Loops and Legs in Quantum Field Theory,
Bastei/Königstein 2000

Exact Results for Fermionic Two-Loop Contributions to Muon Decay in the Standard Model

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3. Treatment of the γ_5–problem
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1. Introduction

- Muon decay in Fermi Model

  4-Fermion interaction $\mu^- \to e^- \, \nu_\mu \, \bar{\nu}_e$ with coupling constant G_F:

$$\Gamma_\mu = \frac{G_\mathrm{F}^2 \, m_\mu^5}{192\pi^3} \, F\left(\frac{m_\mathrm{e}^2}{m_\mu^2}\right) (1 + \Delta q) \, ,$$

  $\Delta q =$ QED corrections.

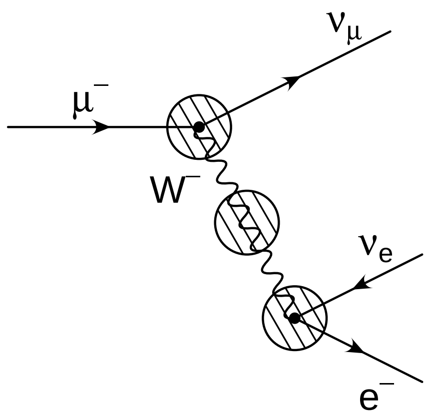
  $\to$ Definition of G_F

- Muon decay in Standard Model

  Corrections due to massive gauge bosons and loop contributions

$$G_\mathrm{F} = \frac{e^2}{8 s_\mathrm{W}^2 M_\mathrm{W}^2} \, (1 + \Delta r)$$

$$\left(s_\mathrm{W}^2 = 1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)$$



- Theoretical evaluation of Δr allows prediction of M_W-M_Z relation:

$$M_\mathrm{W}^2 = M_\mathrm{Z}^2 \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\alpha\pi}{\sqrt{2} G_\mathrm{F} M_\mathrm{Z}^2} (1 + \Delta r)} \right).$$

- High precision experimental input parameters:

$M_\mathrm{W} = (80.419 \pm 0.038)$ GeV — today
$\pm\, 0.015$ GeV — LHC
$\pm\, 0.006$ GeV — Tesla

$G_\mathrm{F} = (1.16637 \pm 0.00001)\, 10^{-5}$ GeV^{-2}
including 2-loop QED corrections
[T. v.Ritbergen, R. Stuart '98]

$M_\mathrm{Z} = (91.1867 \pm 0.0021)$ GeV

$m_\mathrm{t} = (174.3 \pm 5.1)$ GeV — today
$\pm\, 1.5$ GeV — LHC
$\pm\, 0.2$ GeV — Tesla

- Comparision prediction $\leftrightarrow$ exp. value of M_W:
$\to$ Constraint on Higgs mass M_H
$\to$ Test of Standard Model

Previous calculations of Δr

One-loop corrections $\mathcal{O}(\alpha)$

[A. Sirlin '80] [W.J. Marciano, A. Sirlin '80]

$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\Delta\rho + \Delta r_{\mathrm{rem}}(M_{\mathrm{H}})$$

$$\propto \log m_f \qquad \propto m_{\mathrm{t}}^2$$

$$\approx 6\% \qquad \approx 3.3\% \qquad \approx 1\%$$

QCD corrections $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ and $\mathcal{O}(\alpha\alpha_{\mathrm{s}}^2)$

[A. Djouadi '88] [F. Halzen, B.A. Kniehl '91]
[L. Avdeev et al. '94]
[K. Chetyrkin, J. Kühn, M. Steinhauser '95]

Resummations of leading fermionic terms

[A. Sirlin '84] [S. Fanciotti, A. Sirlin '91]
[M. Consoli, W. Hollik, F. Jegerlehner '89]

$$1 + \Delta r = \frac{1}{(1-\Delta\alpha)(1+\frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\Delta\rho) - \Delta r_{\mathrm{rem}}} + \ldots$$

