Polarised Bhabha and Möller scatterings

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Outline

Kinematics

$$\ell^{+,-}(k_1,\xi_1^{in}) + \ell^-(k_2,\xi_2^{in}) \to \ell^{+,-}(p_1,\xi_1^{out}) + \ell^-(p_2,\xi_2^{out})$$

The Mandelstam variables

$$\begin{aligned} s &= (k_1+k_2)^2 = (p_1+p_2)^2, \\ t &= (k_1-p_1)^2 = (k_2-p_2)^2, \\ u &= (k_1-p_2)^2 = (k_2-p_1)^2. \end{aligned}$$

The Target System variables

$$E = \frac{k_1 \cdot k_2}{m}; \quad \gamma = \frac{E}{m}; \quad T = E - m; \quad x = \frac{T_{cut}}{T}$$

$$T_{p_2} = E_{p_2} - m; \quad \epsilon = \frac{T_{p_2}}{T}$$

Initial Particles Polarisation Vectors

$$\mathcal{L}_1^{in} = \frac{(s-2m^2)k_1 - 2m^2k_2}{\sqrt{s}m\sqrt{s-4m^2}} \quad , \quad \mathcal{L}_2^{in} = \frac{(2m^2-s)k_2 + 2m^2k_1}{\sqrt{s}m\sqrt{s-4m^2}}$$

$$\mathcal{T}_1^{in} = \mathcal{T}_2^{in} = \frac{tk_2 + uk_1 - (t+u)p_1}{\sqrt{-t}\sqrt{-u}\sqrt{-t-u}} \quad , \quad \mathcal{N}_1^{in} = \mathcal{N}_2^{in} = -\frac{1}{m}\epsilon^{\mu k_{1,2}\mathcal{L}_{1,2}^{in}\mathcal{T}_{1,2}^{in}}$$

Then any polarisation state of initial particle could be expressed as follows

$$\xi_{1,2}^{in} = a_L\mathcal{L}_{1,2}^{in} + a_T\mathcal{T}_{1,2}^{in} + a_N\mathcal{N}_{1,2}^{in}$$

In Target System

$$\begin{aligned}
\mathcal{L}_1^{in} &= \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, 0, 0, 1\right) \\
\mathcal{L}_2^{in} &= (0,0,0,1) \\
\mathcal{T}_1^{in} &= \mathcal{T}_2^{in} = (0, -\cos(\phi), -\sin(\phi), 0) \\
\mathcal{N}_1^{in} &= \mathcal{N}_2^{in} = (0, \quad \sin(\phi), -\cos(\phi), 0)
\end{aligned}$$

Final Particles Polarisation Vectors

$$\mathcal{L}_1^{out} = \frac{(2m^2 - u)p_1 - 2m^2 k_2}{\sqrt{-u}m\sqrt{4m^2 - u}} \quad , \quad \mathcal{L}_2^{out} = \frac{(2m^2 - t)p_2 - 2m^2 k_2}{\sqrt{-t}m\sqrt{4m^2 - t}}$$

$$\mathcal{T}_1^{out} = \frac{-tk_2 - (4m^2 - u)k_1 + sp_1}{\sqrt{s}\sqrt{-t}\sqrt{4m^2 - u}} \quad , \quad \mathcal{T}_2^{out} = \frac{-uk_2 - (4m^2 - t)k_1 + sp_2}{\sqrt{s}\sqrt{-u}\sqrt{4m^2 - t}}$$

$$\mathcal{N}_1^{out} = -\frac{1}{m}\epsilon^{\mu p_1 \mathcal{L}_1^{out} \mathcal{T}_1^{out}} \quad , \quad \mathcal{N}_2^{out} = -\frac{1}{m}\epsilon^{\mu p_2 \mathcal{L}_2^{out} \mathcal{T}_2^{out}}$$

Then any polarisation state of initial particle could be expressed as follows

$$\xi_{1,2}^{out} = a_L \mathcal{L}_{1,2}^{out} + a_T \mathcal{T}_{1,2}^{out} + a_N \mathcal{N}_{1,2}^{out}$$

