Coherent Exclusive Exponentiation

New Theoretical framework for

Precision Predictions of Higher Order

Radiative Corrections

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Outline:

- **Introduction.**
- **Overview of CEEX.**
- **Example numerical results, σ and $A_{\rm FB}$.**
- **Summary, outlook.**

Papers by S. Jadach, B.F.L. Ward and Z. Wąs:

Phys. Lett. B449 (1999) 97, (CERN-TH/98-253), CERN-TH/98-235, subm. to EPJ,

DESY-99-106 (CERN-TH/99-235), 88p., Comput. Phys. Commun. in press

UTHEP-99-09-01 (CERN-TH/2000-087), in preparation.

Slides are on http://home.cern.ch/jadach

INTRODUCTION

EEX = Exclusive EXponentiation

Based on Yennie-Frautschi-Suura (1961)

YFS2, YFS3, KORALZ, $e^+e^- \to f\bar{f} + n\gamma$

YFS3WW, $e^+e^- \to W^+W^- + n\gamma$

YFS3ZZ, $e^+e^- \to ZZ + n\gamma$

BHLUMI, BHWIDE, $e^+e^- \to e^+e^- + n\gamma$

CEEX = Coherent Exclusive EXponentiation

New one, by Jadach-Wąs-Ward (1998), an extension of Yennie-Frautschi-Suura

$\mathcal{KK}$MC, $e^+e^- \to f\bar{f} + n\gamma,\ \ f = \mu, \tau, d, u, s, c, b$

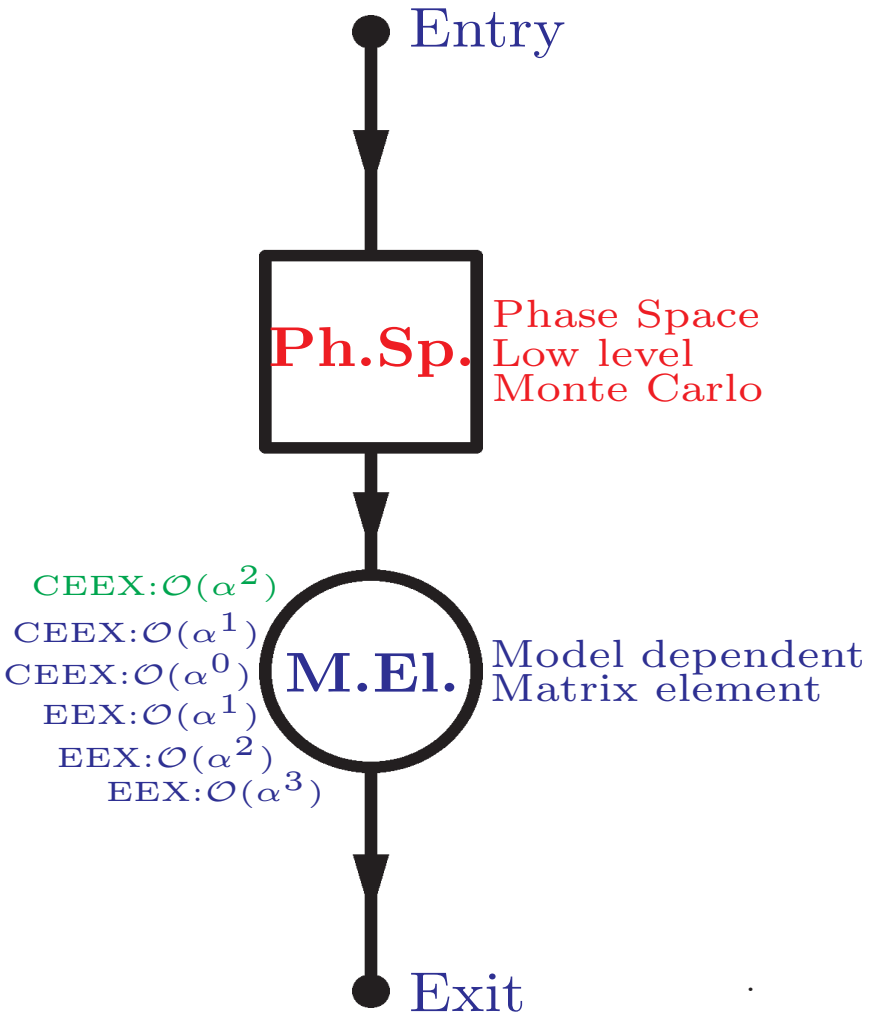
IEX = Inclusive EXponentiation

Naive semianalytical exponentiation,

$\mathcal{KK}$sem, CALASY, Zfitter, Gentle...

INTRODUCTION

General structure of $\mathcal{KK}$ MC (flowchart)



- **Universal Phase-space Monte Carlo simulator**
- **Library of several types of SM/QED matrix elements**
- **Tau decays and hadronization come after**

CEEX features (terminology)

Why COHERENT?

Because: Friendly to Quantum coherence;

Coherence among Feynman diagrams: Narrow resonances,

$\gamma \oplus Z$ exchanges, $t \oplus s$ channels, ISR$\oplus$FSR real emissions, etc.

Complete $\Big| \sum_{diagr.}^{n} \mathcal{M}_i \Big|^2$ rather then often incomplete $\sum_{i,j}^{n^2} \mathcal{M}_i \mathcal{M}_j^*$

Why EXCLUSIVE?

Because: Phase-Space $\times \sum_{spin} \Big|\mathcal{M}\Big|^2$

Contrary to Parton-Showers, Leading-Logs, Structure-Functions

which are all inherently inclusive.

Why EXPONENTIATION?

Infrared (IR) contributions summed to $\mathcal{O}(\alpha^\infty)$

Going **beyond** Yennie-Frautschi-Suura (EEX):

Real IR singularities isolated/reorganized at the **spin amplitude** level, while IR cancellations occur for probabilities (integrated x-sections). Paradox???

Differences with older EEX/YFS? See next slide

EEX= Exclusive EXponentiation = variant of YFS expon.

EEX $\leftrightarrow$ CEEX

$e^-(p_1,\lambda_1)+e^+(p_2,\lambda_2)\to f(q_1,\lambda_1')+\bar{f}(q_2,\lambda_2')+\gamma(k_1,\sigma_1)+\ldots+\gamma(k_n,\sigma_n)$

EEX/YFS very schematically

$$\sigma=\sum_{n=0}^{\infty}\int_{m_\gamma} d\Phi_{n+2}\, e^{Y(m_\gamma)} D_n(q_1,q_2,k_1,\ldots,k_n)$$

$\mathcal{O}(\alpha^1)$ **Example:**

$$D_0=\bar{\beta}_0$$
$$D_1(k_1)=\bar{\beta}_0\tilde{S}(k_1)+\bar{\beta}_1(k_1)$$
$$D_2(k_1,k_2)=\bar{\beta}_0\tilde{S}(k_1)\tilde{S}(k_2)+\bar{\beta}_1(k_1)\tilde{S}(k_2)+\bar{\beta}_1(k_2)\tilde{S}(k_1)$$

Real soft factors:

$$\tilde{S}(k)=\sum_\sigma|\mathfrak{s}_\sigma(k)|^2=|\mathfrak{s}_+|^2(k)+|\mathfrak{s}_-(k)|^2=-\tfrac{\alpha}{\pi}\left(\tfrac{q_1}{kq_1}-\tfrac{q_2}{kq_2}\right)^2$$

