\kappa_{1,1}+\kappa_{1,2})\\
&+\Bigg(\frac{(n-2)(1-\xi)\mathcal{M}_2}{p_1^2p_2^2(p_1^2-p_2^2)}\{(p_1-p_2)^2[p_1^2p_2^2-(p_1p_2)m^2]+2\mathcal{K}m^2\}\\
&+2(n-3)\xi m^2(p_1-p_2)^2(p_1^2-p_2^2)\Bigg)(\kappa_{1,1}-\kappa_{1,2})\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&\tau_2^{(1b)}(p_1^2,p_2^2,p_3^2) = -\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{32\mathcal{K}^2\mathcal{M}_1}\\
&\times\Bigg(2\bigg\{(n-4)(n-3)\xi^2\mathcal{K}[(p_1p_2)+m^2]p_3^2m^2\\
&-\mathcal{M}_1\bigg[\mathcal{K}\{(n-2)(\xi-2)(p_1^2+p_2^2)-[4+2(n-3)\xi](p_1p_2)\\
&+[4(n-3)-2(2n-5)\xi]m^2\}-(n-3)\xi^2\Big(p_1^2p_2^2p_3^2+(n-2)\{(p_1p_2)^2p_3^2\\
&+\mathcal{K}[(p_1p_2)+m^2]\}+(n-1)p_3^2(p_1p_2)m^2\Big)-(n-1)(\xi-2)\mathcal{M}_1p_3^2\bigg]\bigg\}\varphi_1\\
&+(n-2)\mathcal{K}\{[(n-3)\xi^2-(\xi-2)\mathcal{M}_1(p_1^2p_2^2)^{-1}]m^2(p_1p_2)\\
&-(n-3)\xi^2(\mathcal{M}_1-p_1^2p_2^2)\}(\kappa_{1,1}+\kappa_{1,2}-2\widetilde{\kappa})\\
&+2\{[(n-1)(\xi-2)\mathcal{M}_1+(n-3)^2\xi^2\mathcal{K}]m^2\\
&+(n-1)\mathcal{M}_1(p_1p_2)[(n-3)\xi^2+\xi-2]\}[(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{0,3}]\\
&+\mathcal{K}\mathcal{M}_1\{[-4\xi+4-(n-5)\xi^2]n-2(3\xi^2-5\xi+6)\}(\kappa_{1,1}+\kappa_{1,2}-2\kappa_{0,3})\\
&+(n-4)\mathcal{K}[(n-3)\xi^2p_1^2p_2^2+(\xi-2)\mathcal{M}_1+(n-3)\xi^2m^2(p_1p_2)](\kappa_{1,1}+\kappa_{1,2})\\
&+\mathcal{K}\mathcal{M}_1\{(n-2)(\xi-2)m^2[(p_1^2+p_2^2)(p_1p_2)(p_1^2p_2^2)^{-1}-2]\\
&-[(3\xi-2)n-8(\xi-1)](p_1-p_2)^2\}\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\Bigg)
\end{aligned}
$$

Special Cases

We have studied the following special cases

- Quarks on-shell, $p_1^2 = p_2^2 = m^2$
  - Some 'structures' become linearly dependent
    $L_2 \leftrightarrow T_2, \qquad L_3 \leftrightarrow T_1, \qquad T_4 \leftrightarrow T_7$
  - Limit $p_3^2 \to 0$ regular
  - Dirac and Pauli form factors

$$\begin{aligned}\overline{u}(-p_1)\Gamma^\mu u(p_2) &= F_1(p_3^2)\,\overline{u}(-p_1)\gamma^\mu u(p_2)\\ &- \frac{1}{2m}F_2(p_3^2)\,\overline{u}(-p_1)\sigma^{\mu\nu}p_{3_\nu}u(p_2)\end{aligned}$$

$$\begin{aligned}F_1 + F_2 &= \lambda_1 + p_3^2\tau_3 - 2m\tau_5 + \tfrac{1}{2}(p_3^2 - 4m^2)\tau_8\\ \frac{1}{2m}F_2 &= -2m\widetilde{\lambda}_2 + \widetilde{\lambda}_3 - \tau_5 + \tfrac{1}{2}p_3^2\widetilde{\tau}_7 - m\tau_8\end{aligned}$$

  - QED limit: Pokorski
  - $m \to 0$, $\xi \to 0$: Nowak, Praszałowicz, Słomiński
- Zero-momentum limit, $p_3 \to 0$
  - $m \to 0$, Braaten & Leveille
- $n = 3$: Bashir, Kızılersü, Pennington
- Yennie gauge: Adkins, Lymberopoulos, Velkov