Final Particles Polarisation Vectors. Target System

$$\mathcal{L}_1^{out} = \frac{E_{p_1}}{m}\left(\sqrt{1-\frac{m^2}{E_{p_1}^2}}, -\cos(\phi)\sin(\theta_{p_1}), -\sin(\phi)\sin(\theta_{p_1}), \cos(\theta_{p_1})\right)$$

$$\mathcal{L}_2^{out} = \frac{E_{p_2}}{m}\left(\sqrt{1-\frac{m^2}{E_{p_2}^2}}, \quad \cos(\phi)\sin(\theta_{p_2}), \quad \sin(\phi)\sin(\theta_{p_2}), \cos(\theta_{p_2})\right)$$

$$\mathcal{T}_1^{out} = (0, -\cos(\phi)\cos(\theta_{p_1}), -\sin(\phi)\cos(\theta_{p_1}), -\sin(\theta_{p_1}))$$

$$\mathcal{T}_2^{out} = (0, \quad \cos(\phi)\cos(\theta_{p_2}), \quad \sin(\phi)\cos(\theta_{p_2}), -\sin(\theta_{p_2}))$$

$$\mathcal{N}_1^{out} = (0, \quad \sin(\phi), -\cos(\phi), 0)$$

$$\mathcal{N}_2^{out} = (0, -\sin(\phi), \quad \cos(\phi), 0)$$

Total Cross Section

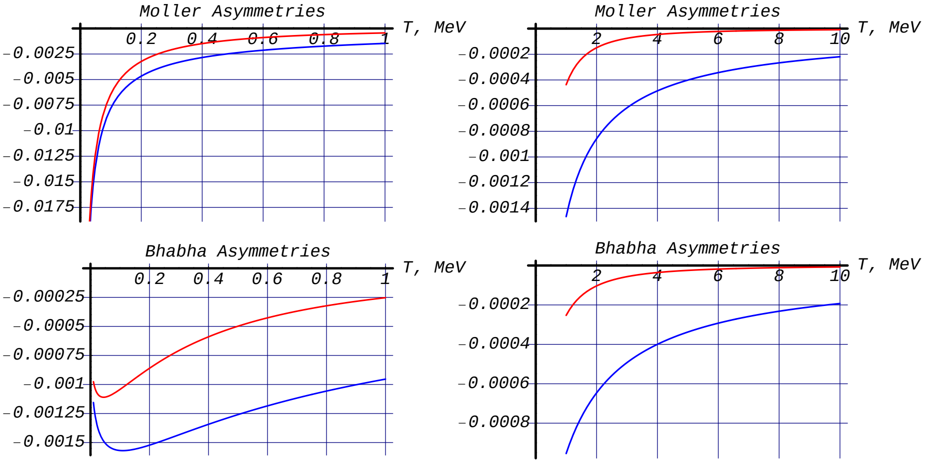
$$\sigma^M_{pol} = \sigma^M_0 \left[1 + z_1 z_2 A^M_L + (x_1 x_2 + y_1 y_2)\, A^M_T\right]$$

$\vec{S_1} = (x_1,\ y_1,\ z_1) \quad \vec{S_2} = (x_2,\ y_2,\ z_2)$

$$\sigma^B_{pol} = \sigma^B_0 \left[1 + z_+ z_- A^B_L + (x_+ x_- + y_+ y_-)\, A^B_T\right]$$

$\vec{S_{pos}} = (x_+,\ y_+,\ z_+) \quad \vec{S_{ele}} = (x_-,\ y_-,\ z_-)$

Total Cross Section Asymmerties



transverse longitudinal

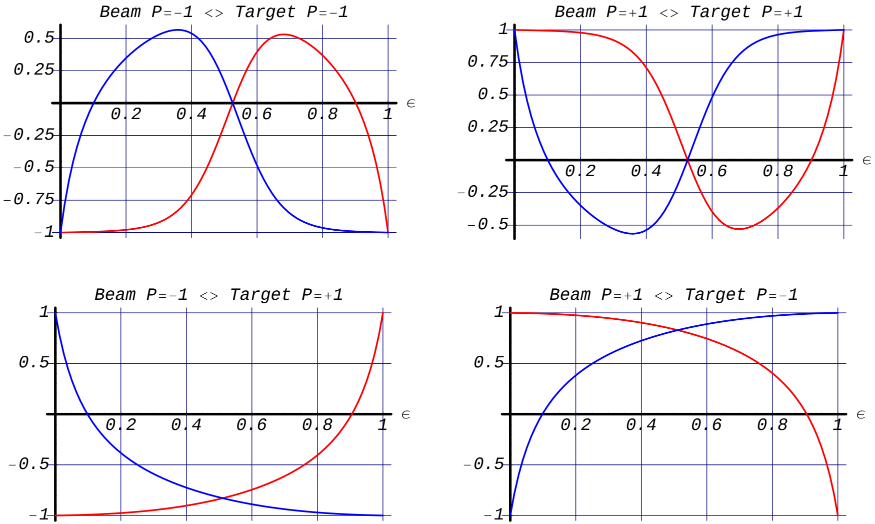
Differential Cross Section

$$\xi = a_L \mathcal{L} + a_T \mathcal{T} + a_N \mathcal{N} \quad \rightarrow \quad \boldsymbol{\xi} = (x, y, z)$$