IR-finite building blocks:

$\bar{\beta}_0=\sum_\lambda|\mathcal{M}_\lambda|^2$, λ = fermion helicities, σ = photon helicity

$$\bar{\beta}_1(k)=\sum_{\lambda\sigma}|\mathcal{M}^{1-\mathrm{PHOT}}_{\lambda\sigma}|^2-\sum_\sigma|\mathfrak{s}_\sigma(k)|^2\sum_\lambda|\mathcal{M}^{\mathrm{Born}}_\lambda|^2$$

Everything in terms of $\sum_{spin}|\ldots|^2$ **!!!!**

CEEX schematically

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} \sum_{\lambda,\sigma_1,\ldots,\sigma_n} |e^{B(m_\gamma)} \mathcal{M}^{\lambda}_{n,\sigma_1,\ldots,\sigma_n}(k_1,\ldots,k_n)|^2$$

$\mathcal{O}(\alpha^1)$ Example:

$\mathcal{M}^{\lambda}_{0} = \hat{\beta}^{\lambda}_{0}$, λ=fermion helicities,

$\mathcal{M}^{\lambda}_{1,\sigma_1}(k_1) = \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma_1}(k_1) + \hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)$

$\mathcal{M}^{\lambda}_{2,\sigma_1,\sigma_2}(k_1,k_2) = \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^{\lambda}_{1,\sigma_2}(k_2)\mathfrak{s}_{\sigma_1}(k_1)$

IR-finite building blocks:

$$\hat{\beta}^{\lambda}_{0} = \left(e^{-B}\mathcal{M}^{\mathrm{Born+Virt.}}_{\lambda}\right)\Big|_{\mathcal{O}(\alpha^1)},$$

$$\hat{\beta}^{\lambda}_{1,\sigma}(k) = \mathcal{M}^{\lambda}_{1,\sigma}(k) - \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma}(k)$$

Everything in terms of $\mathcal{M}$-spin-amplitudes !!!!

CEEX in practice

Practical problems

Deficiencies of YFS **Exclusive Exponentiation** (EEX) technique, as implemented in KORALZ/YFS3, BHLUMI, BHWIDE, KORALW:

- Lack of the ISR/FSR or Up/Down interferences
  (except BHWIDE, but difficult to upgrade...)
- Simplified treatment of spin, neglected transverse polarization
  (except KORALB, but no exponentiation...)
- Approximate matrix element for 2 and 3 hard large p_T photons.
  (Multiphotons are important for searches.)

The CEEX solution

New Coherent Exclusive Exponentiation CEEX solves many (all?):

- ISR$\otimes$FSR interferences are included in a natural way.
  Spin amplitudes for ISR and FSR summed squared numerically.
  (The BHWIDE approach gets cumbersome beyond first order)
- Exact treatment of fermion spin polarizations (transv.$\&$longitudinal).
  Numerical Wigner rotations of spin density matrices available.
  (Necessary for interfacing with decay M.C. simulating fermion decays).
- Exact matrix element for 2 and 3 and more photons using
  Kleiss-Stirling (KS) spinor technique.

MC Implementation of CEEX

Comparison of $\mathcal{KK}$ MC with its predecessors

Feature	KORALB	KORALZ	$\mathcal{KK}$ 4.13	$\mathcal{KK}$ 2000+?
QED type	$\mathcal{O}(\alpha)$	EEX	CEEX, EEX	CEEX, EEX
CEEX(ISR+FSR)	none	none	$\{\alpha, \alpha L; \alpha^2 L^2, \alpha^2 L^1\}$	$\{\ldots\alpha^2 L^1; \alpha^3 L^3\}$
EEX(ISR*FSR)	none	$\{\alpha, \alpha L, \alpha^2 L^2\}$	$\{\alpha, \alpha L, \alpha^2 L^2, \alpha^3 L^3\}$	$\{\ldots\alpha^2 L^2, \alpha^3 L^3\}$
ISR-FSR int.	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)$	$\{\alpha, \alpha L\}_{\rm CEEX}$	$\{\alpha, \alpha L\}_{\rm CEEX}$
Exact bremss.	1 γ	1, 2coll. γ	1, 2, 3coll. γ	up to 3 γ
Electroweak	No Z-res.	DIZET 6.x	DIZET 6.x	New version?
Beam polar.	long+trans.	longit.	long+trans.	long+trans.
τ polar.	long+trans.	longit.	long+trans.	long+trans.
Hadronization	—	JETSET	JETSET	PYTHIA
τ decay	TAUOLA	TAUOLA	TAUOLA	TAUOLA
Inclusive mode	—	No	Yes	Yes
Beamstrahlung	—	No	Yes	Yes
Beam spread	—	No	Yes	Yes
$\nu\nu$ channel	—	Yes	No	Yes
ee channel	—	No	No	Yes
tt channel	—	No	No	yes?
WW channel	—	No	No	yes?

Open Problems

Which problems CEEX does not solve?

CEEX provides better environment for virtual correction, but we still need a real breakthrough in the numerical treatment of the virtual corrections comparable to one which we made for real emissions.

NUMERICAL OUTSORCING:
in the phase space integration and constructing the real photon emission amplitudes we have fully exploited $\sim 10^3$ progress in the CPU power over last decade 1990/2000. Another 10^3 in CPU will come. Virtual corrections are next front!

1980-87 ph-space $n = 1, 2, ...\infty$ photons (MonteCarlo)

1985-99 amplits. $n = 2, 3, ...\infty$ real phot. (WeylSpinors)

2000-15 amplits. $n = 2, 3, ...\infty$ loops!!! (???)

Bhabha Problem

'Notorious Bhabha': Problems & Requirements

- **Going beyond $\mathcal{O}(\alpha^1)$ mandatory, precision $< 0.5\%$.**

  Hence, up to four particles in the final state, and two loop virtual corrections.

- **Monte Carlo is mandatory: Why?**

  1. 4-body final states,
  2. nontrivial experimental cuts,
  3. calorimetric measurements of electrons and photons.

- **Exponentiation is mandatory: Why?**

  1. To have realistic MC multi-photons, as in experiment.
  2. To avoid **negative x-sections**. In $\mathcal{O}(\alpha^1)$ the minimum real photon energy k_0 cannot be small enough because MC weight (diff. xsect.) becomes negative, (events with 0-real and 1-virtual photon). Moreover keeping $k_0 > 0.005$ causes technical error $\sim 0.5\%$. Going to $\mathcal{O}(\alpha^2)$ without expon. does not help. Weights < 0 (diff. xsect. < 0) still there, now for events with 1-real and 1-virtual photon. (Surprise to some people during 1990 LEP workshop).
  3. Construction of complete $\mathcal{O}(\alpha^2)$ MC will take time, it is desirable to split it into **stages**. If exponentiation adopted then one deals with the IR-finite building-blocks, it is therefore possible to have an intermediate working solution with the incomplete $\mathcal{O}(\alpha^2)$, for example without sub-sub-leading terms, or without difficult virtual corrections, before available.
  4. Solutions based on the leading-logs (LL) combined with $\mathcal{O}(\alpha^1)$ are plagued with the problem of the **LL-scale**. Within LL there is no way to say whether one use s or t-scale in the LL QED correction. In reality transition from s to t scale occur smoothly when going from s to t angles. LL ignores this fact. Another problem: the LL scale is not defined for $\gamma - Z$ interference where Z is s-channel **resonance** and gamma is t-channel exchange.

