Fermion Pair Production with ZFITTER at e^+e^- Colliders

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LOOPS AND LEGS IN QUANTUM FIELD THEORY
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In collaboration with
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D. Bardin, P. Christova, L. Kalinovskaya (JINR Dubna),
A. Arbuzov (Uni Torino), and S. Jadach (CERN)

Fermion Pair Production

with

ZFITTER

at e^+e^- Colliders

Contents:

- History
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  - ⋆ Hard-Photon Phase Space with Acollinearity Cut
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- Conclusions

Fermion pairs and radiative corrections

⋆ High precision measurements to the SM $\to M_H$, M_{SUSY}

⋆ Limits for New Physics:

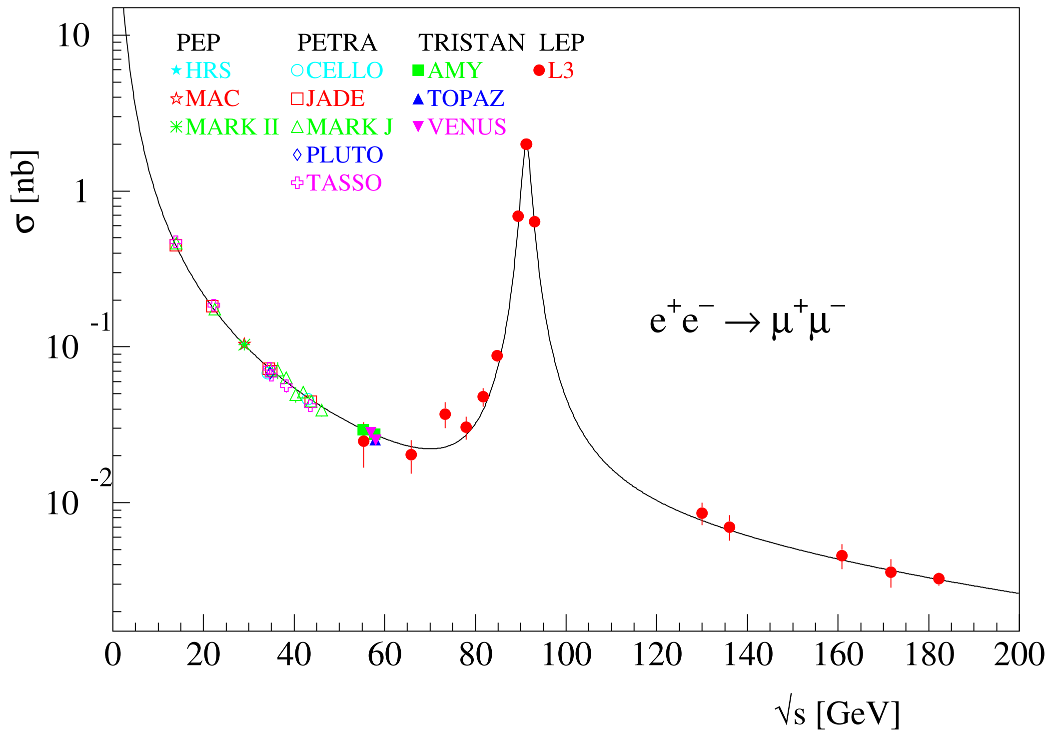
- Z'-bosons (e.g.: E_6 GUT): limits on $M_{Z'}$, $\theta_{Z-Z'}$:
  $M_{Z'} > 270 - 805$ GeV, $\theta_{Z-Z'} < 0.003$
- Contact interactions: $\Lambda > O(1.6 \ldots 9.3\,\text{TeV})$
- Exchange of Leptoquarks, SUSY particles: lower limits on
  λ_{ijk} for $m_{\tilde{\nu}} \approx 90 \ldots 220$ GeV and $m_{LQ} \approx 120 \ldots 430$ GeV
- Extra dimensions, Kaluza-Klein towers, graviton exchange, etc.:
  $M_{Pl} = \mathcal{R}^n M_S^{n+2} \to M_S > 0.7 \ldots 1$ TeV

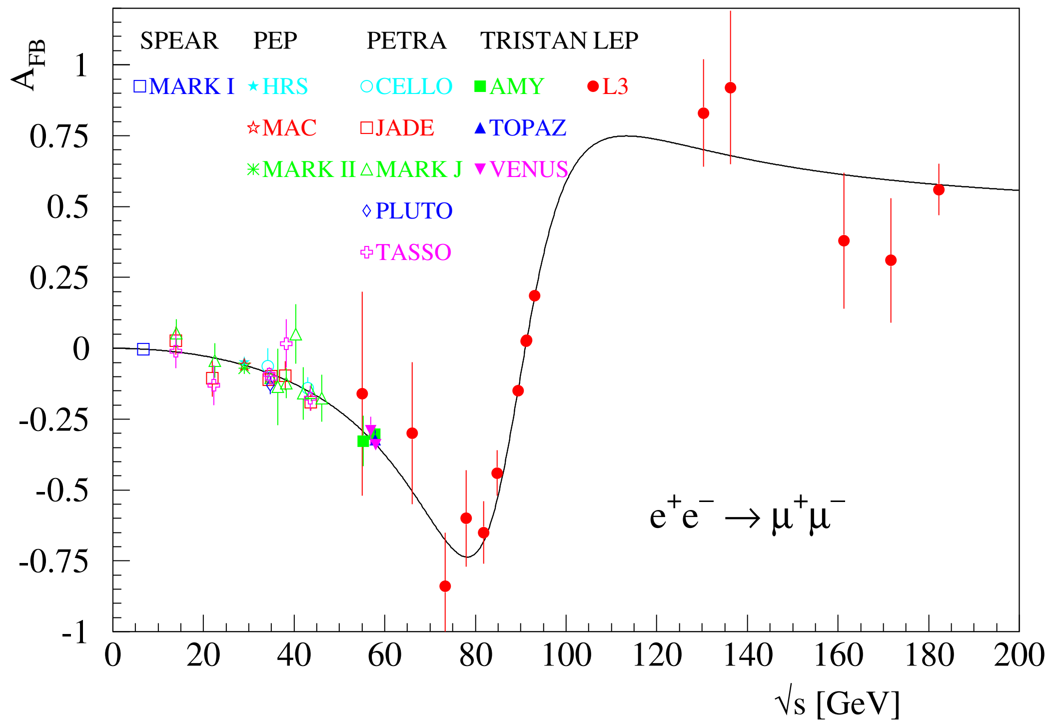
LEP 1:	M. Acciarri et al. (L3) [hep-ex/0002046]
	D. Abbaneo et al. (LEP) CERN-EP-2000-016
LEP 2:	G. Abbiendi et al. (OPAL) [hep-ex/9908008]
Extra Dim.:	M. Acciarri et al. (L3) PLB470 (1999) 281

⋆ Estimated precisions for single cross section contributions:

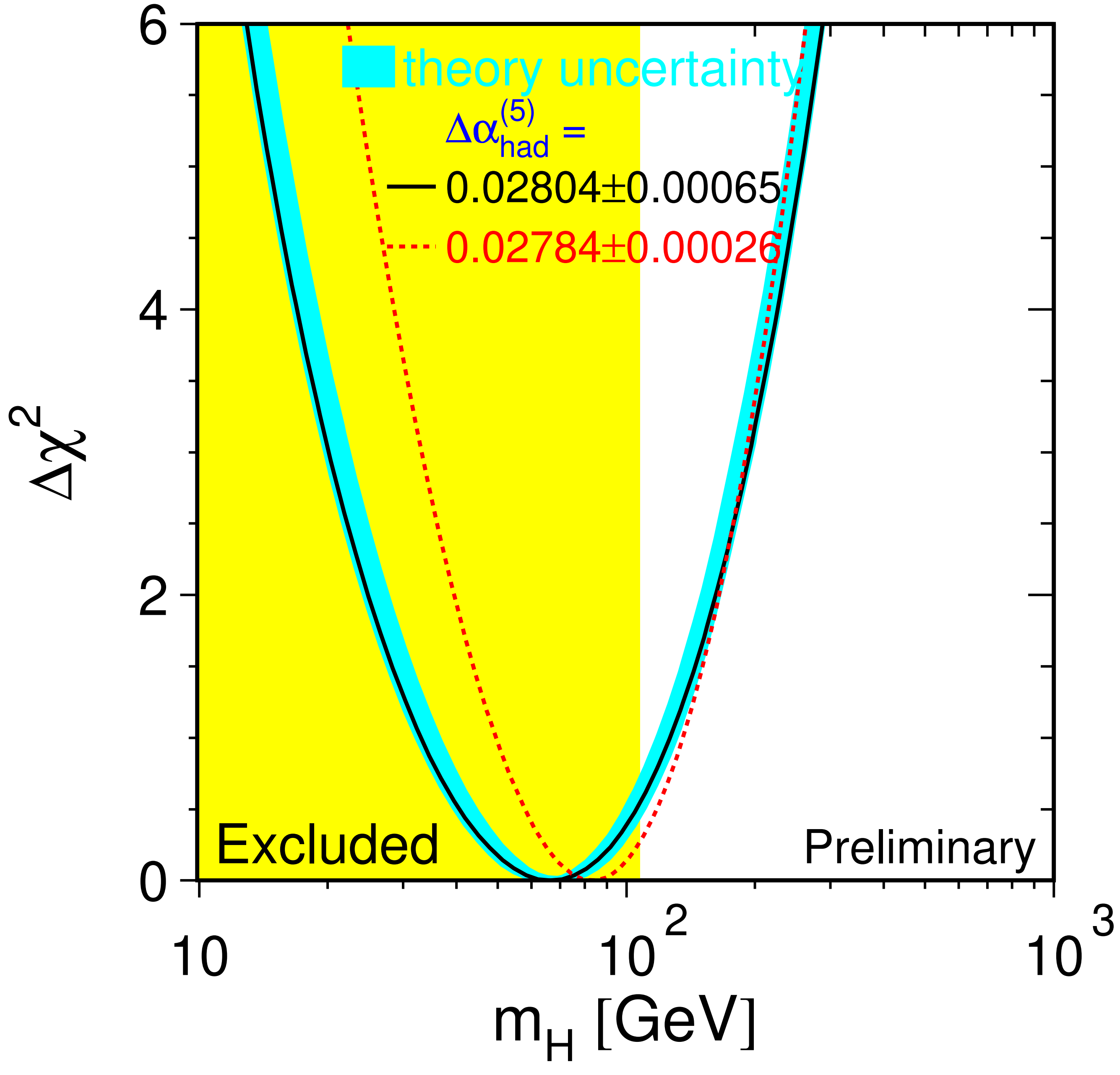
LEP 1: $\delta\sigma/\sigma \approx 0.01\%$ $\to$ ‘Giga Z’ at LC
LEP 2: $\delta\sigma/\sigma \approx 0.1 - 1\%$ $\to$ LC high energy modes