$$\frac{d\sigma(\xi_1^{in}, \xi_2^{in}, \xi_1^{out}, \xi_2^{out})}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left[1 + \boldsymbol{\xi}_1^{\mathbf{in}} \mathbf{A}_0 \boldsymbol{\xi}_2^{\mathbf{in}\,T} \right] \times$$
$$\left[\boldsymbol{\xi}_1^{\mathbf{out}} \left(\mathbf{A}_{11} \boldsymbol{\xi}_1^{\mathbf{in}\,T} + \mathbf{A}_{12} \boldsymbol{\xi}_2^{\mathbf{in}\,T} \right) + \boldsymbol{\xi}_2^{\mathbf{out}} \left(\mathbf{A}_{21} \boldsymbol{\xi}_1^{\mathbf{in}\,T} + \mathbf{A}_{22} \boldsymbol{\xi}_2^{\mathbf{in}\,T} \right) \right.$$
$$\left. + \boldsymbol{\xi}_1^{\mathbf{out}} \mathbf{C}_0 \boldsymbol{\xi}_2^{\mathbf{out}\,T} + \boldsymbol{\xi}_1^{\mathbf{out}} \mathbf{C}_1(\xi_1^{in}, \xi_2^{in}) \boldsymbol{\xi}_2^{\mathbf{out}\,T} \right]$$