Bhabha Problem

CEEX can help!

Why? The phase space integration of 2 and more photons, done routinely since long, without any approximation as in BHLUMI and KORALZ and KKMC. The only problem is with matrix element, and here an important progress: CEEX has many features needed to solve the $\mathcal{O}(\alpha^2)$ Bhabha problem:

1. **No problem with 2-photon 4-body phase space.**
2. **No problem with negative cross sections.**
3. **No problems with the LL scale. Controlled automatically by the constructive/destructive interferences. Smooth transition between s to t-scales, when going to small angles.**
4. **No problem with Z resonance and its interference with the rest, Γ/M cancellations under control.**
5. **No problem with dividing development of the project into stages. All virtual corrections enter as IR-free objects, can be added one by one as soon as they are available. (Penta-boxes etc., 2-loop boxes etc.)**
6. **Exact 2γ emission amplitudes using spinor methods.**
7. **Electroweak $\mathcal{O}(\alpha^1)$ corrs. enter consistently/naturally.**
8. **Beam polarizations implemented trivially.**

Full Scale CEEX

Full Scale CEEX $\mathcal{O}(\alpha^r)$, r=1,2, master formula

Polarized total x-section:

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_n(p_a+p_b; p_c, p_d, k_1, \ldots, k_n)\; e^{2\alpha \Re B_4} \sum_{\sigma_i,\lambda,\bar{\lambda}} \sum_{i,j,l,m=0}^{3}$$

$$\hat{\varepsilon}_a^i \hat{\varepsilon}_b^j \sigma^i_{\lambda_a \bar{\lambda}_a} \sigma^j_{\lambda_b \bar{\lambda}_b} \mathfrak{M}_n^{(r)} \begin{pmatrix} p\, k_1\, k_2 \\ \lambda\, \sigma_1\, \sigma_2 \end{pmatrix} \cdots \begin{matrix} k_n \\ \sigma_n \end{matrix} \Big) \Big[\mathfrak{M}_n^{(r)} \Big(\begin{matrix} p\, k_1\, k_2 \\ \bar{\lambda}\, \sigma_1\, \sigma_2 \end{matrix} \cdots \begin{matrix} k_n \\ \sigma_n \end{matrix} \Big) \Big]^{\star} \sigma^l_{\bar{\lambda}_c \lambda_c} \sigma^m_{\bar{\lambda}_d \lambda_d} \hat{h}_c^l \hat{h}_c^m$$

CEEX amplitudes:

$$\mathfrak{M}_n^{(1)} \begin{pmatrix} p\, k_1 \\ \lambda\, \sigma_1 \end{pmatrix} \cdots \begin{matrix} k_n \\ \sigma_n \end{matrix} \Big) = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^{n} \mathfrak{s}_{[i]}^{\{\wp_i\}} \left\{ \beta_0^{(1)} \begin{pmatrix} p \\ \lambda \end{pmatrix}; X_\wp \Big) + \sum_{j=1}^{n} \frac{\beta^{(1)}_{1\{\wp_j\}} \begin{pmatrix} p\, k_j \\ \lambda\, \sigma_j \end{pmatrix}; X_\wp \Big)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} \right\}$$

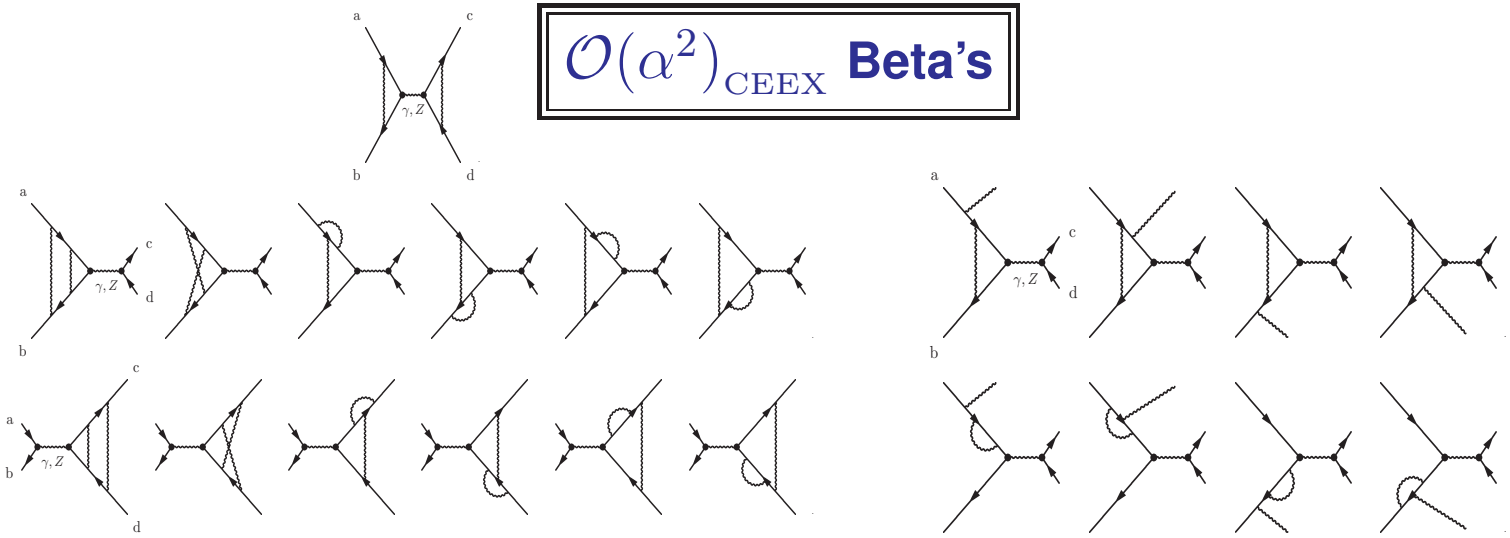
$$\mathfrak{M}_n^{(2)} \begin{pmatrix} p\, k_1 \\ \lambda\, \sigma_1 \end{pmatrix} \cdots \begin{matrix} k_n \\ \sigma_n \end{matrix} \Big) = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^{n} \mathfrak{s}_{[i]}^{\{\wp_i\}}$$

$$\times \left\{ \beta_0^{(2)} \begin{pmatrix} p \\ \lambda \end{pmatrix}; X_\wp \Big) + \sum_{j=1}^{n} \frac{\beta^{(1)}_{2\{\wp_j\}} \begin{pmatrix} p\, k_j \\ \lambda\, \sigma_j \end{pmatrix}; X_\wp \Big)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} + \sum_{1 \le j < l \le n} \frac{\beta^{(2)}_{2\{\wp_j,\wp_l\}} \begin{pmatrix} p\, k_j\, k_l \\ \lambda\, \sigma_j\, \sigma_l \end{pmatrix}; X_\wp \Big)}{\mathfrak{s}_{[j]}^{\{\wp_j\}} \mathfrak{s}_{[l]}^{\{\wp_l\}}} \right\}$$

For details see next slides.