$\longrightarrow$ treatment of radiative corrections
at LEP/LC with per mil level accuracy or better





Muon pair production cross sections and forward-backward asymmetries.



χ^2 distribution of global fit to electroweak data (preliminary).

De-convoluted, realistic observables

Due to the many radiative corrections, the measured cross sections $\sigma(s)$ deviate considerably from the Born cross sections $\sigma_0(s')$ which describe the physics phenomenon to be studied.

In many cases, one may arrange the formulae with the following general structure, introducing some radiator(s) $\rho(s'/s)$:

$$\sigma(s) = \int \frac{ds'}{s} \; \sigma_0(s') \; \rho(s'/s)$$

$s' = \left(p_f + p_{\bar{f}}\right)^2$: invariant mass sq. of the final state fermions $f, \bar{f}$

Such radiators depend on:

- the process studied (e.g. lepton pair production)
- the measured observable (e.g. σ_{tot}, A_{FB})
- cut conditions (e.g. s' cut - i.e. cut on photon energy)
- kind of corrections (e.g. only initial state radiation)
- the order in perturbation theory (e.g. second order LLA)

The photonic corrections consist of Born-like virtual+soft parts S, photonic box parts B, and hard parts H $\quad (v = 1 - R,\ R = s'/s)$.

A more realistic cross section formula is e.g. (ZFITTER):

$$\sigma_T(c) = \int_0^{1-4m_f^2/s} dv$$

$$\left\{ \left[\sigma_T^0(s',c) \left(1 + \bar{S}^{ini}\right) \beta_e v^{\beta_e - 1} + \sigma_T^0(s') \bar{H}_T^{ini}(v,c) \right] \bar{R}_T^{fin}(v) \right.$$

$$\left. + \, \sigma_{FB}^0(s,s') \left[H_T^{int}(v,c) - \frac{\sigma_{FB}^0(s)}{\sigma_{FB}^0(s,s')} H_T^{int,sing}(v,c) \right] \right\}$$

$$+ \, \sigma_{FB}^0(s,c) \bar{S}_T^{int} + \sum_{m,n=\gamma,Z} \sigma_{FB}^0(s,s,m,n) B_T(c,m,n).$$

Example: Classical result for initial state radiation ($c = 1$):

$$\sigma_T^{ini}(s) = \int_0^{1-4m_f^2/s} dR \, \sigma_T^0(s') \, \rho_T^{ini}(R),$$

$$\rho_T^{ini}(R) = \left(1 + \bar{S}\right) \beta_e v^{\beta_e - 1} + \bar{H}_T^{ini}(R),$$

$$\bar{S} = \frac{3}{4}\beta_e + \frac{\alpha}{\pi} Q_e^2 \left(\frac{\pi^2}{3} - \frac{1}{2} \right) + h.o. \quad , \quad \beta_e = \frac{2\alpha}{\pi} Q_e^2 \left(\ln \frac{s}{m_e^2} - 1 \right),$$

$$\bar{H}_T^{ini}(R) = \left[\bar{H}_{BM}(R) - \frac{\beta_e}{v} \right] + h.o. \quad , \quad \bar{H}_{BM}(R) = \frac{1}{2} \frac{1+R^2}{v} \beta_e.$$

See: Bonneau, Martin NPB 27 (1971), Greco et al. NPB 101 (1975)

The ZFITTER concept

$$\sigma \sim \int \frac{ds'}{s} \; \sigma_0(s') \;\; \rho(s'/s)$$

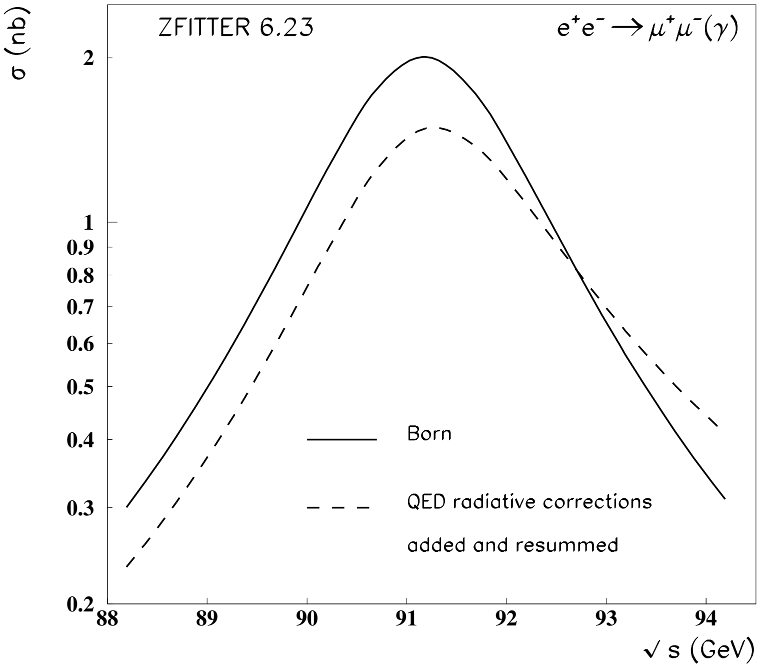
One-dimensional numeric integration for 3 treatments of photonic corr's.:

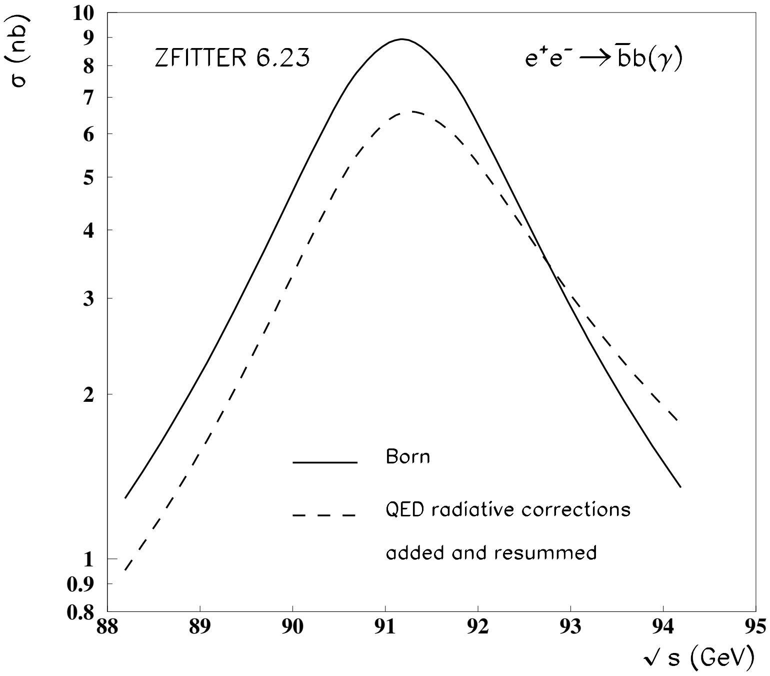
- no cut
- cut on s' and on $\cos\vartheta$
- cut on θ_{acol} and on $E_{\min}$ and on $\cos\vartheta$

Analytic effective Born $\sigma_0(s')$:

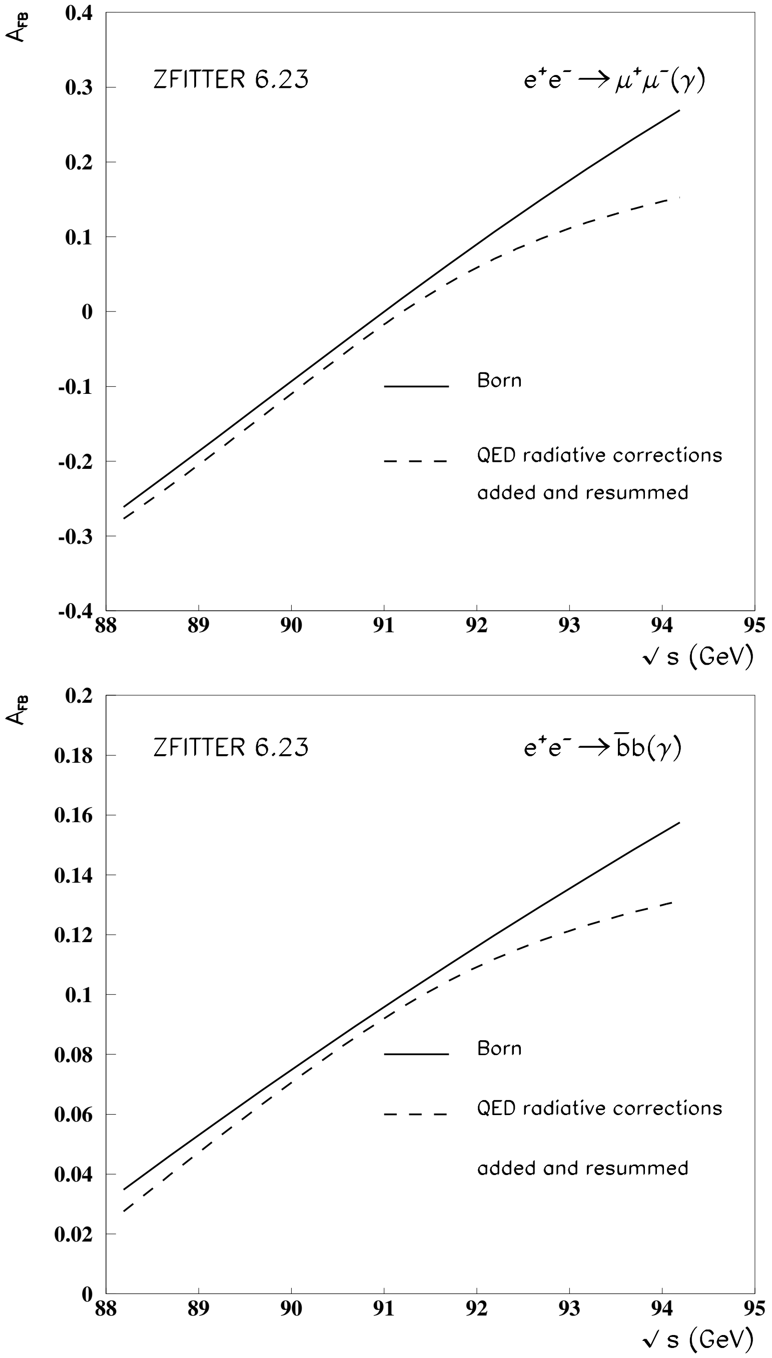
- Standard Model
- Model Independent
- others, e.g. with a heavy Z' (add code ZEFIT)

'Canonical' references for ZFITTER:
Bardin et al., Christova et al.:
DESY 99-070 [hep-ph/9908433] – for ZFITTER v. 6.21 $\longrightarrow$ CPC
CERN-TH. 6443/92 [hep-ph/9412201] – for ZFITTER v. 4.5
A.B. Arbuzov [hep-ph/9907500] – LL $O(\alpha^3) + O(\alpha^4)$ ini.-state QED pair corr.'s
PLB 456 (1999) 264 – QED corr's. with acollinearity and energies' cut
NPB 351 (1991) 1 – QED corr's. to angular distribution, $d\sigma/d\cos\theta$
PLB 255 (1991) 290 – QED corr's. to σ_T and A_{FB} with cuts
ZfPC 44 (1989) 493 – weak corr's. to Z lineshape
http://www.ifh.de/theory/publist.html

