Progress towards $2 \to 2$ scattering at two loops

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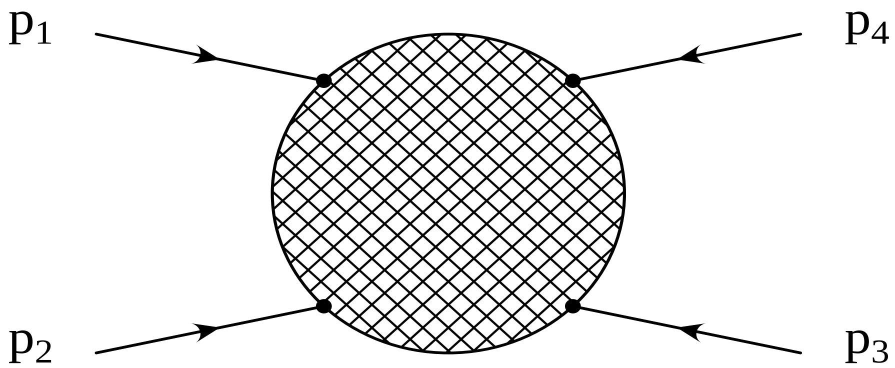
with Maria Elena Tejeda-Yeomans

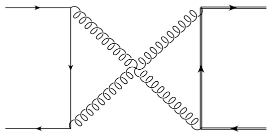


- Overview
- Application: $q\bar{q} \to Q\bar{Q}$
  - Auxiliary diagram
  - Tensor Integrals
  - Results
  - Unresolved issues
- Conclusion

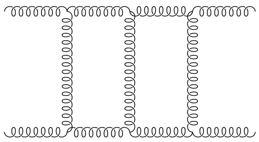
$2 \to 2$ scattering processes

Process	Tree	One loop	Two loops
$gg \to gg$	4	63 + 9	1030 + 501
$q\bar{q} \to gg$	3	24 + 5	346 + 217
$q\bar{q} \to Q\bar{Q}$	1	6 + 4	81 + 105
$\gamma\gamma \to \gamma\gamma$	0	6	96 + 48
$e^+e^- \to e^+e^-$	2	8 + 2	62 + 40

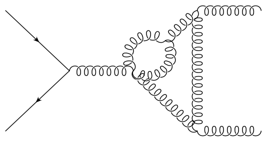
- Products of one loop graphs such as



- Genuine new topologies such as planar double box

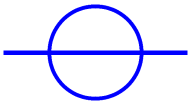
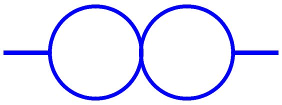
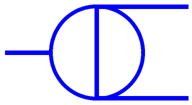


- Self energy insertions such as

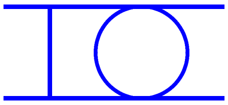
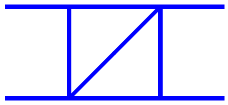
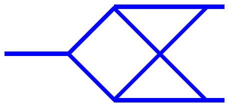


Master Integrals

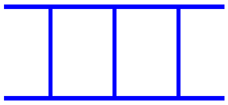
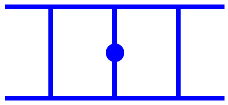
- The trivial topologies

 (s)  (s)  (s)

- The less trivial topologies

 (s,t)  (s,t)  (s)

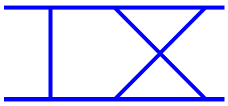
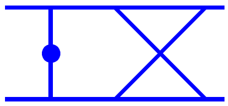
- The non-trivial topologies
  - The planar boxes

 (s,t)  (s,t)

$\Longrightarrow$ Smirnov, hep-ph/9905323

$\Longrightarrow$ Smirnov and Veretin, hep-ph/9907385

  - The non-planar boxes

 (s,t)  (s,t)

$\Longrightarrow$ Tausk, hep-ph/9909506

$\Longrightarrow$ Anastasiou, Gehrmann, Oleari, Remiddi and Tausk, hep-ph/0003261