Full Scale CEEX

$\mathcal{O}(\alpha^2)_{\rm CEEX}$ Beta's



$$\beta_0^{(l)}\left({}^{p}_{\lambda};P\right)=\mathfrak{M}_0^{(l)}\left({}^{p}_{\lambda};P\right),\ l=0,1,2,$$

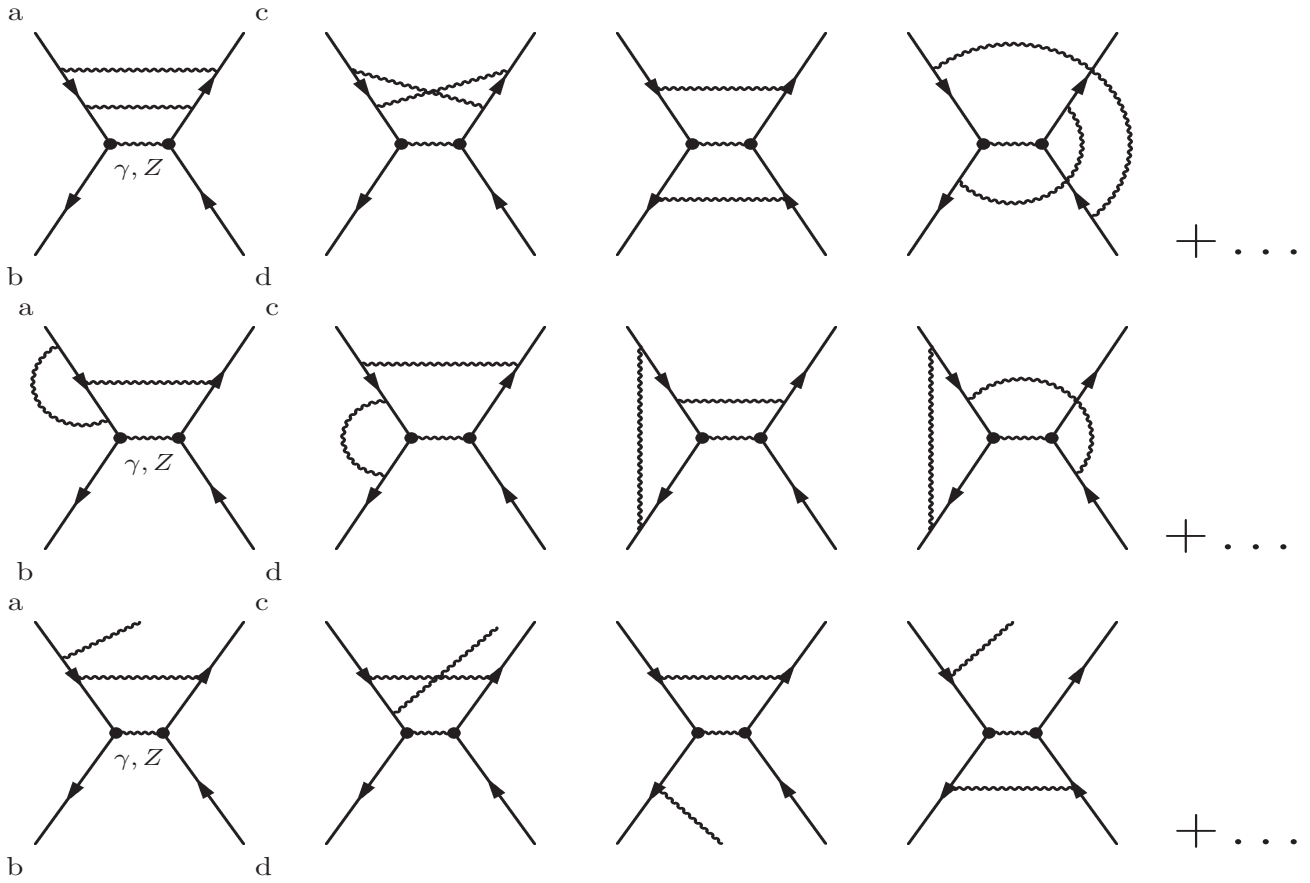
$$\beta_{1\{I\}}^{(1+l)}\left({}^{p\,k_1}_{\lambda\,\sigma_1};P-k_1\right)=\mathfrak{M}_{1\{I\}}^{(1+l)}\left({}^{p\,k_1}_{\lambda\,\sigma_1};P-k_1\right)-\beta_0^{(l)}\left({}^{p}_{\lambda};P-k_1\right)\mathfrak{s}_{\sigma_1}^{\{I\}}(k_1),\ l=0,1,$$

$$\beta_{1\{F\}}^{(1+l)}\left({}^{p\,k_1}_{\lambda\,\sigma_1};P\right)=\mathfrak{M}_{1\{F\}}^{(1+l)}\left({}^{p\,k_1}_{\lambda\,\sigma_1};P\right)-\beta_0^{(l)}\left({}^{p}_{\lambda};P\right)\mathfrak{s}_{\sigma_1}^{\{F\}}(k_1),\ l=0,1,$$

$$\begin{aligned}\beta_{2\{\omega_1,\omega_2\}}^{(2)}\left({}^{p\,k_1\,k_2}_{\lambda\,\sigma_1\,\sigma_2};X_\omega\right)&=\mathfrak{M}_{2\{\omega_1,\omega_2\}}^{(2)}\left({}^{p\,k_1\,k_2}_{\lambda\,\sigma_1\,\sigma_2};X_\omega\right)\\&-\beta_{1\{\omega_1\}}^{(1)}\left({}^{p\,k_1}_{\lambda\,\sigma_1};X_\omega\right)\mathfrak{s}_{\sigma_2}^{\{\omega_2\}}(k_2)-\beta_{1\{\omega_2\}}^{(1)}\left({}^{p\,k_2}_{\lambda\,\sigma_2};X_\omega\right)\mathfrak{s}_{\sigma_1}^{\{\omega_1\}}(k_1)\\&-\beta_0^{(0)}\left({}^{p}_{\lambda};X_\omega\right)\mathfrak{s}_{\sigma_1}^{\{\omega_1\}}(k_1)\mathfrak{s}_{\sigma_2}^{\{\omega_2\}}(k_2),\end{aligned}$$