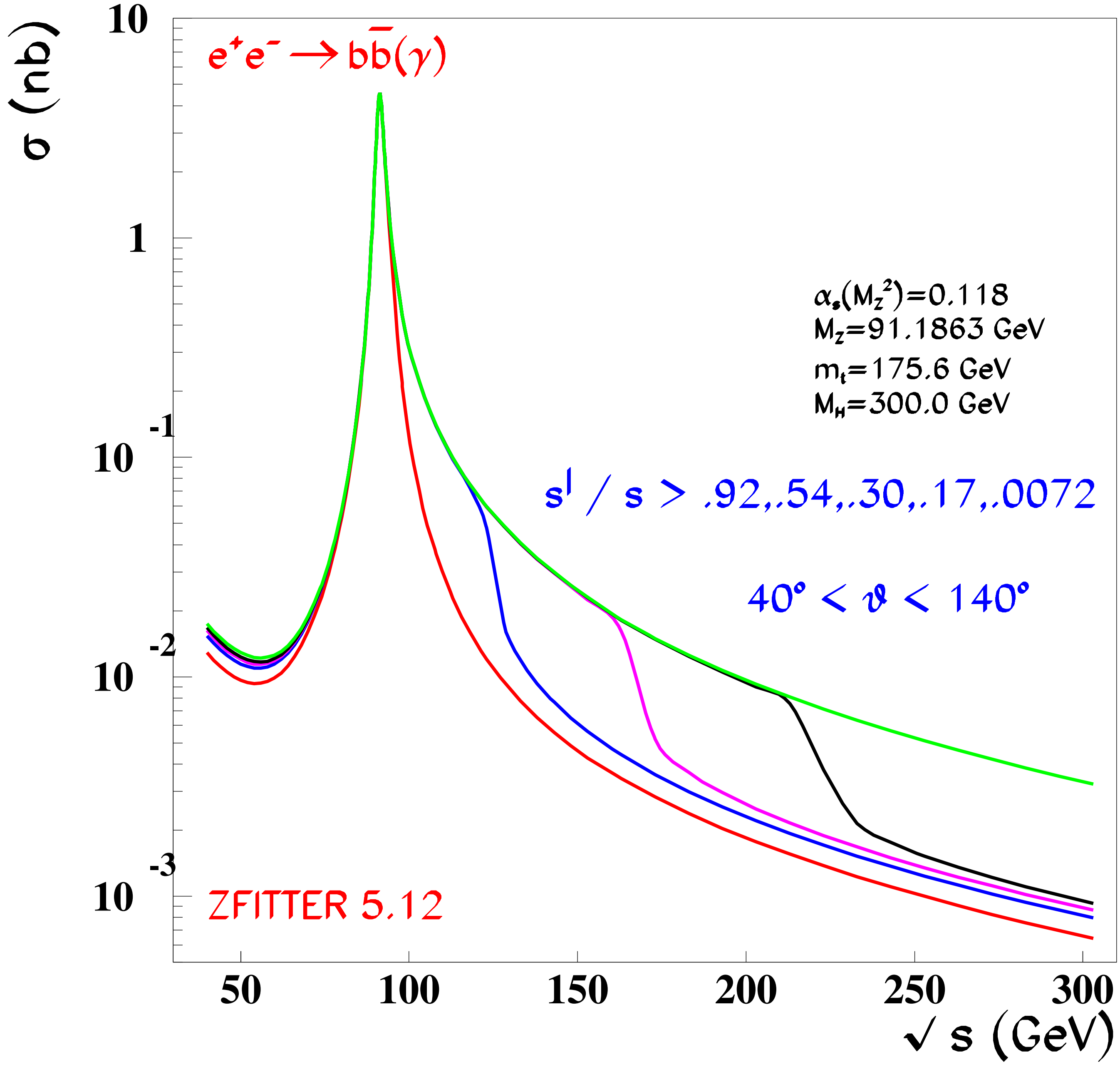




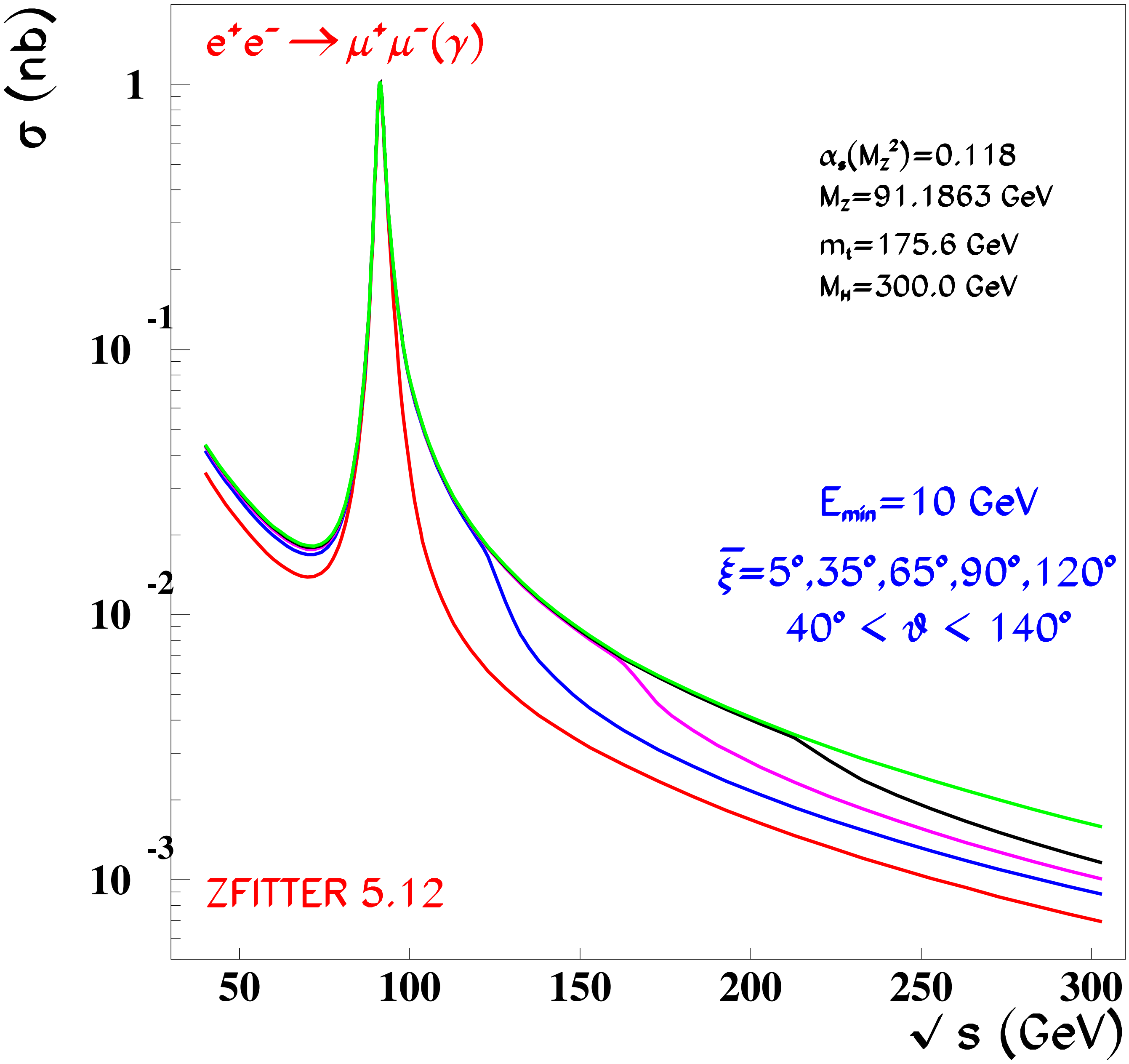
Born and QED corrected Z lineshapes for myon and b quark pair production calculated with ZFITTER v.6.23. QED corrections include full $O(\alpha^2)$ and leading logarithmic $O(\alpha^3)$ corrections with soft- and virtual-photon exponentiation for initial-state Bremsstrahlung.



Born and QED corrected forward-backward asymmetries at the Z peak for myon and b quark pair production calculated with ZFITTER v.6.23. QED corrections include full $O(\alpha^2)$ and leading logarithmic $O(\alpha^3)$ corrections with soft- and virtual-photon exponentiation for initial-state Bremsstrahlung.



b-quark pair production cross-sections with different s' cuts



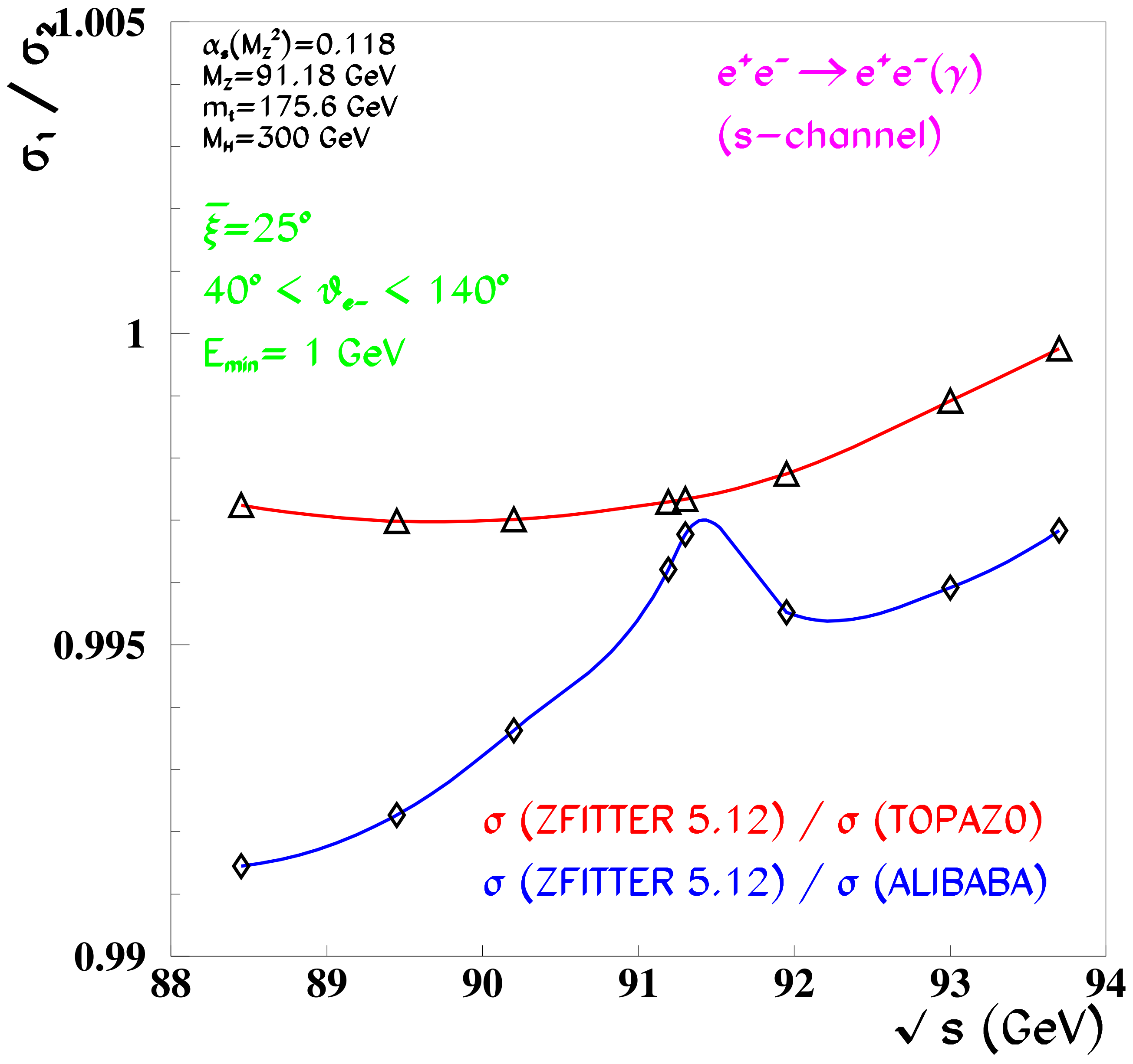
Muon pair production cross-sections with different acollinearity cuts

Experimental demands and different 2-fermion codes

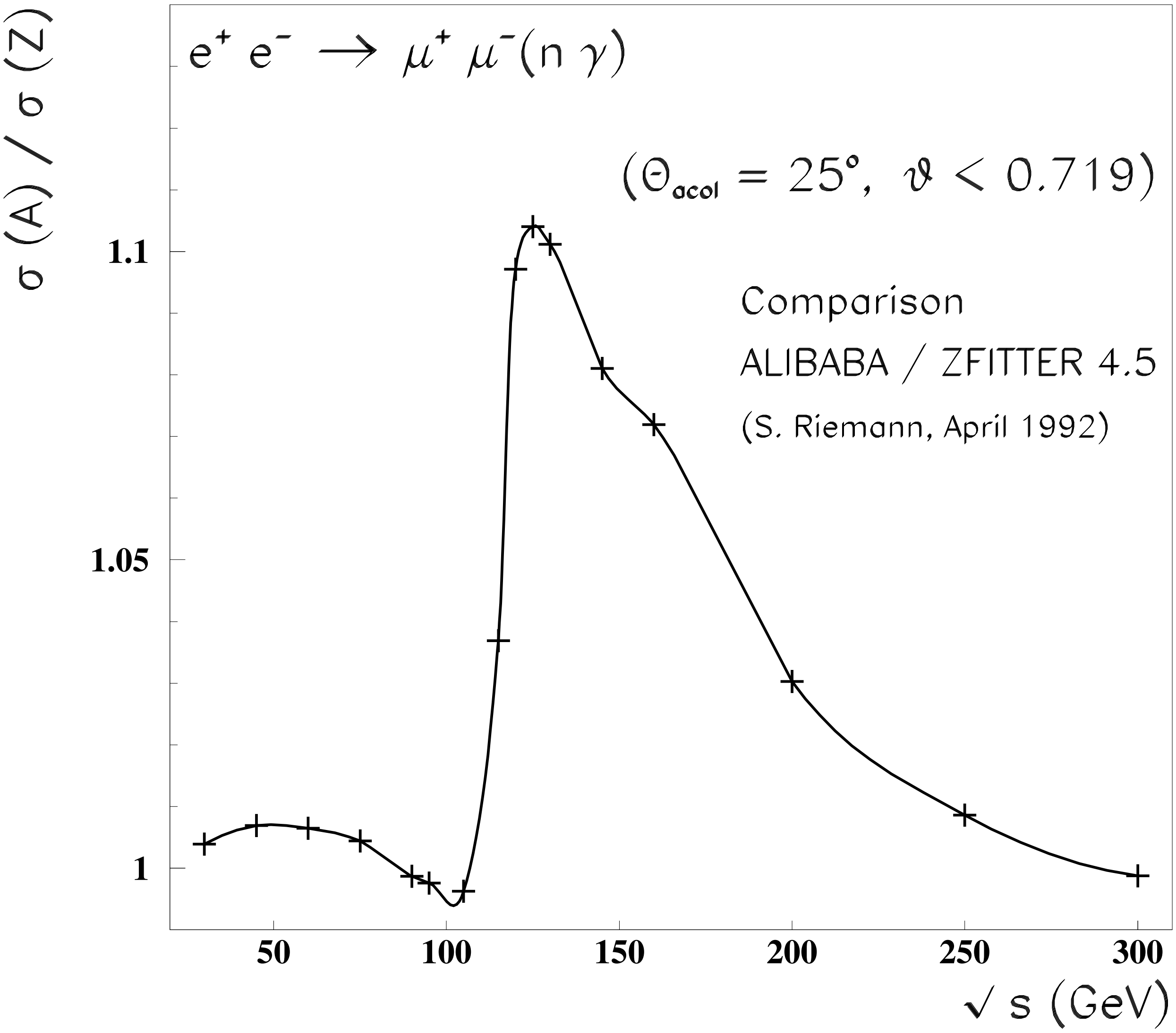
- Demands from experiment:
  - level of accuracy at the Z peak: $O(< 10^{-4} \ldots 10^{-3})$
    e.g. $M_Z = 91.1867 \pm 0.0021$; $\sin^2\vartheta_{eff} = 0.23189 \pm 0.00024$
    CERN EP/99-15 (Feb. 1999)
  - expected level of accuracy at LEP 2, LC: $O(3 \cdot 10^{-3})$
  - codes with realistic cuts, inclusion of higher orders necessary
  - quick, numerically precise and stable codes necessary for $\sqrt{s} = 30 - 500, 800$ GeV
- Situation with codes now:
  e.g. ZFITTER (semi-ana.) vs. TOPAZ0, KK2f, ALIBABA (num.)
  - s' cut:
    * perfect agreement at LEP 1 ($M_Z \pm 3$ GeV): $O(< 10^{-4})$ (PCP Report)
    * LEP 2 ($\sqrt{s} = 189$ GeV): deviations less than $1 \ldots 2 \cdot 10^{-3}$ (LEP2 MCWS, March 2000)
  - Acoll. cut: nice agreement for INI at LEP 1, but agreement deteriorates with INTF: $O(\approx 3 \cdot 10^{-3})$; large discrepancies at higher energies ($\sqrt{s} = 100 \ldots 200$ GeV): $O(\text{few } \%)$