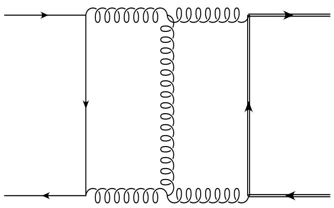
Application: $q\bar{q} \to Q\bar{Q}$

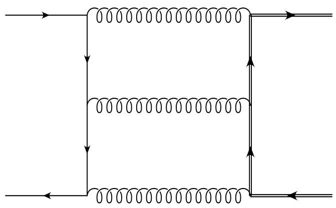
- The interference of tree level amplitude $\mathcal{M}_0$ with two-loop graphs $\mathcal{M}_2$ has the following colour structure

$$\mathcal{M}_2\mathcal{M}_0^\dagger + \mathcal{M}_2^\dagger\mathcal{M}_0 = g_s^6\left(N^2-1\right)$$
$$\times \quad \left(AN^2 + B + C\frac{1}{N^2} + DN_FN + E\frac{N_F}{N}\right)$$

where N is the number of colours

- The leading colour amplitude A contains only planar diagrams such as



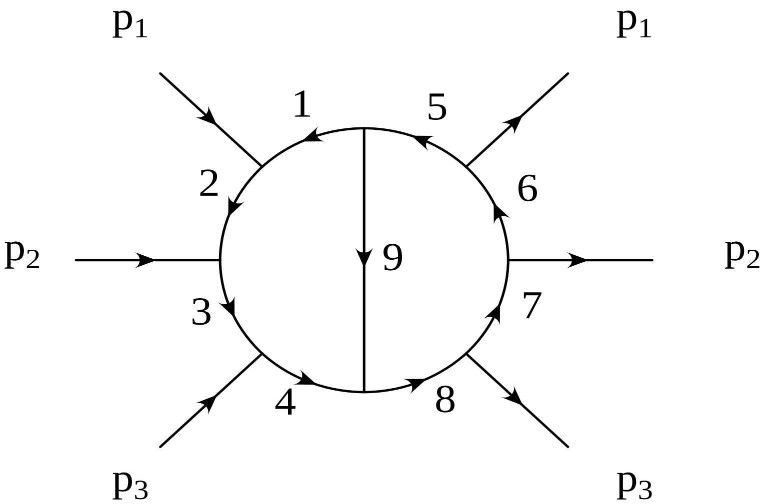


- The amplitudes suppressed by powers of N $(B, C, D$ and $E)$ contain the non-planar graphs

$\Longrightarrow$ focus on leading colour amplitudes as proof of principle

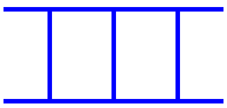
Auxiliary diagram

- The general auxiliary planar diagram with propagator i raised to the power ν_i



$$I^D(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,\nu_7,\nu_8,\nu_9)$$

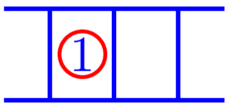
e.g. the planar box integral



$$= I^D(1,1,1,0,1,0,1,1,1)$$

- Tensor integrals appear with negative values of ν_i

e.g. the planar box integral with one irreducible numerator on the left hand loop



$$= I^D(1,1,1,-1,1,0,1,1,1)$$

General procedure

- Use QGRAF to generate the Feynman diagrams

  $\Longrightarrow$ compute interference with tree level

  $\Longrightarrow$ identify combinations of auxiliary integrals

- Exchange tensor integrals ($\nu_i < 0$) for auxiliary integrals in higher dimension with higher powers of propagators ($\nu_i \geq 0$)

- Apply the 10 IBP identities such as..

$$
\begin{aligned}
&s_{12}\nu_1 \mathbf{1}^+ I^D = \\
&\qquad -(D-\nu_1-\nu_2-2\nu_3-\nu_4-\nu_9) I^D \\
&\qquad + \left((\nu_1 \mathbf{1}^+ + \nu_2 \mathbf{2}^+ + \nu_4 \mathbf{4}^+)\mathbf{3}^- + \nu_9 \mathbf{9}^+ (\mathbf{3}^- - \mathbf{7}^-) \right) I^D
\end{aligned}
$$

  ..to reduce general auxiliary integrals to combinations of the Master Topologies ($\nu_i \neq 1$)

- Specific IBP identities to reduce Master Topologies to Master Integrals ($\nu_i = 1$)

  $\Longrightarrow$ but not planar boxes

Planar box graphs

Want to stay in $D = 4$ to do the tensor integrals

- Apply Gehrmann, Remiddi, hep-ph/9912329 approach

  $\Longrightarrow$ generate many identities

  $\Longrightarrow$ solve system directly

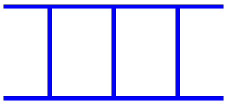
$r \backslash s$	0	1	2	3	4
7	1	2	3	4	5
8	1 + 6	14	21	28	35
9	28	56	84	112	140
10	84	168	252	336	420

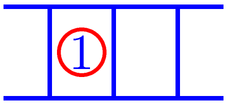
  $r =$ sum of positive ν_i, $s =$ (-) sum of negative ν_i