Bhabha Differential Cross Section Longitudinal Asymmetry

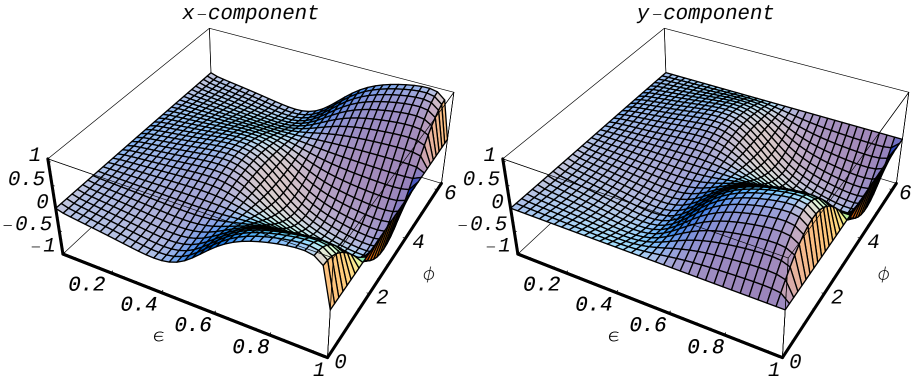
z-component



positrons electrons

Bhabha Differential Cross Section Transverse Asymmetry

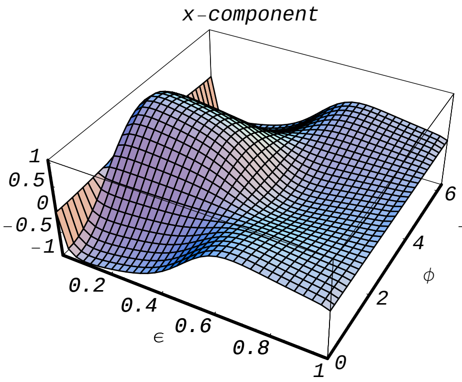
positrons

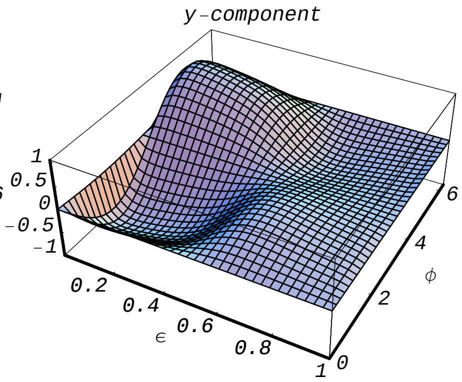


Beam $P = (0, 0, +1)$ Target $P = (0, 0, +1)$

Bhabha Differential Cross Section Transverse Asymmetry

electrons

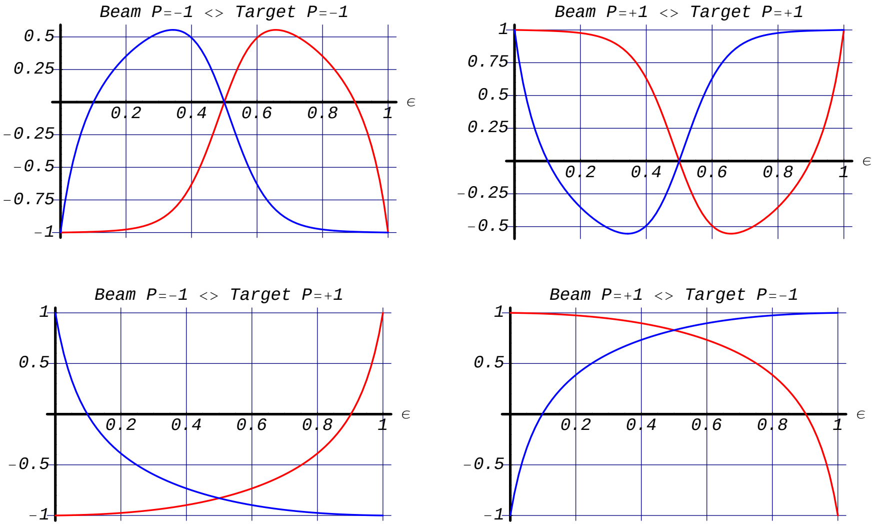




Beam $P = (0, 0, +1)$ Target $P = (0, 0, +1)$

Möller Differential Cross Section Longitudinal Asymmetry

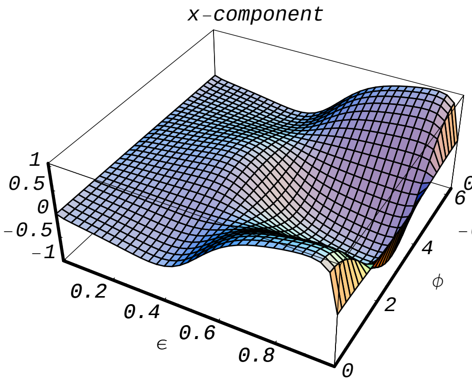
z-component



beam electrons target electrons

Möller Differential Cross Section Transverse Asymmetry

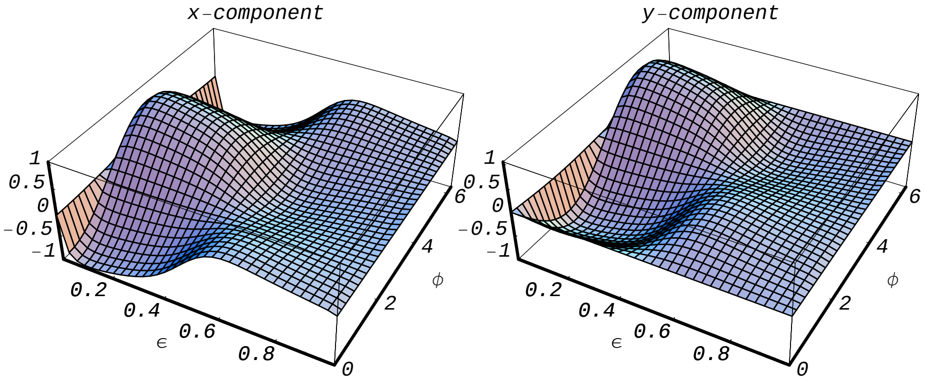
beam electrons



Beam $P = (0, 0, +1)$ Target $P = (0, 0, +1)$

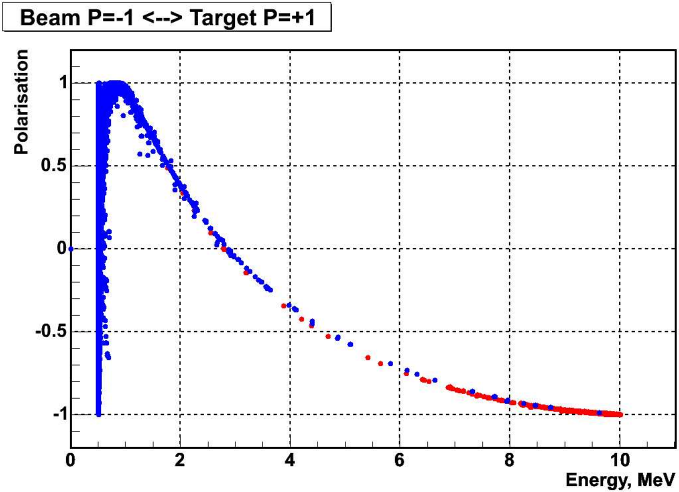
Möller Differential Cross Section Transverse Asymmetry