References:

Christova et al. DESY 98-184, hep-ph/9812412
Christova et al. PLB456, DESY 99-015, hep-ph/9902408
Bardin et al. PCP Report, hep-ph/9902452
Christova et al. DESY 99-037, hep-ph/9908289
Jack, Riemann hep-ph/9909401
Jack DESY 99-166, hep-ph/9911296
Christova et al. LC-TH-2000-008, hep-ph/0002054



Ratios of s-channel parts of Bhabha scattering: ZFITTER v.5.12 vs. TOPAZ0 and ALIBABA (both latter from PLB 425, 1998).



S. Riemann, April 1992, unpublished:
Muon pair production cross section ratio with acollinearity cut –
ALIBABA_1990 (v.1/2) vs. ZFITTER 4.5 (1992)

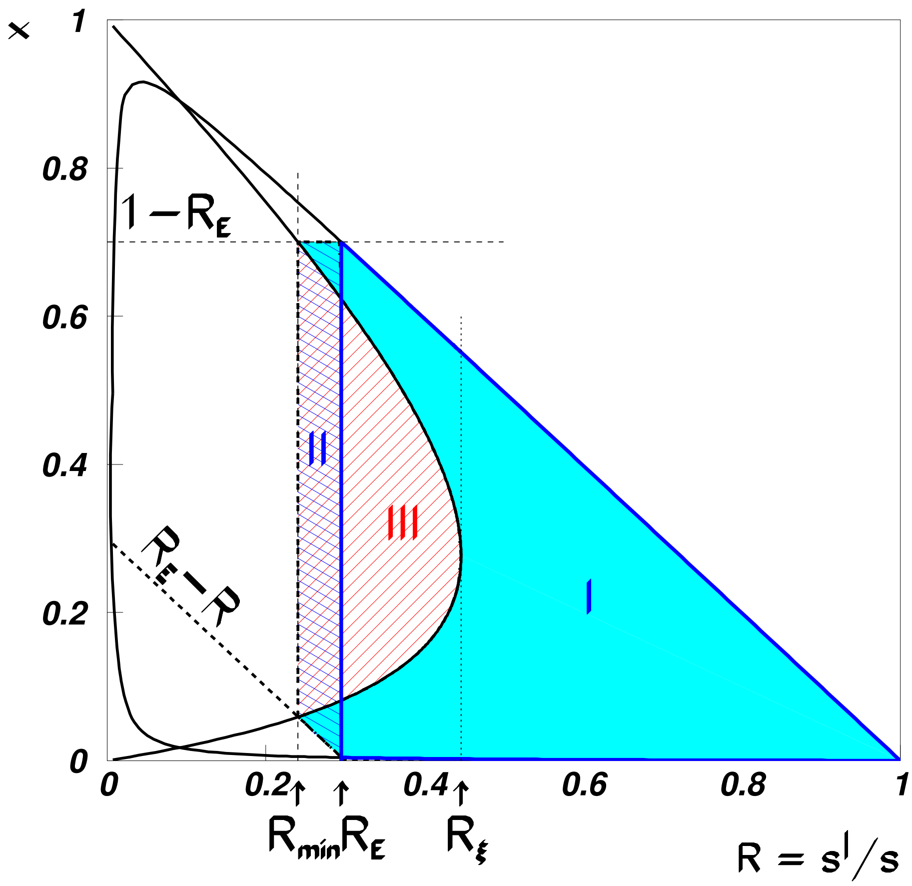
QED corrections with cuts: New Results

P. Christova, M. Jack, T. Riemann: PLB 456 (1999) 264

Cuts on:

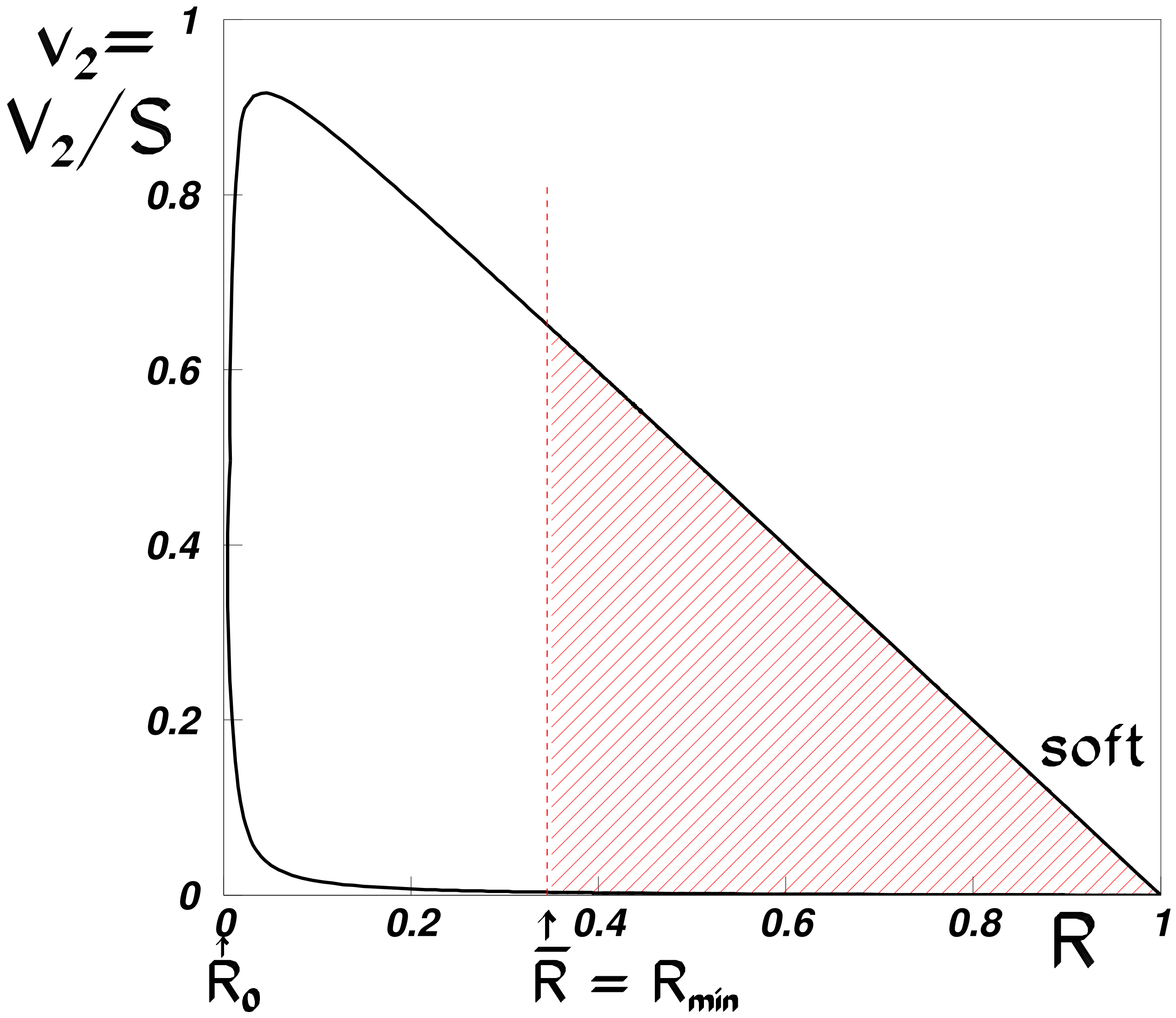
- maximal acollinearity angle: $0 \le \xi (= \theta_{acol}) \le \xi_{\max}$
- minimal energies of final-state fermions: $E_{f,\bar{f}} \ge E_{\min}$
- min./max. scattering angle of one fermion: $\vartheta_{\min} \le \vartheta \le \vartheta_{\max}$

$$\sigma^h = \sigma^h(I) + \sigma^h(II) - \sigma^h(III)$$



Phase space with acollinearity cut and (lower) muon energy cuts
See e.g.: G. Passarino NPB 204 (1982)

$$\sigma^h = \sigma^h(I)$$



Phase space with s' cut

The cross-section is a sum over three regions in phase space:

$$\sigma(s) = \left[\int_{\mathrm{I}} + \int_{\mathrm{II}} - \int_{\mathrm{III}}\right] ds' dx d\cos\vartheta \frac{d\sigma^0(A)}{ds' dx d\cos\vartheta}.$$