Full Scale CEEX

$\mathcal{O}(\alpha^2)_{\rm CEEX}$ Shopping list



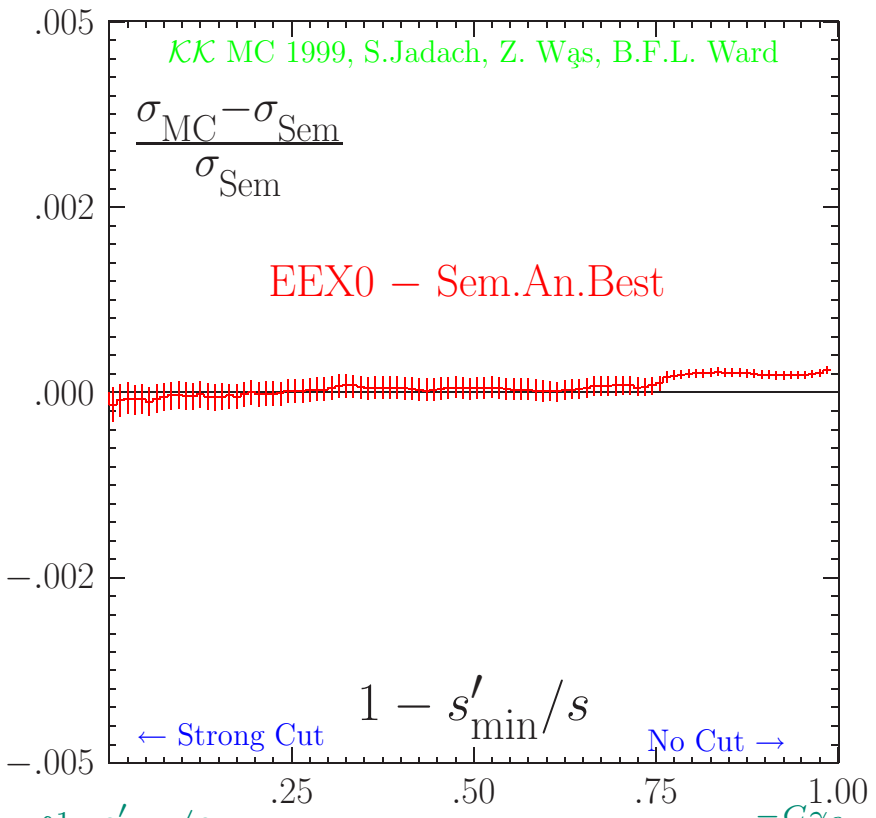
Never ever combine with real emission! (Keep photon mass and publish.)

Never ever square and spin sum! In the M.C. real emission is added anyway.

Numerical Results: Techn. precision

Baseline test of the Low level MC

$|$ Matrix Element$|^2$ = Born $\times$ Product of soft factors



$$\sigma_{\rm Sem.} = \int_0^{1-s'_{\rm min}/s} dv\ \sigma^f_{\rm Born}(s(1-u)(1-v))\ \frac{e^{-C\gamma_e}}{\Gamma(1+\gamma_e)}$$
$$e^{\frac{1}{4}\gamma_e+\frac{\alpha}{\pi}\left(\frac{1}{2}+\frac{\pi^2}{3}\right)}\ \gamma_e v^{\gamma_e-1}\left[1-\frac{\gamma_e\ln(1-v)}{4}-\frac{\alpha}{\pi}\frac{\ln^2(1-v)}{2}+0\,\gamma_e^2\right]$$
$$\frac{e^{-C\gamma_f}}{\Gamma(1+\gamma_f)}e^{\frac{1}{4}\gamma_f-\frac{1}{2}\gamma_f\ln(1-u)+\frac{\alpha}{\pi}\left(\frac{1}{2}+\frac{\pi^2}{3}\right)}\gamma_f u^{\gamma_f-1}\left[1-\frac{\gamma_f\ln(1-u)}{4}\right]$$

Techn.Error=$|$MC−Sem.Anl.$|$ $\leq 2\cdot 10^{-4}$

Numerical Results: Physical Precision

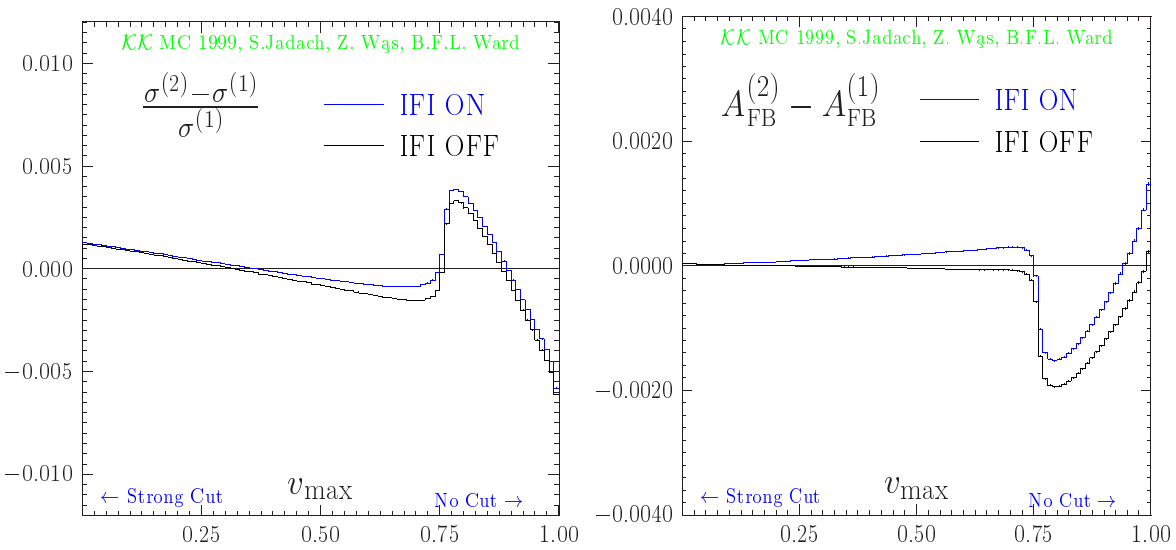
PP = neglected higher orders and subleadings

Our basic estimate: PP = $\frac{1}{2}\left\{\mathcal{O}(\alpha^2)_{\rm CEEX} - \mathcal{O}(\alpha^1)_{\rm CEEX}\right\}$

$\mathcal{KK}$ MC results for $e^+e^- \to \mu^+\mu^-$ at 189GeV

Photon Energy Cut: $v = 1 - s'/s < v_{\rm max}$; $s' = m^2_{f\bar{f}}$

Angular Cut: $|\cos\theta| < 1$



The above result imply for Physical Precision:

$\frac{\delta\sigma}{\sigma} \leq 0.2\%, \qquad \delta A_{FB} \leq 0.1\%,$

even including radiative-return on-shell Z.

This is confirmed by more tests, see later.