Expansions in M_{H}, m_{t} of the $\mathcal{O}(\alpha^2)$ contributions

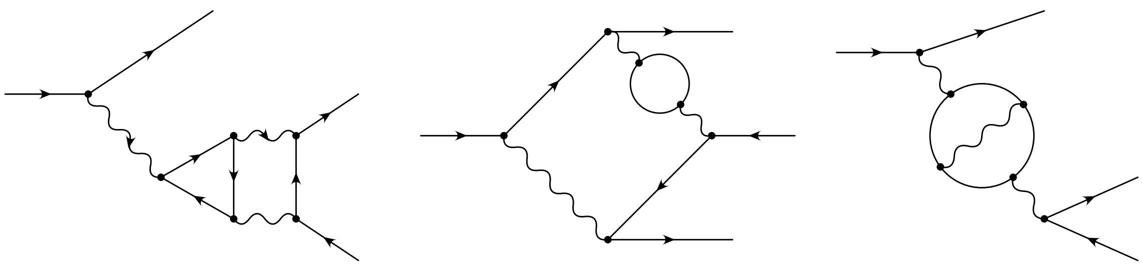
[J. v.d.Bij, M. Veltman '84]
[J. v.d.Bij, F. Hoogeveen '87]
[R. Barbieri et al. '93]
[J. Fleischer, O.V. Tarasov, F. Jegerlehner '95]
[G. Degrassi, P. Gambino, A. Vicini '96]

Leading and NLO terms of m_{t}-expansion of same order

$\rightarrow$ Full $\mathcal{O}(\alpha^2)$ calculation desirable

This work:

Exact two-loop corrections to Δr with fermionic loops



Calculational methods

- Neglect light fermion masses and external momenta
  $\rightarrow$ Reduction to vacuum diagrams

- On-shell renormalization
  $\rightarrow$ Self-energy diagrams with $p \neq 0$

- Split off QED contributions
  $\rightarrow$ Treatment in the Fermi Model

- General R_ξ-gauge

- Generate Feynman diagrams and counterterm insertions with *FeynArts*
  [J. Küblbeck, M. Böhm, A. Denner '90]
  [H. Eck, J. Küblbeck '92] [T. Hahn '98]

- Tensor integral reduction for general 2-loop 2-point functions with *TwoCalc*
  [G. Weiglein et al. '94]
  $\rightarrow$ set of standard scalar integrals

- Analytical formulae for
  - One-loop integrals
    [G .'t Hooft, M. Veltman '79]
  - Two-loop vacuum integrals
    [A.I. Davydychev, J.B. Tausk '94]

- One-dimensional integral representations for two-loop self-energy integrals
  [S. Bauberger et al. '95]

separation of QED contributions

Interplay between QED ($\Delta\omega$) and EW (Δr) corrections:

$$\Gamma_\mu = \Gamma_{\mu,0}(1+\Delta\omega)^2\,(1+\Delta r)^2$$

$$= \Gamma_{\mu,0}\Big[1 + 2(\Delta\omega^{(\alpha)} + \Delta r^{(\alpha)}) + (\Delta\omega^{(\alpha)} + \Delta r^{(\alpha)})^2$$