In the three regions the parameter $A(R)$ differs ($R = \frac{s'}{s}$):

$$\begin{aligned} A_{\mathrm{I}}(R) &= [1 - 4m_f^2/(s \cdot R)]^{1/2} \approx 1, \\ A_{\mathrm{II}}(R) &= (1 + R - 2R_E)/(1 - R), \\ A_{\mathrm{III}}(R) &= [1 - R(1 - R_\xi)^2/(R_\xi(1 - R)^2)]^{1/2}, \end{aligned}$$

with

$$\begin{aligned} R_E &= 2E_{min}/\sqrt{s}, \\ R_\xi &= (1 - \sin(\xi^{\max}/2))/(1 + \sin(\xi^{\max}/2)). \end{aligned}$$

For each of the three regions, it is generically
e.g. for initial-state radiation:

$$\begin{aligned} \sigma_T^{ini}(s, \xi^{\max}, E_{min}) &= \frac{\alpha}{\pi} Q_e^2 \int dR\, \sigma_T^0(s')\, \rho_T^{ini}(R, A(R)), \\ A_{FB}^{ini}(s, \xi^{\max}, E_{min}) &= \frac{\alpha}{\pi} Q_e^2 \int dR\, A_{FB}^0(s')\, \rho_{FB}^{ini}(R, A(R)). \end{aligned}$$

For $A \to 1$, regions II and III do not contribute and $\rho_{T,FB}^{ini}(R, A)$ approach the well-known results derived in Bonneau, Martin (1971).
If one looks into the limit $c = 1$ (no acceptance cut), but arbitrary acollinearity cut $A(R)$, the hard flux functions are rather compact.

With s' cut see: D. Bardin et al. PLB 229 (1989).
With acol. cut see: P. Christova et al. DESY 99-015, hep-ph/9902408

Initial-State Radiation

Hard parts $\bar{H}^{ini}_{T,FB}$ of $\rho^{ini}_{T,FB}$ for full acceptance $0 \le \vartheta \le 180°$ ($R \equiv s'/s$):

$$\frac{4}{3}\cdot \bar{H}^{ini}_{T}(R,A) = \left(A+\frac{A^3}{3}\right)\frac{1+R^2}{1-R}\left(\ln\frac{s}{m_e^2}-1\right)+(A-A^3)\frac{2R}{1-R},$$

$$\begin{aligned}\bar{H}^{ini}_{FB}(R;A\ge A_0) = {} & \frac{1+R^2}{1-R}\left[\frac{4R}{(1+R)^2}\left(\ln\frac{s(1+R)^2}{4m_e^2R}-1\right)\right. \\ & -\frac{1}{(1+R)^2}\left[y_+y_-\ln|y_+y_-|+4R\ln(4R)\right] \\ & \left.-(1-A^2)\left(\ln\frac{s}{4m_e^2(1+A)^2R}-1\right)\right] \\ & +\frac{4A(1-A)R}{1-R},\end{aligned}$$

$$\begin{aligned}\bar{H}^{ini}_{FB}(R;A< A_0) = {} & \frac{1+R^2}{1-R}\left[-\frac{y_+y_-}{(1+R)^2}\ln\left|\frac{y_+}{y_-}\right|+(1-A^2)\ln\frac{1+A}{1-A}\right] \\ & +\frac{8AR^2}{(1+R)(1-R)}.\end{aligned}$$

$$A = A_{I,II,III}(R;\xi_{\max},E_{\min},s_{\min})$$

$$y_\pm = (1+R)\cdot(A_0\pm A),\qquad A_0(R)=(1-R)/(1+R).$$

Results with cuts on maximal acollinearity $\xi_{\max}$, minimal energies E_{min}, minimal/maximal acceptance $\vartheta_{\min,\max}$ and minimal inv. mass squared $s_{\min}$ available.

Photonic Initial-Final State Interference

Numerical comparison of TOPAZ0, ZFITTER-5.20 and ZFITTER-new (versions v.6.x)

Shown are the relative/absolute contributions due to the initial-final state interference:

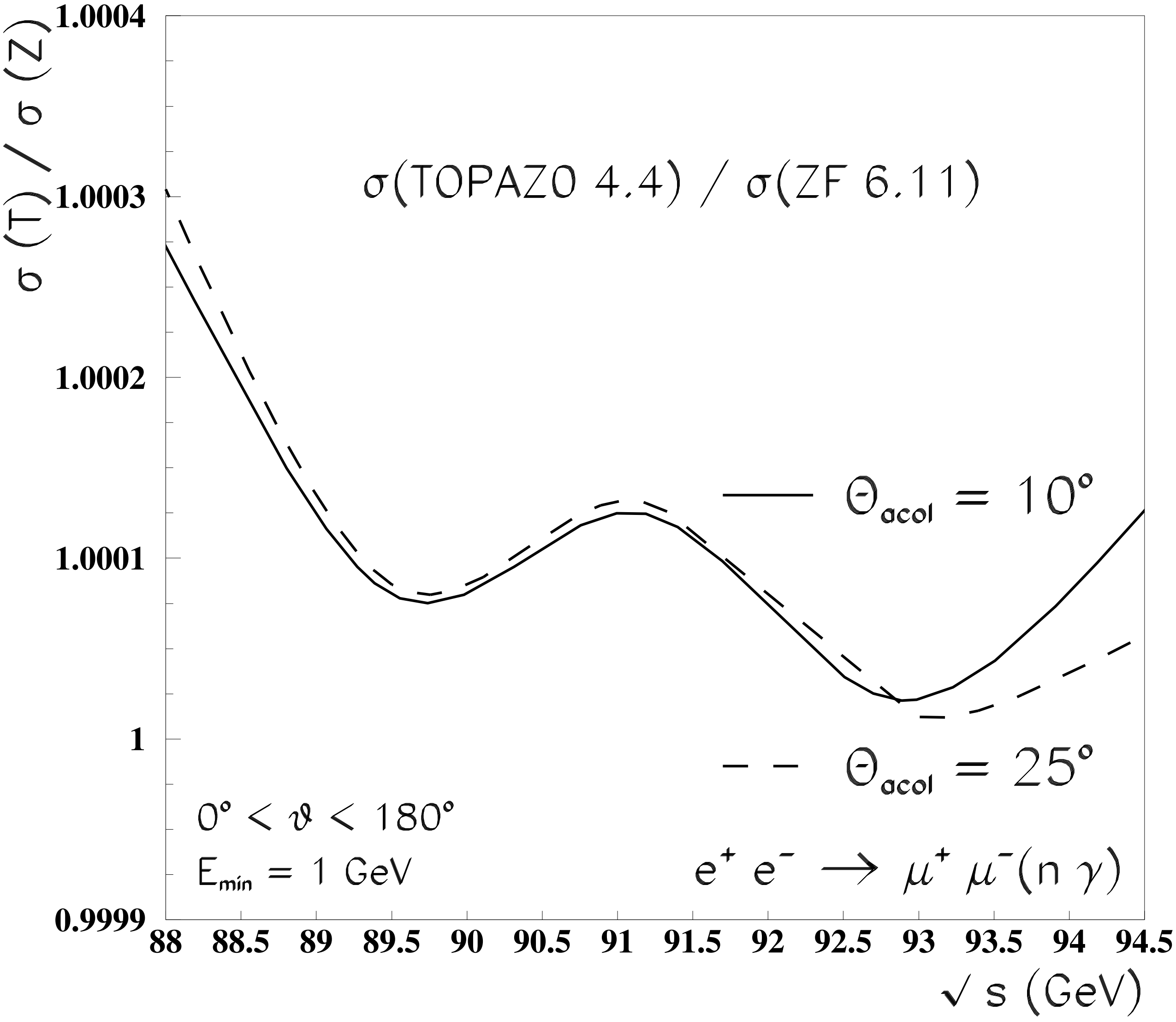
Example: $\xi^{\max} = 10^\circ$, $E_{\min} = 1$ GeV, $0^\circ \leq \theta \leq 180^\circ$

$$\delta\sigma^{int} = \frac{\sigma^{all} - \sigma^{no-interf}}{\sigma^{all}} \times 1000 \quad per \quad mil$$