  For each value of r, s get 10 IBP identities in terms of integrals with up to $r = r + 1$, $s = s + 1$ and simpler pinched integrals

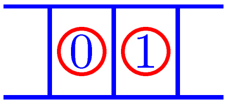
  $\Longrightarrow$ eliminate unknowns using linear algebra

- A more natural set of master integrals for tensors are

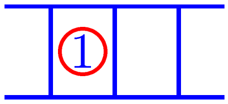
$r = 7, s = 0$  (s, t)

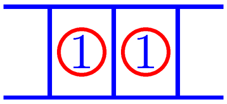
$r = 7, s = 1$  (s, t)

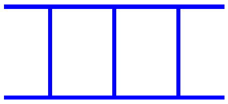
Planar box tensor integrals

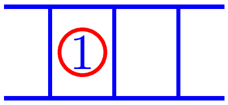


$=$

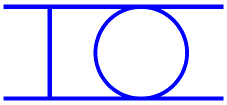


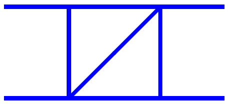


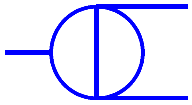
$= \dfrac{st}{2}$  (s,t)

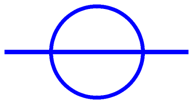
$- \dfrac{3s}{2}$ 

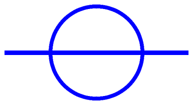
(s,t)

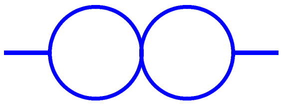
$+ \dfrac{8(D-3)}{(D-4)}$  (s,t)

$- \dfrac{(7s+9t)}{s}$  (s,t)

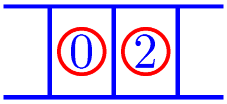
$+ \dfrac{17(D-3)(3D-10)}{2s(D-4)^2}$  (s)

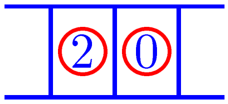
$- \dfrac{2(3D-8)(3D-10)}{s^2(D-4)^3}$  (s)

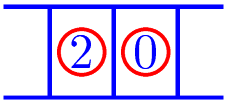
$+ \dfrac{9(3D-10)(3D-8)(D-3)}{st(D-4)^3}$  (t)

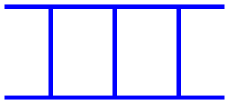
$+ \dfrac{2(D-3)((2D-5)s+2(D-3)t)}{s^2(D-4)^2}$  (s)

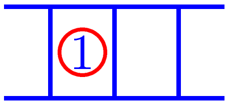
Planar box tensor integrals

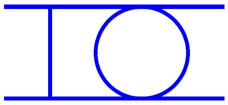


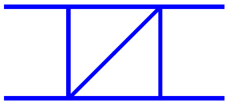
$=$


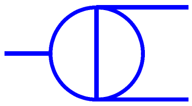


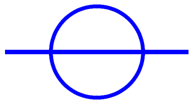
$= \dfrac{(D-4)st}{2(D-3)}$ 

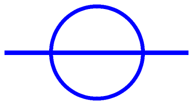
$- \dfrac{(3(D-4)s-2t)}{2(D-3)}$ 

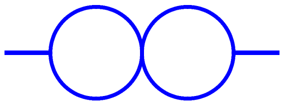
$+ \; 8$  (s,t)

$- \dfrac{(7s+9t)(D-4)}{s(D-3)}$  (s,t)

$+ \dfrac{(13s-2t)(3D-10)}{2s^2(D-4)}$  (s)

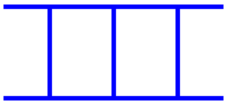
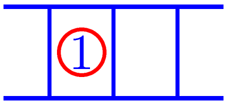
$+ \dfrac{2(s+2t)(3D-8)(3D-10)}{2s^3(D-3)(D-4)^2}$  (s)

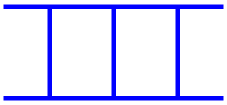
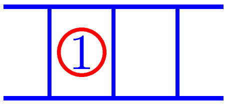
$+ \dfrac{9(3D-8)(3D-10)}{st(D-4)^2}$  (t)

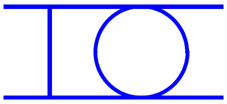
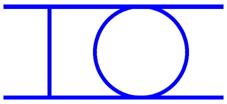
$+ \dfrac{2((2D-7)s+2(D-4)t)}{s^2(D-4)}$  (s)