$$+ 2\Delta\omega^{(\alpha)}\Delta r^{(\alpha)} \qquad \text{mix QED/weak}$$

$$+ 2\Delta\omega^{(\alpha^2)} \qquad \text{QED } \mathcal{O}(\alpha^2)$$

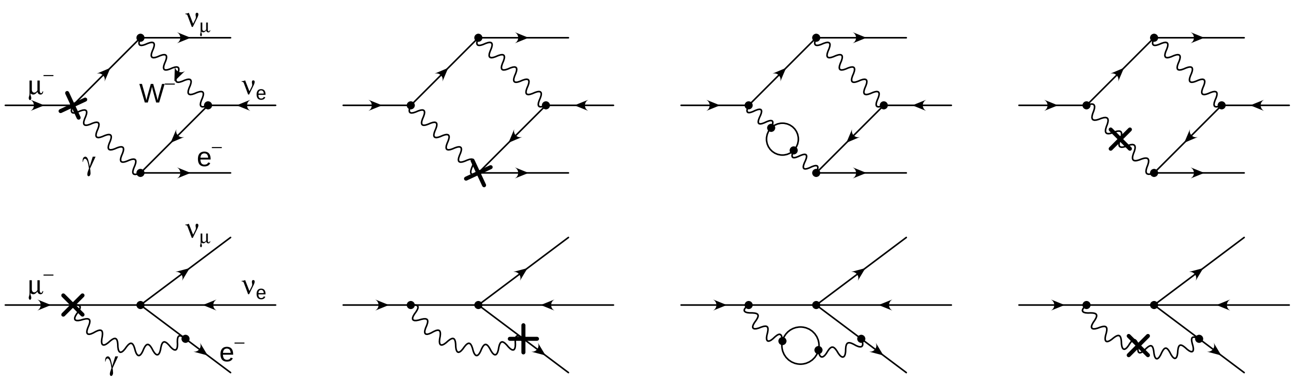
$$+ 2\Delta r^{(\alpha^2)} \qquad \cancel{\text{QED } \mathcal{O}(\alpha^2)} \textit{ and } \cancel{\text{mix QED/weak}}$$

$$+ \mathcal{O}(\alpha^3)\Big].$$

Subtract QED contributions in Δr:

- Field renormalization of external fermions
- Box contributions

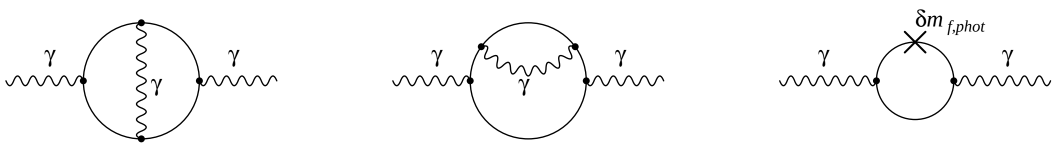
  $\rightarrow$ difference between SM and Fermi Model



Difficulties in Fermi Model diagrams with Fierz identities in D dimensions

- Charge renormalization:

  QED contributions with light fermions in photon vacuum polarization are part of $\Delta\alpha$



2. On-shell renormalization in $\mathcal{O}(\alpha^2)$ and gauge invariance

Definition of the on-shell mass

$\mathcal{O}(\alpha^2)$ mass counterterms required

$\rightarrow$ Difference between mass defintion via

- real pole of propagator

$$\Re\left\{(D^{\rm V}_{\rm T})^{-1}(\widetilde{M}^2_{\rm V})\right\} = 0$$

- real part of complex pole of propagator

$$(D^{\rm V}_{\rm T})^{-1}(\mathcal{M}^2_{\rm V}) = 0, \qquad \mathcal{M}^2_{\rm V} = M^2_{\rm V} - iM_{\rm V}\Gamma_{\rm V}$$

$\rightarrow$ Two-loop counterterm for W mass

$$\delta M^2_{\rm W(2)} = \Re\left\{\Sigma^{\rm W}_{\rm T(2)}(M^2_{\rm W})\right\} - \delta Z^{\rm W}_{(1)}\delta M^2_{\rm W(1)} + \Im\left\{\Sigma^{\rm W\prime}_{\rm T(1)}(M^2_{\rm W})\right\}\Im\left\{\Sigma^{\rm W}_{\rm T(1)}(M^2_{\rm W})\right\}$$

S-matrix theory: complex pole is gauge invariant
[A. Sirlin '91] [R. Stuart '91] [H. Veltman '94]
[M. Passera, A. Sirlin '98] [P. Gambino, P. Grassi '99]

Check with explicit calculation:

Counterterm for $\delta s_{\mathrm{W}(2)}$ only gauge invariant with mass defintion via complex pole.