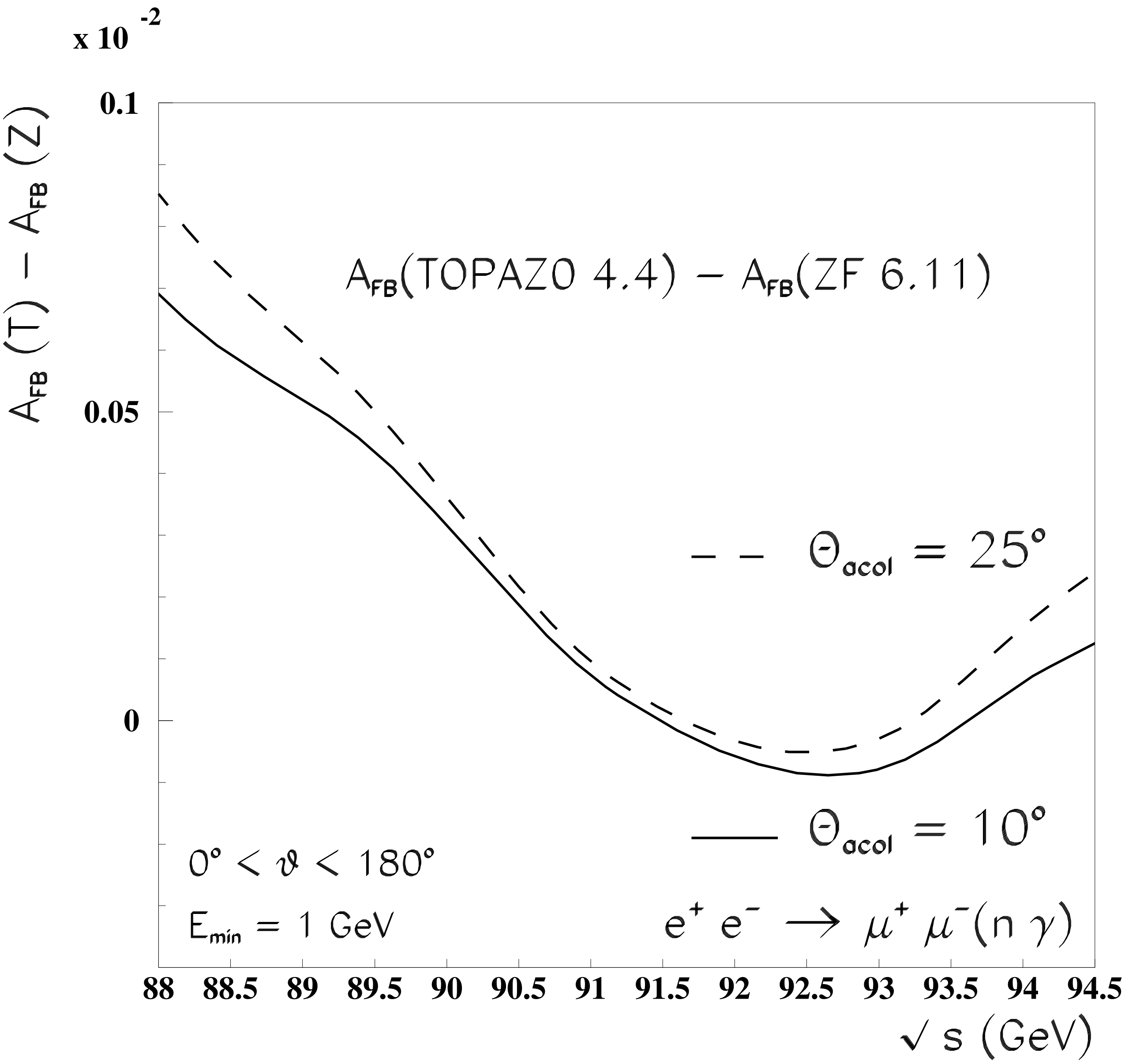
$\delta\sigma$	$M_Z - 3.0$	$M_Z - 1.8$	M_Z	$M_Z + 1.8$	$M_Z + 3.0$
zf520	-3.48	-2.14	0.01	1.36	1.72
topaz0	-7.16	-4.43	-0.07	2.49	3.17
zf-new	-7.16	-4.40	-0.03	2.60	3.27
t0 - zf520	-3.68	-2.29	-0.08	1.13	1.45
t0 - zf-new	0.00	-0.03	-0.04	-0.11	-0.10

$$\delta A_{FB}^{int} = (A_{FB}^{all} - A_{FB}^{no-interf}) \times 1000 \quad per \quad mil$$

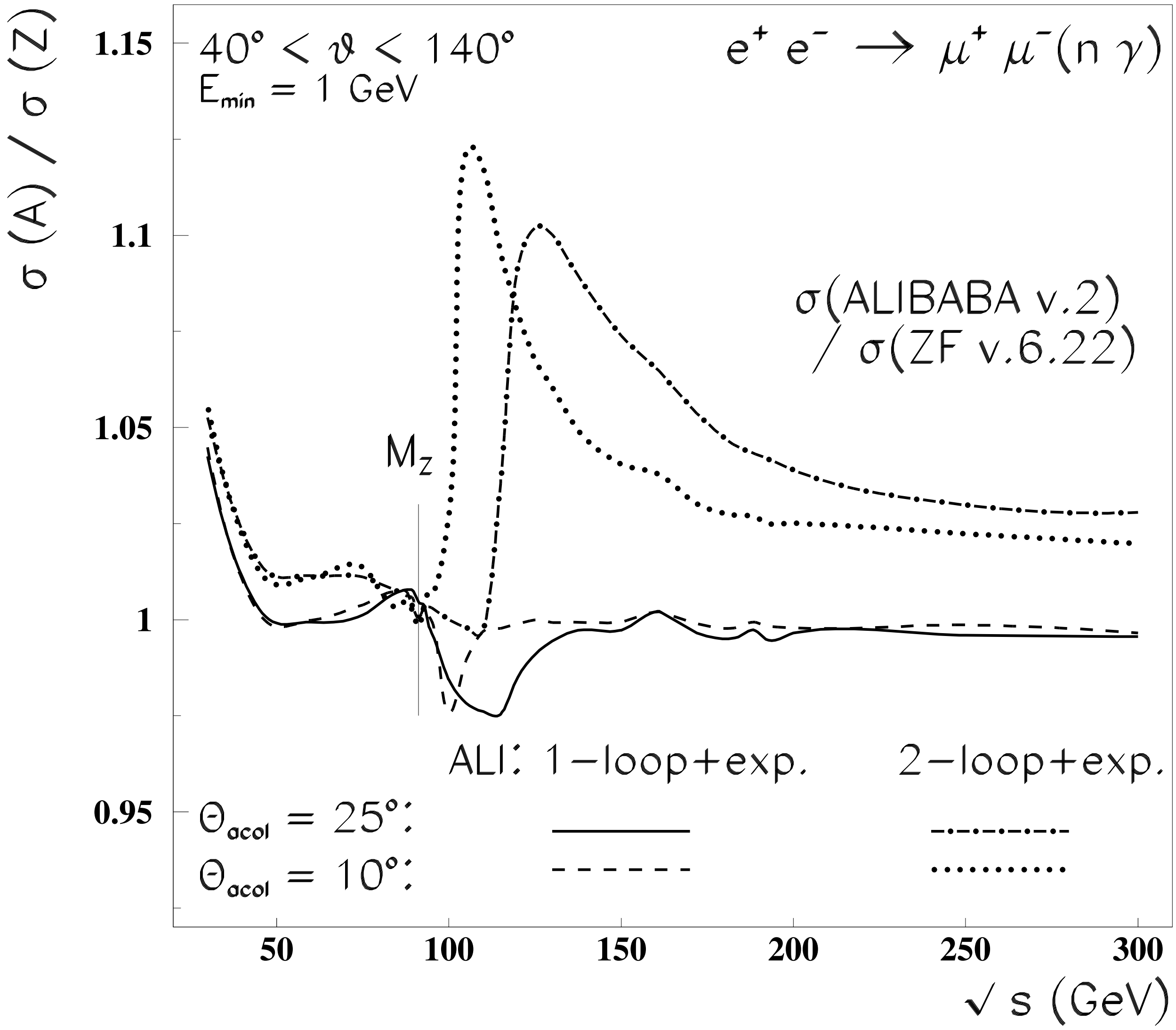
δA_{FB}	$M_Z - 3.0$	$M_Z - 1.8$	M_Z	$M_Z + 1.8$	$M_Z + 3.0$
zf520	1.71	1.28	0.45	-0.11	-0.12
topaz0	2.92	2.49	0.55	-1.27	-1.71
zf-new	2.75	2.27	0.60	-1.03	-1.56
t0 - zf520	1.21	1.21	0.10	-1.16	-1.59
t0 - zf-new	0.17	0.22	-0.05	-0.24	-0.15