Absolute prediction for σ and A_{FB}

Process: $e^-e^+ \to f\bar{f}$, $f = \mu^-$, **at 189GeV.**

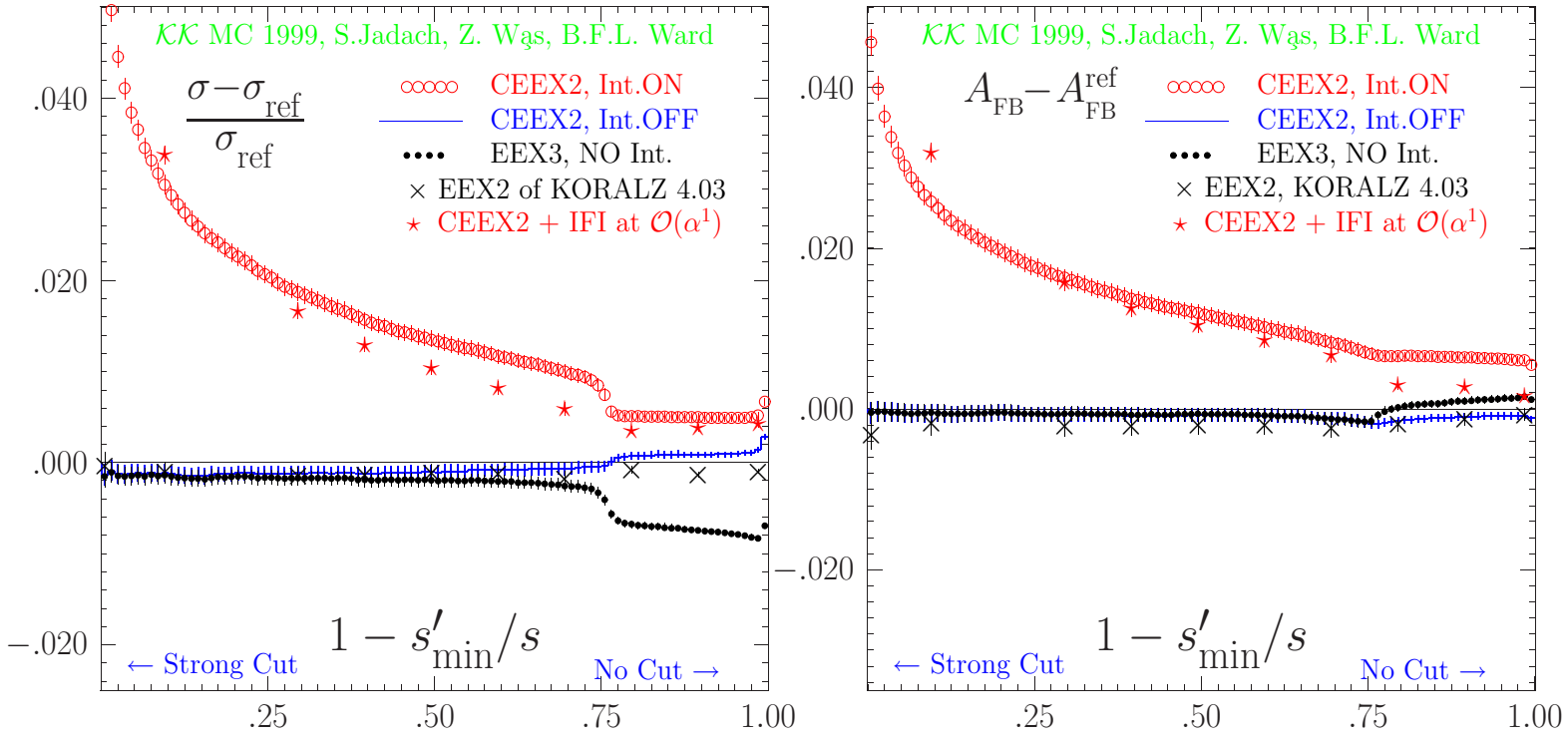
Energy cut: $v < v_{\max}$, **where** $v = 1 - s'/s$, $s' = M^2_{f\bar{f}}$.

Scattering angle for $A_{\rm FB}$ **is** $\theta = \theta^\bullet$. **No cut in** θ. **E-W corr. in** $\mathcal{KK}$ **according to DIZET 6.x.** $\mathcal{O}(\alpha^3)_{\rm LL}$ **EEX3 matrix element in** $\mathcal{KK}$ **MC (No ISR⊗FSR interf.)**

$\mathcal{KK}$**sem is semianalytical part of** $\mathcal{KK}$. (Angle $\theta^\bullet$ is from Phys. Rev. D41, 1425 (1990).)

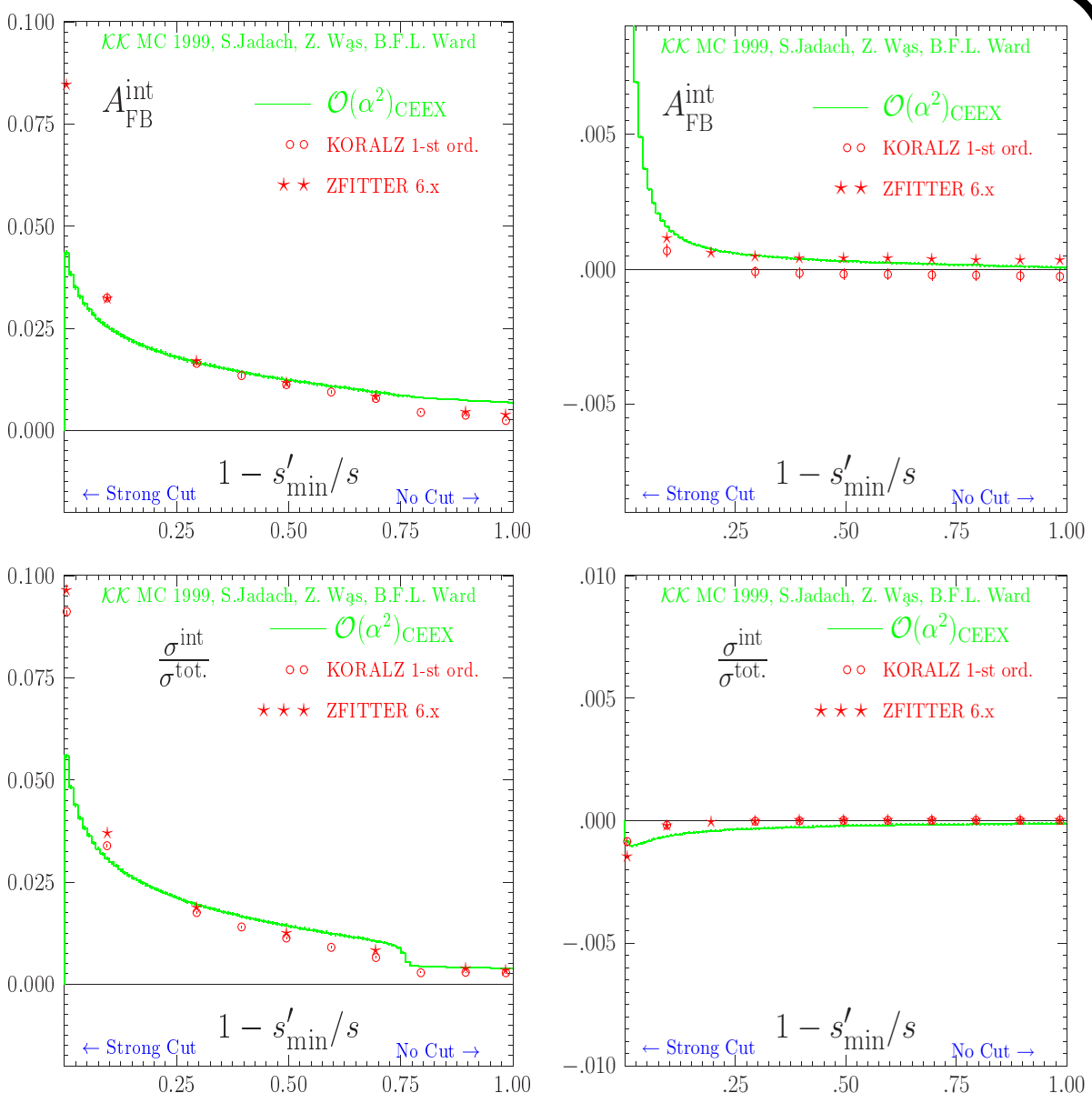
$v_{\max}$	$\mathcal{KK}$sem Refer.	$\mathcal{O}(\alpha^3)_{\rm EEX3}$	$\mathcal{O}(\alpha^2)_{\rm CEEX}$ intOFF	$\mathcal{O}(\alpha^2)_{\rm CEEX}$	KORALZ	KORALZ Interf.
	$\sigma(v_{\max})$ [pb], $\mathcal{KK}$ M.C. and KORALZ 1-st order					
0.01	1.6712 ± 0.0000	1.6687 ± 0.0020	1.6669 ± 0.0020	1.7657 ± 0.0024	0.9639 ± 0.0009	0.0912 ± 0.0005
0.10	2.5198 ± 0.0000	2.5164 ± 0.0023	2.5147 ± 0.0023	2.5943 ± 0.0027	2.1919 ± 0.0010	0.0339 ± 0.0004
0.30	3.0616 ± 0.0000	3.0565 ± 0.0024	3.0578 ± 0.0024	3.1183 ± 0.0029	2.7690 ± 0.0010	0.0175 ± 0.0003
0.50	3.3747 ± 0.0000	3.3682 ± 0.0025	3.3740 ± 0.0025	3.4219 ± 0.0029	3.0565 ± 0.0010	0.0113 ± 0.0003
0.70	3.7225 ± 0.0000	3.7131 ± 0.0025	3.7257 ± 0.0025	3.7629 ± 0.0030	3.3649 ± 0.0010	0.0065 ± 0.0003
0.90	7.1434 ± 0.0000	7.0904 ± 0.0024	7.1538 ± 0.0024	7.1795 ± 0.0029	6.3558 ± 0.0010	0.0029 ± 0.0001
0.99	7.6145 ± 0.0000	7.5511 ± 0.0024	7.6642 ± 0.0024	7.6888 ± 0.0029	6.7004 ± 0.0010	0.0028 ± 0.0001
	$A_{\rm FB}(v_{\max})$, $\mathcal{KK}$ M.C. and KORALZ 1-st order					
0.01	0.5654 ± 0.0000	0.5650 ± 0.0014	0.5650 ± 0.0014	0.6111 ± 0.0016	0.5765 ± 0.0013	0.1201 ± 0.0013
0.10	0.5664 ± 0.0000	0.5660 ± 0.0011	0.5660 ± 0.0011	0.5922 ± 0.0012	0.5784 ± 0.0006	0.0324 ± 0.0006
0.30	0.5692 ± 0.0000	0.5687 ± 0.0009	0.5686 ± 0.0009	0.5855 ± 0.0011	0.5818 ± 0.0005	0.0164 ± 0.0005
0.50	0.5744 ± 0.0000	0.5738 ± 0.0009	0.5737 ± 0.0009	0.5862 ± 0.0010	0.5868 ± 0.0005	0.0112 ± 0.0005
0.70	0.5864 ± 0.0000	0.5852 ± 0.0008	0.5853 ± 0.0008	0.5944 ± 0.0009	0.5972 ± 0.0004	0.0078 ± 0.0004
0.90	0.3105 ± 0.0000	0.3115 ± 0.0004	0.3108 ± 0.0004	0.3177 ± 0.0005	0.3260 ± 0.0002	0.0037 ± 0.0002
0.99	0.2851 ± 0.0000	0.2867 ± 0.0004	0.2844 ± 0.0004	0.2903 ± 0.0004	0.3039 ± 0.0002	0.0024 ± 0.0002