Note:

Breit-Wigner parametrization with running width corresponds to real pole mass $\widetilde{M}_{\mathrm{V}}$

$$M_{\mathrm{W}} = \widetilde{M}_{\mathrm{W}}^{\mathrm{exp}} - 26.4 \text{ MeV}$$
$$M_{\mathrm{Z}} = \widetilde{M}_{\mathrm{Z}}^{\mathrm{exp}} - 34.0 \text{ MeV}$$

Gauge fixing and ghost sector

Keep gauge fixing term invariant under renormalization

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{2}\Big((F^{\gamma})^2 + (F^{\mathrm{Z}})^2 + F^{+}F^{-} + F^{-}F^{+}\Big),$$

$$F^{\gamma} = \left(\xi_1^{\gamma}\right)^{-\frac{1}{2}} \partial_\mu A_0^\mu + \frac{\xi^{\gamma \mathrm{Z}}}{2} \partial_\mu Z_0^\mu,$$

$$F^{\mathrm{Z}} = \left(\xi_1^{\mathrm{Z}}\right)^{-\frac{1}{2}} \partial_\mu Z_0^\mu + \frac{\xi^{\mathrm{Z}\gamma}}{2} \partial_\mu A_0^\mu - \left(\xi_2^{\mathrm{Z}}\right)^{\frac{1}{2}} M_{\mathrm{Z}0}\, \chi_0,$$

$$F^{\pm} = \left(\xi_1^{\mathrm{W}}\right)^{-\frac{1}{2}} \partial_\mu W_0^{\pm\mu} \mp i \left(\xi_2^{\mathrm{W}}\right)^{\frac{1}{2}} M_{\mathrm{W}0}\, \phi_0^{\pm}.$$

Fields and masses are renormalized

$\Rightarrow$ Gauge parameters to be renormalized, too:

$$\begin{pmatrix} (\xi^\gamma)^{-\frac{1}{2}} & 0 \\ 0 & (\xi^{\mathrm{Z}})^{-\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} (\xi_1^\gamma)^{-\frac{1}{2}} & \frac{1}{2}\xi^{\gamma\mathrm{Z}} \\ \frac{1}{2}\xi^{\mathrm{Z}\gamma} & (\xi_1^{\mathrm{Z}})^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} (Z^{\gamma\gamma})^{\frac{1}{2}} & \frac{1}{2}\delta Z^{\gamma\mathrm{Z}} \\ \frac{1}{2}\delta Z^{\mathrm{Z}\gamma} & (Z^{ZZ})^{\frac{1}{2}} \end{pmatrix}$$

$$\xi^{\mathrm{Z}} = \xi_2^{\mathrm{Z}} \frac{M_{\mathrm{Z}}^2 + \delta M_{\mathrm{Z}}^2}{M_{\mathrm{Z}}^2} Z^\chi$$

$$\xi^{\mathrm{W}} = \frac{1}{Z^{\mathrm{W}}} \xi_1^{\mathrm{W}} \qquad\qquad \xi^{\mathrm{W}} = \xi_2^{\mathrm{W}} \frac{M_{\mathrm{W}}^2 + \delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2} Z^\phi.$$

$\xi^\gamma, \xi^{\mathrm{Z}}, \xi^{\mathrm{W}}$: Renormalized gauge parameters

$\rightarrow$ Determines renormalization of ghost sector

$$\mathcal{L}_{\mathrm{FP}} = \sum_{a,b=\gamma,\mathrm{Z},\pm} \bar{u}^a \frac{\delta F^a}{\delta \theta^b} u^b.$$

Benefits:

- No counterterm contributions from the gauge fixing sector
- Finite buildings blocks e.g. selfenergies
- Rernormalized result in general R_ξ–gauge

3. Treatment of the γ_5–problem

Problem: Dirac-algebra for γ_5

$$\{\gamma_5, \gamma_\alpha\} = 0 \qquad \text{für} \qquad \alpha = 1, \ldots, 4 \qquad (1)$$

$$\mathrm{Tr}\{\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\} = 4i\epsilon_{\mu\nu\rho\sigma}, \qquad (2)$$

cannot be translated to D Dimensions.