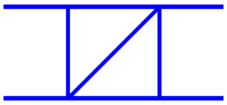
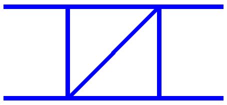
Leading Colour Matrix Elements
$q\bar{q} \to Q\bar{Q}$

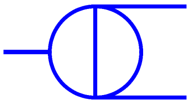
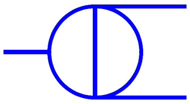
The matrix elements have the following structure

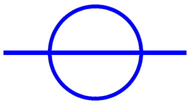
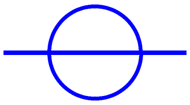
$A \;=\; +a_1$  (s,t) $+\,a_2$  (s,t)

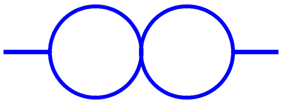
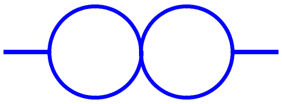
$+a_3$  (t,s) $+\,a_4$  (t,s)

$+a_5$  (s,t) $+\,a_6$  (t,s)

$+a_7$  (s,t) $+\,a_8$  (t,s)

$+a_9$  (s) $+\,a_{10}$  (t)

$+a_{11}$  (s) $+\,a_{12}$  (t)

$+a_{13}$  (s) $+\,a_{14}$  (t)

Coefficients of planar box integrals

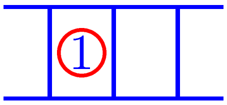
$$a_1 = -\frac{(55D^2 - 281D + 328)}{32(D-3)}\, s^3 t - \frac{31}{8}\, st^3 - \frac{(17D^2 + 40D - 314)}{32(D-3)}\, s^2t^2$$

$$a_2 = \frac{(141D^2 - 707D + 792)}{32(D-3)}\, s^3 + \frac{(163D - 530)}{16(D-3)}\, st^2 + \frac{(63D^2 + 12D - 766)}{32(D-3)}\, s^2 t$$

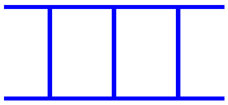
$$a_3 = -\frac{(3D^3 + 16D^2 - 196D + 368)}{64(D-3)}\, s^2t^2 - \frac{(D-2)(9D-28)(3D^2 - 18D + 32)}{128(D-3)^2}\, st^3 - 4\, t^4$$

$$a_4 = \frac{(18D^4 - 165D^3 + 308D^2 + 708D - 1904)}{64(D-3)^2}\, st^2 + \frac{3(27D^4 - 300D^3 + 1028D^2 - 944D - 512)}{128(D-3)^2}\, t^3$$

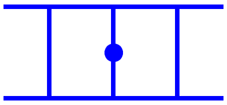
Relationship between Master Integrals



$$= -\frac{(3D-14)s}{2(D-4)}$$



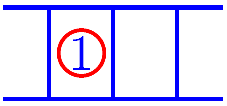
$$-\frac{(D-6)st}{2(D-4)(D-5)}$$



$+$ pinchings

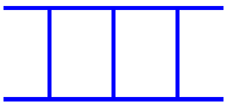
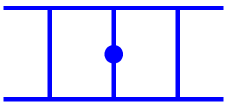
$\Longrightarrow$ apparent problem

- ...the $1/\epsilon^4$ poles and descendents cancel completely so that

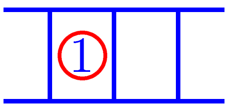


$$\sim \frac{1}{\epsilon^4}$$

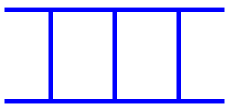
as they should...

- ..but the finite parts are controlled by the $\mathcal{O}(\epsilon)$ parts of  and 

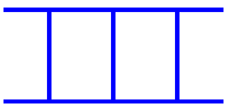
- In terms of derivatives



$$= -\frac{(s(D-5)-t)}{(D-4)}$$



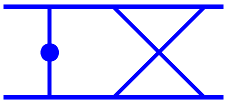
$$+\frac{ut}{(D-4)}\,\frac{\partial}{\partial t}$$



$+$ pinchings

Conclusions

- With paper of Anastasiou et al, the last unknown Master Integral for lightlike $2 \to 2$ scattering is known



(s, t)

- Trial run of $q\bar{q} \to Q\bar{Q}$ at leading colour
  - Tensor reduction OK
  - Size of expressions and Feynman diagrams OK
  - Analytic results in terms of Master Integrals OK
  - Expansion in $D-4$ and pole structure not checked yet
- But, it appears that the $\mathcal{O}(\epsilon)$ contribution to the planar box graph integral is needed.