Absolute prediction for σ and A_{FB}



Absolute prediction for total cross section an charge asymmetry. They are for $\mu^+\mu^-$ final state at $\sqrt{s}=189$GeV. Results are plotted as a function of the cut-off on the total photon energy $v_{\rm max}=1-s'_{\rm min}/s$. The "reference" σ and $A_{\rm FB}$ in first column are from $\mathcal{KK}$sem semi-analytical program.

ISR⊗FSR interference



The s'-cut dependence of $\delta\sigma$ and δA_{FB} due to ISR⊗FSR interference. No θ-cut. For the ISR⊗FSR KORALZ is the best reference and starting point

Pure $\mathcal{O}(\alpha^1)$ (no exponentiation) ISR⊗FSR in KORALZ was verified very precisely $\sim 0.01\%$ with special analytical calculation, S.Jadach Z.Wąs, Phys. Rev. D41, 1425 (1990).

Recent Results for Quarks

$\mathcal{KK}$MC versus $\mathcal{KK}$sem, WT-ed events

Inclusive run of $\mathcal{KK}$MC, $q = d, u, s, c, b$.

Energy: $\sqrt{s}$= 189GeV. FSR is off.

The only cut $v = 1 - s'/s < v_{\max}$.

$v_{\max}$	$\mathcal{KK}$sem Refer.	$\mathcal{O}(\alpha^3)_{\rm EEX3}$	$\mathcal{O}(\alpha^2)_{\rm CEEX}$ intOFF
	$\sigma(v_{\max})$ [pb]		
0.01	14.9309 ± 0.0000	14.9276 ± 0.0078	14.9298 ± 0.0078
0.10	19.5171 ± 0.0000	19.5093 ± 0.0086	19.5135 ± 0.0086
0.20	21.2573 ± 0.0000	21.2487 ± 0.0088	21.2542 ± 0.0088
0.30	22.4477 ± 0.0000	22.4370 ± 0.0089	22.4436 ± 0.0089
0.40	23.4602 ± 0.0000	23.4454 ± 0.0090	23.4539 ± 0.0090
0.50	24.4711 ± 0.0000	24.4515 ± 0.0091	24.4639 ± 0.0091
0.60	25.7152 ± 0.0000	25.6890 ± 0.0092	25.7082 ± 0.0092
0.70	28.2545 ± 0.0000	28.2038 ± 0.0092	28.2482 ± 0.0093
0.80	92.7078 ± 0.0000	91.8508 ± 0.0088	92.8080 ± 0.0098
0.90	95.3717 ± 0.0000	94.4731 ± 0.0087	95.4779 ± 0.0097
0.99	97.0035 ± 0.0000	96.0726 ± 0.0086	97.1242 ± 0.0096
	$A_{\rm FB}(v_{\max})$		
0.01	0.6358 ± 0.0000	0.6357 ± 0.0006	0.6357 ± 0.0006
0.10	0.6358 ± 0.0000	0.6358 ± 0.0005	0.6358 ± 0.0005
0.20	0.6357 ± 0.0000	0.6357 ± 0.0005	0.6357 ± 0.0005
0.30	0.6356 ± 0.0000	0.6355 ± 0.0005	0.6355 ± 0.0005
0.40	0.6352 ± 0.0000	0.6351 ± 0.0005	0.6351 ± 0.0005
0.50	0.6343 ± 0.0000	0.6341 ± 0.0004	0.6341 ± 0.0004
0.60	0.6316 ± 0.0000	0.6313 ± 0.0004	0.6313 ± 0.0004
0.70	0.6169 ± 0.0000	0.6163 ± 0.0004	0.6162 ± 0.0004
0.80	0.2600 ± 0.0000	0.2599 ± 0.0001	0.2586 ± 0.0001
0.90	0.2432 ± 0.0000	0.2434 ± 0.0001	0.2420 ± 0.0001
0.99	0.2354 ± 0.0000	0.2359 ± 0.0001	0.2344 ± 0.0001

Satisfactory agreement $< 0.2\%$ for Z-excl. cuts.