$\to$ DReg. could violate chiral symmetries

Consider several schemes:

- naively anti-commuting

$$\{\gamma_5, \gamma_\alpha\} = 0 \qquad \text{für} \qquad \alpha = 1, \ldots, D$$

$$\mathrm{Tr}\{\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\} = 0,$$

- Breitenlohner-Maison

[G. 't Hooft, M. Veltman '72]
[P. Breitenlohner, D. Maison '77]

Separate treatment of the first four dimensions and the remaining dimensions

$$\gamma_\mu = \bar{\gamma}_\mu + \hat{\gamma}_\mu,$$

- mixed scheme

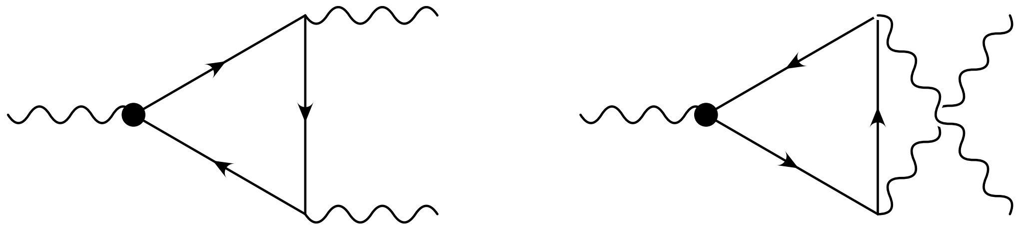
Rules (1) *and* (2) despite inconsistency

γ_5–schemes might violate Slavnov-Taylor-identities.
$\to$ Restoration with extra counterterms.

Remaining scheme dependence associated with ϵ-tensor expressions.
If these break symmetry relations $\to$ anomalies

In Standard Model:

Triangle diagrams are sensitive to γ_5–problem:



Breitenlohner-Maison regularization: Standard Model is renormalizable, *anomaly-free*
[D.J. Gross, R. Jackiw '72] [G. Costa et al. '77]

$\to$ no divergent contributions to ϵ-tensor terms

Verification of Slavnov-Taylor-identities:

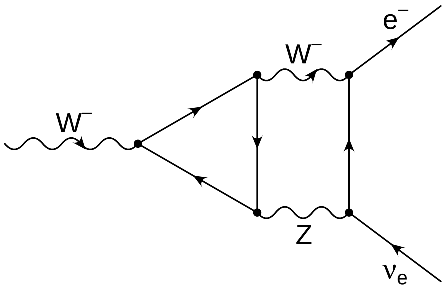
naive/mixed scheme : fulfilled

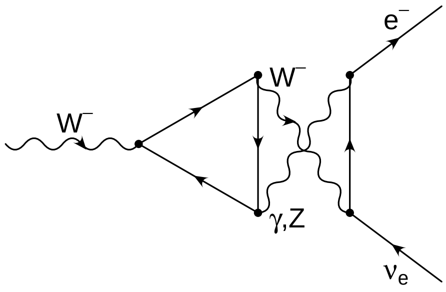
Breitenlohner-Maison : extra counterterms
$\to$ very difficult for two loops

Explicit calculation of 1-loop triangle diagrams:

$$\Gamma^{\Delta,\ \mathrm{BM}}_{\mu\nu\rho} = \Gamma^{\Delta,\ \mathrm{mix}}_{\mu\nu\rho} + \mathcal{O}(D-4)$$
$$\neq \Gamma^{\Delta,\ \mathrm{naive}}_{\mu\nu\rho}$$

Triangle graphs in $\mathcal{O}(\alpha^2)$ vertex corr. to Δr:





Analysis of divergency structure of second loop:
Renormalizability forbids additional divergency contributions to ϵ-tensor terms

$$\Gamma^{\mathrm{CC},\ \mathrm{BM}}_{\mu(2)} = \Gamma^{\mathrm{CC},\ \mathrm{mix}}_{\mu(2)} + \mathcal{O}(D-4)$$