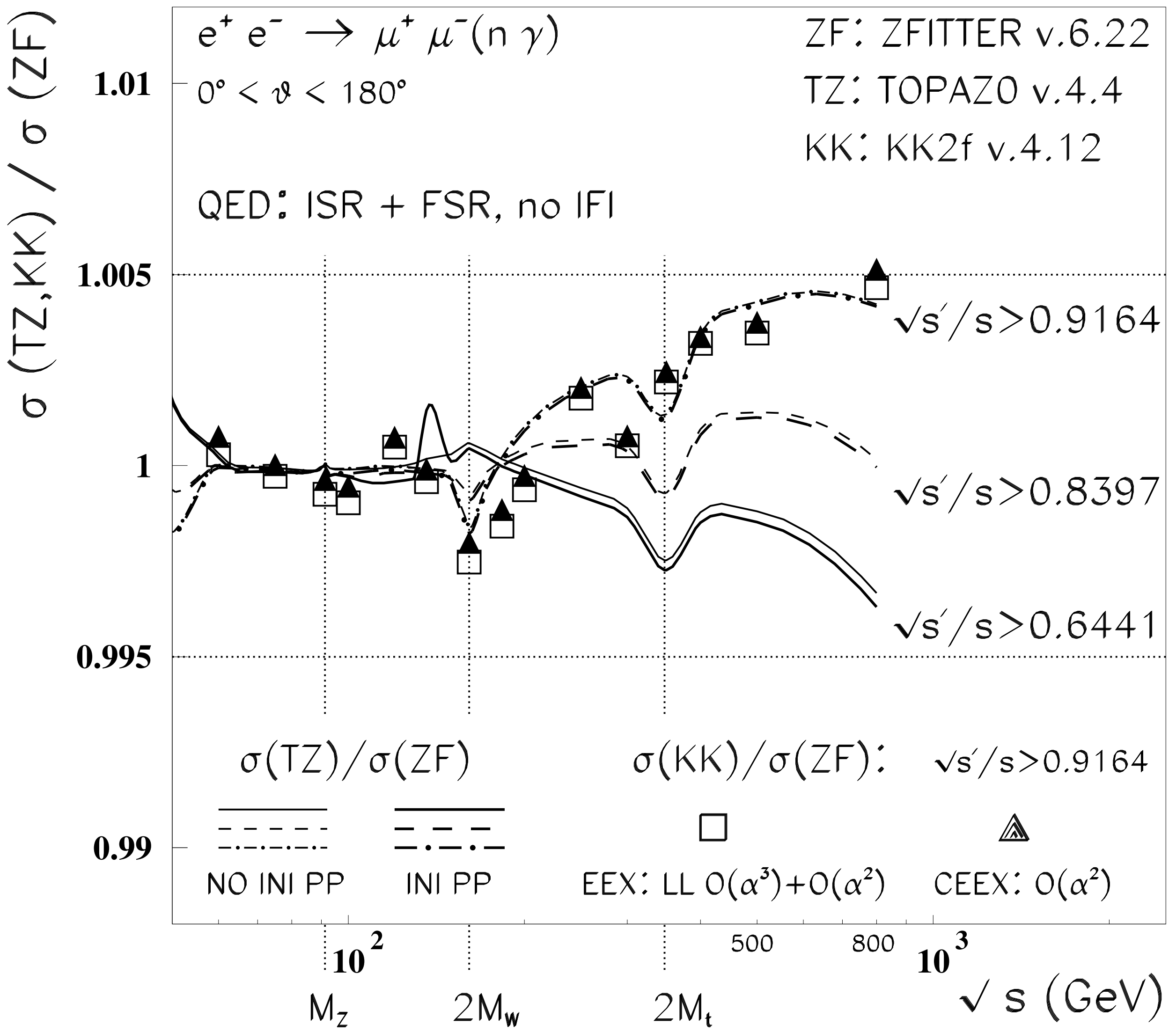
Ratios of cross sections for muon pair production: ZFITTER v. 6.11 vs. TOPAZO v. 4.4 for different acollinearity cuts at the Z resonance



Differences of forward-backward asymmetries for muon pair production: ZFITTER v. 6.11 vs. TOPAZO v. 4.4 for different acollinearity cuts at the Z resonance.



Muon-pair production cross-section ratios with $\theta_{\rm acol}^{\rm max} = 10^\circ, 25^\circ$ and $\theta_{\rm acc} = 40^\circ$: ALIBABA v.2 (1990) versus ZFITTER v.6.22 (Flag setting: ISPP=0).



Cross section ratios for muon-pair production with different s' cuts
for codes ZFITTER v.6.22, TOPAZ0 v.4.4, KK2f v.4.12 (1999)
from 60 to 800 GeV c.m. energy; without initial-final state interference
(INI PP: initial state pair production; LL: leading logarithmic terms).

Bardin, Kalinovskaya and Christova, Leike, Riemann and Hollik:

Top Pair-Production Project for the Linear Collider
A Preliminary Result:

The hard-photon radiators to the
initial-final state QED interference with final-state masses

Hoefer, Leike, Riemann (2000): analytical and 3-fold numerical calc.

$$\begin{aligned}\left(\frac{d\sigma}{ds'}\right)^{int}_{ZZ} &= -16\,\alpha^3\,Q_eQ_f\,\Re e[\Delta_Z(s)\Delta_Z^*(s')]\,v_ea_ev_fa_f\,\frac{1}{s^2}\,\frac{s+s'}{s-s'}\,\beta_f \\ &\quad\times\left[1-\left(\frac{m_f^2}{s'}+\frac{m_f^2}{s}\right)\,\frac{L_f}{\beta_f}\right]+\mathcal{O}(m_e^2)\end{aligned}$$

$$\left(\frac{d\sigma}{ds'}\right)^{int}_{Z\gamma} = -4\,\alpha^3\,Q_e^2Q_f^2\,\Re e\Delta_Z(s')\,a_ea_f\,\frac{1}{s^2}\,\frac{s+s'}{s-s'}\,\beta_f\left[1-2\frac{m_f^2}{s'}\,\frac{L_f}{\beta_f}\right]+\mathcal{O}(m_e^2)$$

$$\left(\frac{d\sigma}{ds'}\right)^{int}_{\gamma\gamma} = 0$$

with

$$\begin{aligned}\Delta_Z(s) &= \frac{G_{NC}M_Z^2}{2\sqrt{2}\pi\alpha}\,\frac{s}{s-M_Z^2+iM_Z\Gamma_Z} \\ L_f &= \log\left(\frac{1+\beta_f}{1-\beta_f}\right) \quad , \quad \beta_f = \sqrt{1-\frac{4m_f^2}{s'}}\end{aligned}$$

- Logarithmic mass terms $\frac{m_f^2}{s}L_f \sim m_f^2\ln m_f^2$ may reach order of several percent
- Use at Linear Collider, but also at DAPHNE

Conclusions

Impressing High-Precision Measurements at LEP, SLC
to the Electroweak Sector of the SM
with stringent bounds on $M_H^{\mathtt{SM,MSSM}}$ + 'New Physics'

$\longrightarrow$ 2-Fermion Codes as High-Precision Tools
containing
20 Years of Precision Calculations
to Electroweak, QCD, and QED Rad. Corr.'s

* *

Precision Studies of 2-Fermion Codes:
New Results to QED Flux Functions with Cuts / with Masses Presented for ZFITTER
$\longrightarrow$ good ... very good agreement of 2-fermion codes
ZFITTER, TOPAZ0, ALIBABA, KK2f:

LEP1: $< 0.01\% \ldots 0.1\%$ LEP2: $\approx 0.1\% \ldots 0.2\%$
LC: $\approx 0.5\%$

* *

LEP and SLC have (nearly) finished, LC is coming !
$\longrightarrow$ Work in progress:

- $\star$ Top pairs above threshold (EW form factors, QED)
- $\star$ QED higher orders, Beamstrahlung etc.
- $\star$ MSSM applications

$\vdots$

'Canonical' references for ZFITTER:

Bardin et al., Christova et al.:

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PLB 229 (1989) 405 – QED corr's. to A_{FB}
ZfPC 44 (1989) 493 – weak corr's. to Z lineshape
http://www.ifh.de/theory/publist.html

TOPAZ0:

Montagna et al.:

CPC 117 (1999) [hep-ph/9804211] – for TOPAZ0 v. 4.4
ZfPC 76 (1997) 45 – analysis of hard-photon NL corr.'s
PLB 406 (1997) 243 – $O(\alpha^3)$ QED radiator
PLB 309 (1993) 436 – final-state corr.'s with realistic cuts
PLB 231 (1989) 492 – higher order QED to A_{FB}
ZfPC 39 (1988) 479 – second order QED (structure function)
PLB 196 (1987) 551 – soft photons and second order QED
NPB 204 (1982) 237 – hard-photon phase space with acollinearity cut
http://www.to.infn.it/~giampier/topaz0.html

KK2f/KORALZ:

Jadach et al.:

CERN-TH 99-235, DESY 99-106 [hep-ph/9912214] – for KK2f v. 4.11
CERN-TH 99-119 [hep-ph/9905205] – for KORALZ v. 4.0 - 4.04
CERN-TH 99-217 (1999) – initial-final state interference in the Z lineshape
PLB 456 (1999) 77 – initial-state corrections at the Z peak
PLB 449 (1999) 97 – coherent exclusive exponentiation
PLB 280 (1992) 129 – light-pair corr.'s to the Z lineshape
PLB 257 (1991) 173 – first + higher order QED to A_{FB}
ZfPC 49 (1991) 577 – electron QED non-singlet structure function
PRD 41 (1990) 1425 – soft-photon resummation
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