target electrons



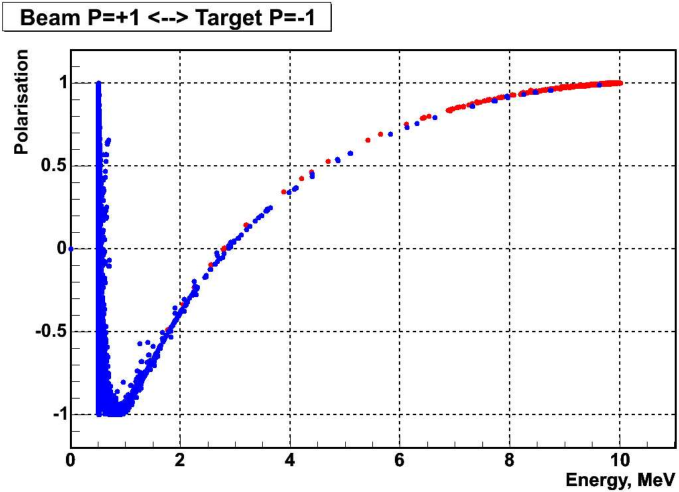
Beam $P = (0, 0, +1)$ Target $P = (0, 0, +1)$

Polarisation transfer in Bhabha scattering



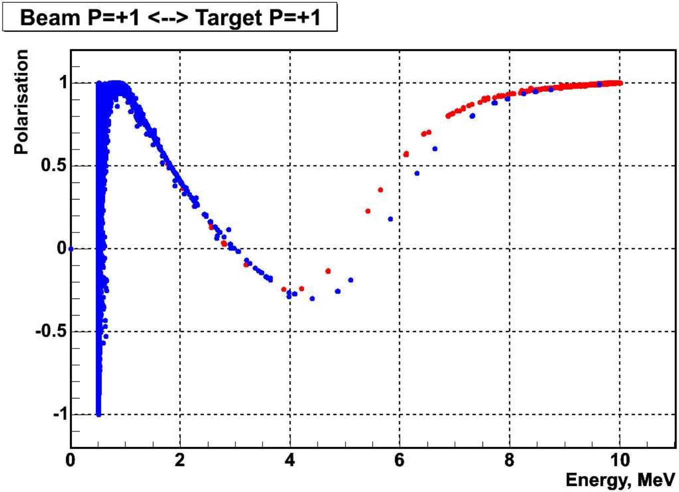
positrons electrons

Polarisation transfer in Bhabha scattering



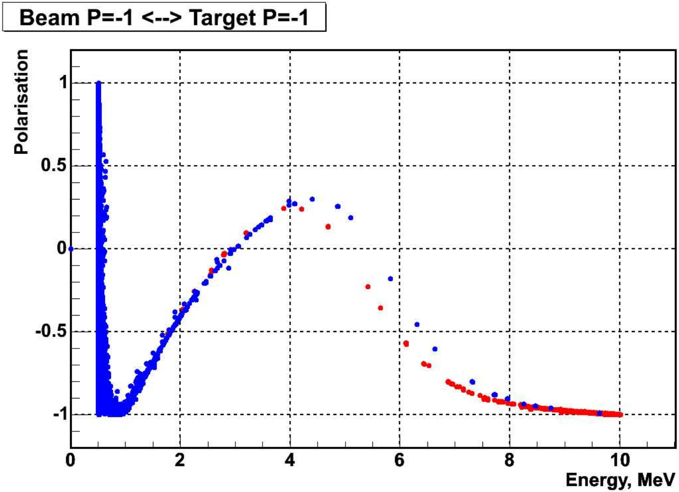
positrons electrons

Polarisation transfer in Bhabha scattering



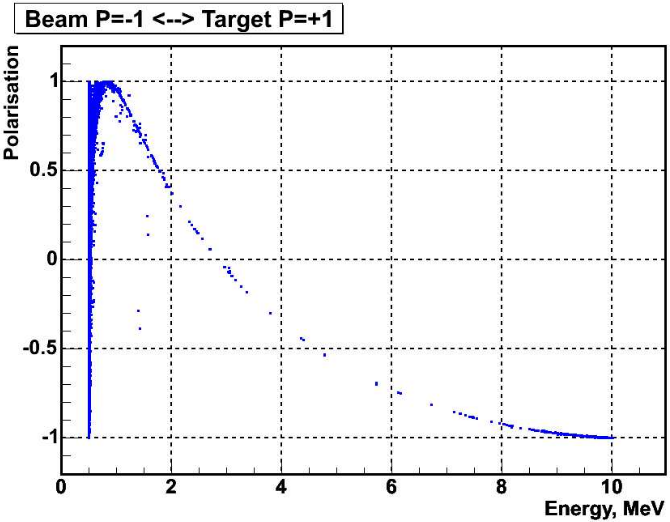
positrons electrons