$\rightarrow$ Mixed scheme technically easy
and correct for our calculation

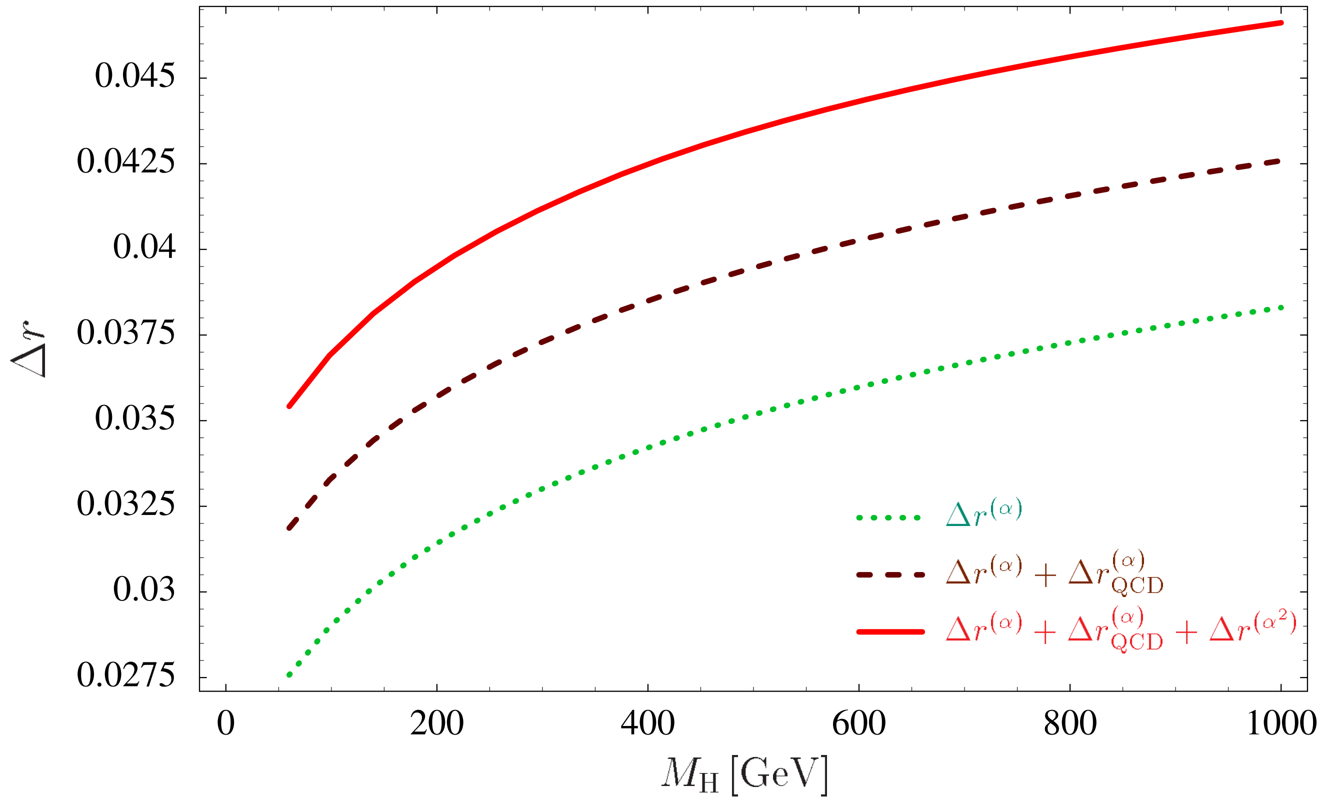
Problems only with two triangles in a diagram
$\rightarrow$ three loops