The same for Z-inclusive cuts.

Comparisons with Zfitter just started.

Recent Results for Quarks

$\mathcal{KK}$MC versus KORALZ 4.02 and 4.04

WT-ed events. $\sqrt{s}$= 189GeV. FSR is off.

$\mathcal{KK}$MC: inclusive run $q = d, u, s, c, b$.

The only cut $v = 1 - s'/s < v_{\max}$.

No.	(a) $\mathcal{O}(\alpha^2)_{\rm CEEX}$ intOFF	(b) KORALZ 4.02	(c) KORALZ 4.04	(b-a)/a	(c-a)/a
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.1$				
u	3.1761 ± 0.0034	3.1640 ± 0.0198	3.1831 ± 0.0099	-0.0038 ± 0.0063	0.0022 ± 0.0033
d	4.9917 ± 0.0045	5.0101 ± 0.0247	4.9860 ± 0.0123	0.0037 ± 0.0050	-0.0011 ± 0.0026
s	3.1753 ± 0.0034	3.1639 ± 0.0198	3.1831 ± 0.0099	-0.0036 ± 0.0063	0.0025 ± 0.0033
c	4.9918 ± 0.0045	5.0098 ± 0.0247	4.9860 ± 0.0123	0.0036 ± 0.0050	-0.0012 ± 0.0026
b	3.1786 ± 0.0034	3.1036 ± 0.0194	3.1274 ± 0.0097	-0.0236 ± 0.0062	-0.0161 ± 0.0032
all	19.5135 ± 0.0087	19.4515 ± 0.0488	19.4656 ± 0.0243	-0.0032 ± 0.0025	-0.0025 ± 0.0013
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.2$				
u	3.4653 ± 0.0035	3.4523 ± 0.0207	3.4697 ± 0.0103	-0.0037 ± 0.0061	0.0013 ± 0.0031
d	5.4305 ± 0.0047	5.4465 ± 0.0258	5.4225 ± 0.0128	0.0030 ± 0.0048	-0.0015 ± 0.0025
s	3.4644 ± 0.0035	3.4522 ± 0.0207	3.4697 ± 0.0103	-0.0035 ± 0.0061	0.0015 ± 0.0031
c	5.4301 ± 0.0047	5.4463 ± 0.0258	5.4225 ± 0.0128	0.0030 ± 0.0048	-0.0014 ± 0.0025
b	3.4640 ± 0.0035	3.3880 ± 0.0203	3.4083 ± 0.0101	-0.0219 ± 0.0059	-0.0161 ± 0.0031
all	21.2542 ± 0.0090	21.1854 ± 0.0510	21.1927 ± 0.0254	-0.0032 ± 0.0024	-0.0029 ± 0.0013
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.3$				
u	3.6654 ± 0.0036	3.6587 ± 0.0213	3.6738 ± 0.0106	-0.0018 ± 0.0059	0.0023 ± 0.0031
d	5.7258 ± 0.0047	5.7361 ± 0.0265	5.7124 ± 0.0132	0.0018 ± 0.0047	-0.0023 ± 0.0024
s	3.6645 ± 0.0036	3.6587 ± 0.0213	3.6738 ± 0.0106	-0.0016 ± 0.0059	0.0025 ± 0.0031
c	5.7238 ± 0.0047	5.7359 ± 0.0265	5.7124 ± 0.0132	0.0021 ± 0.0047	-0.0020 ± 0.0024
b	3.6641 ± 0.0036	3.5894 ± 0.0209	3.6086 ± 0.0104	-0.0204 ± 0.0058	-0.0152 ± 0.0030
all	22.4436 ± 0.0091	22.3787 ± 0.0524	22.3809 ± 0.0261	-0.0029 ± 0.0024	-0.0028 ± 0.0012
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.9$				
u	19.5444 ± 0.0054	19.5391 ± 0.0499	19.3552 ± 0.0247	-0.0003 ± 0.0026	-0.0097 ± 0.0013
d	18.6747 ± 0.0059	18.6464 ± 0.0487	18.5010 ± 0.0242	-0.0015 ± 0.0026	-0.0093 ± 0.0013
s	19.5439 ± 0.0054	19.5394 ± 0.0499	19.3552 ± 0.0247	-0.0002 ± 0.0026	-0.0097 ± 0.0013
c	18.6673 ± 0.0059	18.6464 ± 0.0487	18.5010 ± 0.0242	-0.0011 ± 0.0026	-0.0089 ± 0.0013
b	19.0477 ± 0.0053	19.1046 ± 0.0488	18.9289 ± 0.0242	0.0030 ± 0.0026	-0.0062 ± 0.0013
all	95.4779 ± 0.0125	95.4759 ± 0.1101	94.6413 ± 0.0546	0.0000 ± 0.0012	-0.0088 ± 0.0006
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.99$				
u	19.7046 ± 0.0054	19.6951 ± 0.0095	19.5137 ± 0.0048	-0.0005 ± 0.0006	-0.0097 ± 0.0004
d	19.2592 ± 0.0059	19.2389 ± 0.0120	19.0889 ± 0.0060	-0.0011 ± 0.0007	-0.0088 ± 0.0004
s	19.7043 ± 0.0054	19.6954 ± 0.0095	19.5137 ± 0.0048	-0.0004 ± 0.0006	-0.0097 ± 0.0004
c	19.2518 ± 0.0059	19.2390 ± 0.0120	19.0889 ± 0.0060	-0.0007 ± 0.0007	-0.0085 ± 0.0004
b	19.2042 ± 0.0054	19.2577 ± 0.0092	19.0852 ± 0.0046	0.0028 ± 0.0006	-0.0062 ± 0.0004
all	97.1242 ± 0.0126	97.1261 ± 0.0235	96.2903 ± 0.0118	0.0000 ± 0.0003	-0.0086 ± 0.0002

$\mathcal{KK}$MC agrees with KORALZ 4.02 and 4.04

generally better than 1%, except b quark.

CEEX Numerical Results

Summary on numerical results

- **Technical Precision** $\frac{\delta\sigma}{\sigma} \leq 2 \cdot 10^{-4}$
- **Physical Precision:**

  $\frac{\delta\sigma}{\sigma} \leq 0.2\%$ **and** $\delta A_{FB} \leq 0.1\%$
- **ISR$\otimes$FSR interf. under control at LEP2 energies**

CEEX - Summary and Outlook

CEEX offers clear upgrade path for the precision calculations of the h.o. radiative corrections in the SM/QED, with the Monte Carlo implementation. It goes beyond (extends) the classical Yennie-Frautschi-Suura scheme. $\mathcal{KK}$ MC based on CEEX offers clear upgrade path for the MC event generators to Linear Collider era. CEEX is firmly based on spin amplitudes. The main profits are:

- **Interferences ISR$\otimes$FSR included and under firm control**
- **All kind of coherence effects, including narrow resonances.**
- **Complete treatment of spin (also transverse) for beams and final (unstable) fermions.**
- **Exact M.E. for 2 high hard photons (3 photons pending).**
- **Future extension to all-angle second order Bhabha is possible.**

First official version 4.13 of $\mathcal{KK}$ MC available. It works for fermion pair production at LEP, LC's, τ and b factories, μ-colliders.

Check **http://home.cern.ch/jadach** for slides, programs.