Polarisation transfer in Bhabha scattering

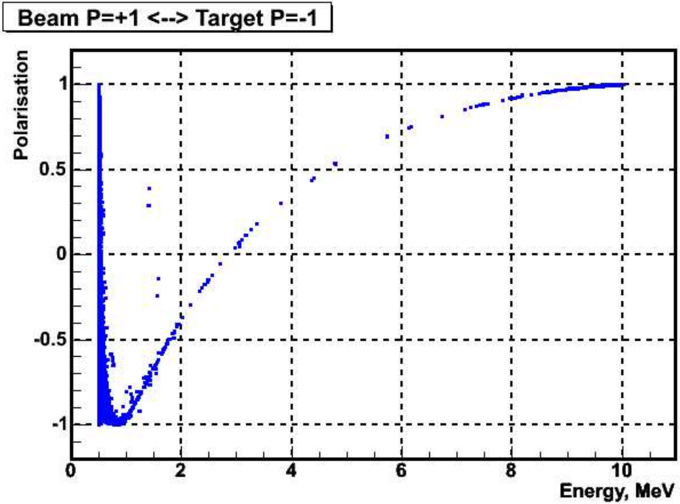


positrons electrons

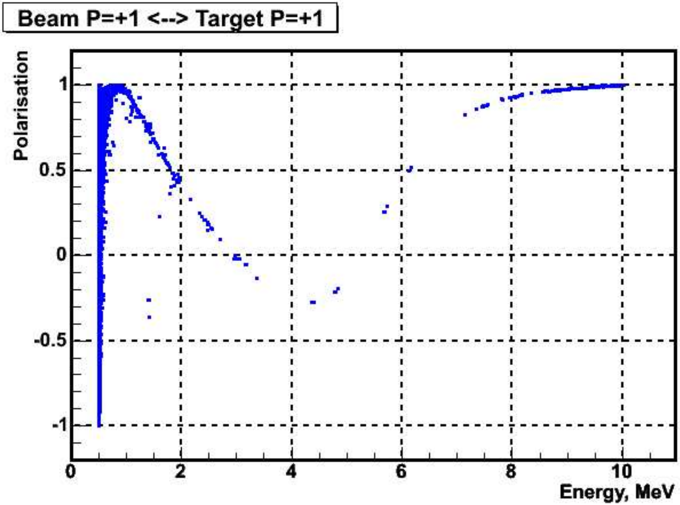
Polarisation transfer in Möller scattering



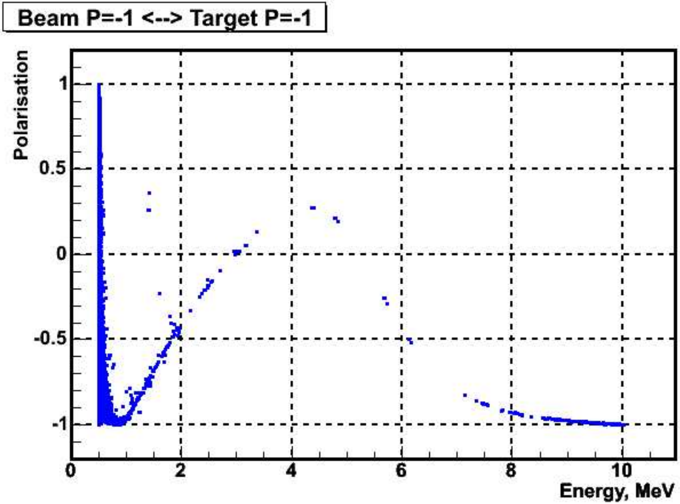
Polarisation transfer in Möller scattering



Polarisation transfer in Möller scattering



Polarisation transfer in Möller scattering



Next steps

- Bremstrahlung
- Compton
- Pair production
- Annihilation