4. Results and analysis

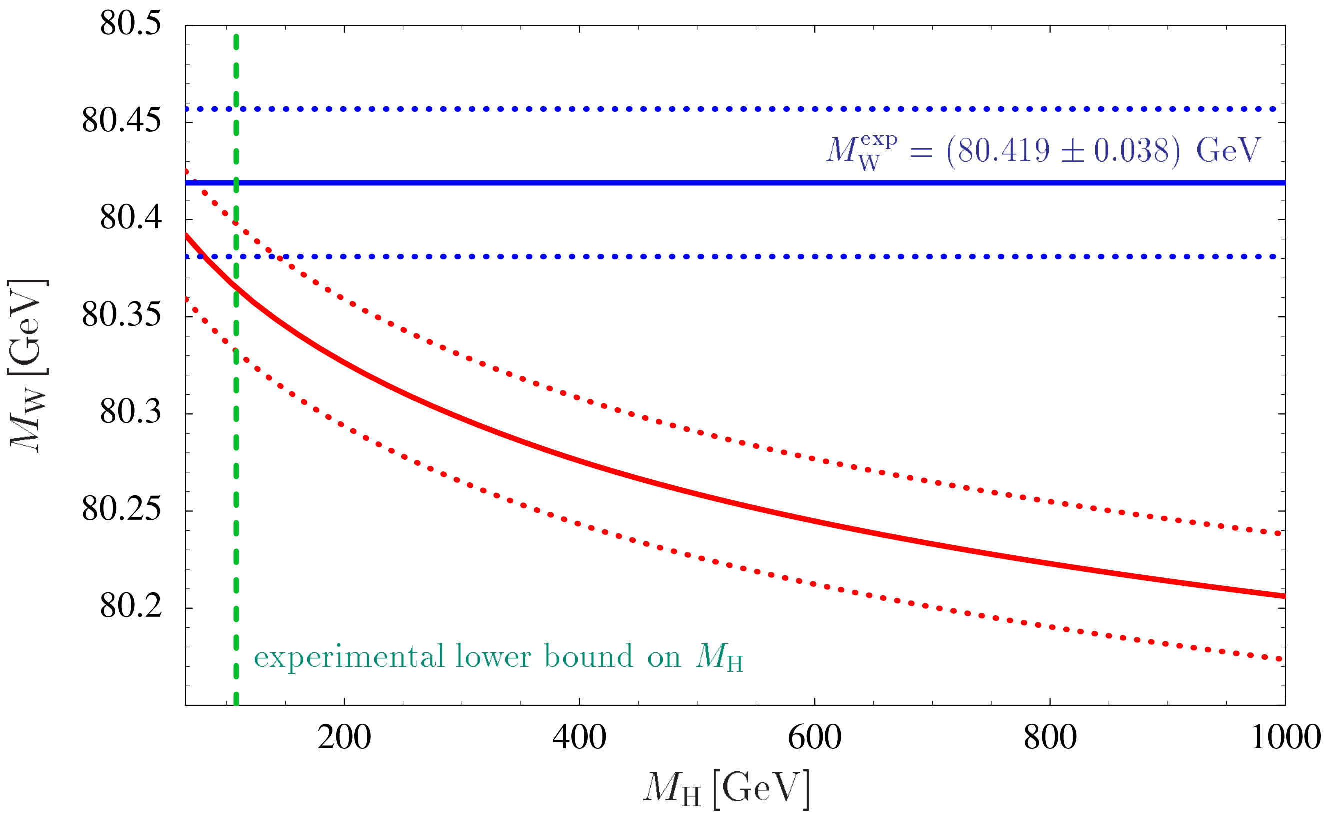
Result for Δr in $\mathcal{O}(\alpha^2)$ with fermion loops

- UV- und IR-finiteness checked
- Gauge parameter independence checked with general R_ξ–gauge at algebraic level
- Correct treatment of γ_5 verified

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)}$$



Prediction for M_W:



Comparision with NLO-expansion in m_t:

[G. Degrassi, P. Gambino, A. Vicini '96]
[G. Degrassi, P. Gambino, A. Sirlin '97]

M_H [GeV]	M_W^{expa} [GeV]	$\widetilde{M}_W$ [GeV]	δM_W [MeV]
65	80.4039	80.4009	3.0
100	80.3805	80.3785	2.0
300	80.3061	80.3068	−0.7
600	80.2521	80.2540	−1.9
1000	80.2129	80.2155	−2.6

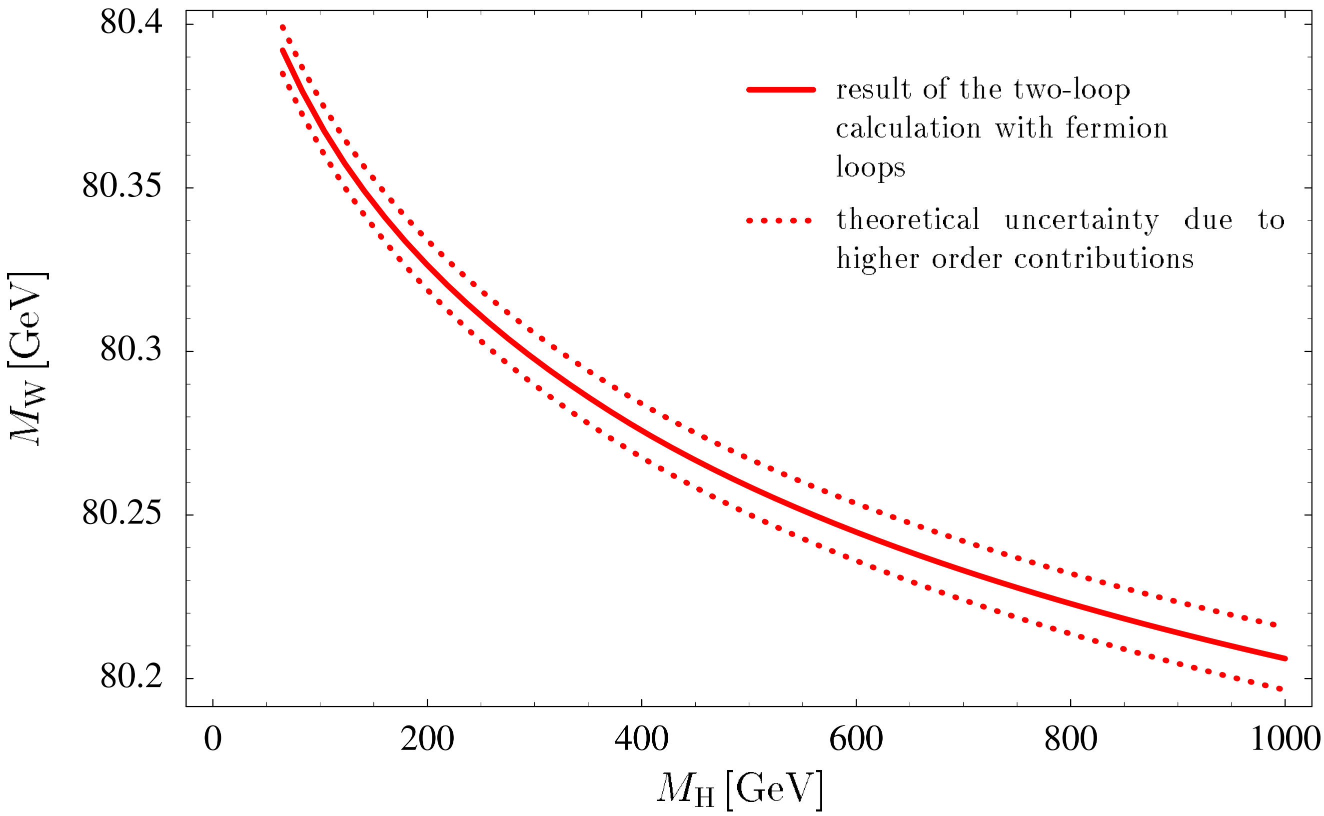
→ good agreement

Estimation of theoretical uncertainty

- Higher order ($\mathcal{O}(\alpha^3)$) EW corrections
  estimation from renorm. scheme dependence

- Bosonic $\mathcal{O}(\alpha^2)$ contributions
  estimation from resummation

- Higher QCD corrections
  $\delta_{\mathrm{QCD}} M_{\mathrm{W}} \approx \pm 5$ MeV [A. Sirlin '95]



Theory uncertainty for light Higgs masses:

$$\boxed{\delta M_W \approx 7 \text{ MeV}}$$

Conclusions

- Exact two-loop calculation in the electroweak Standard Model involving general vacuum and self-energy contributions

- Theoretical uncertainty from unknown higher-order corrections under control

- Improvement of M_{H}-prediction from radiative corrections

- Outlook: Evaluation of other precision observables at two-loop level?