

First results on $e^+e^- \rightarrow 3$ jets at NNLO

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Precision studies of QCD at e^+e^- colliders are based on measurements of event shapes and jet rates. To match the high experimental accuracy, theoretical predictions to next-to-next-to-leading order (NNLO) in QCD are needed for a reliable interpretation of the data. We report the first calculation of NNLO corrections ($\mathcal{O}(\alpha_s^3)$) to three-jet production and related event shapes, and discuss their phenomenological impact.

1 Introduction

Measurements at LEP and at earlier e^+e^- colliders have helped to establish QCD as the theory of strong interactions by directly observing gluon radiation through three-jet production events. The LEP measurements of three-jet production and related event shape observables are of a very high statistical precision. The extraction of α_s from these data sets relies on a comparison of the data with theoretical predictions. Comparing the different sources of error in this extraction, one finds that the experimental error is negligible compared to the theoretical uncertainty. There are two sources of theoretical uncertainty: the theoretical description of the parton-to-hadron transition (hadronisation uncertainty) and the uncertainty stemming from the truncation of the perturbative series at a certain order, as estimated by scale variations (perturbative or scale uncertainty). Although the precise size of the hadronisation uncertainty is debatable and perhaps often underestimated, it is certainly appropriate to consider the scale uncertainty as the dominant source of theoretical error on the precise determination of α_s from three-jet observables. From the planned luminosity of the ILC, one would expect measurements of event shapes comparable in statistical quality to what was obtained at LEP, thus allowing for precision QCD studies at ILC energies.

So far the three-jet rate and related event shapes have been calculated [1,2] up to the next-to-leading order (NLO), improved by a resummation of leading and subleading infrared logarithms [3,4] and by the inclusion of power corrections [5].

QCD studies of event shape observables at LEP [6] are based around the use of NLO parton-level event generator programs [7]. As expected, the current error on α_s from these observables [8] is dominated by the theoretical uncertainty. Clearly, to improve the determination of α_s , the calculation of the NNLO corrections to these observables becomes mandatory. We present here the first NNLO calculation of three-jet production and related event shape variables.

2 Calculation

Three-jet production at tree-level is induced by the decay of a virtual photon (or other neutral gauge boson) into a quark-antiquark-gluon final state. At higher orders, this process receives corrections from extra real or virtual particles. The individual partonic channels that contribute through to NNLO are shown in Table 1. All of the tree-level and loop amplitudes associated with these channels are known in the literature [9–12].

For a given partonic final state, jets are reconstructed according to the same definition as in the experiment, which is applied to partons instead of hadrons. At leading order, all three final state partons must be well separated from each other. At NLO, up to four partons can be present in the final state, two of which can be clustered together, whereas at NNLO, the final state can consist of up to five partons, such that as many as three partons can be clustered together. The more partons in the final state, the better one expects the matching between theory and experiment to be.

LO	$\gamma^* \rightarrow q \bar{q} g$	tree level
NLO	$\gamma^* \rightarrow q \bar{q} g$	one loop
	$\gamma^* \rightarrow q \bar{q} g g$	tree level
	$\gamma^* \rightarrow q \bar{q} q \bar{q}$	tree level
NNLO	$\gamma^* \rightarrow q \bar{q} g$	two loop
	$\gamma^* \rightarrow q \bar{q} g g$	one loop
	$\gamma^* \rightarrow q \bar{q} q \bar{q}$	one loop
	$\gamma^* \rightarrow q \bar{q} q \bar{q} g$	tree level
	$\gamma^* \rightarrow q \bar{q} g g g$	tree level

Table 1: Partonic contributions to three-jet final states in perturbative QCD.

The two-loop $\gamma^* \rightarrow q \bar{q} g$ matrix elements were derived in [9] by reducing all relevant Feynman integrals to a small set of master integrals using integration-by-parts [13] and Lorentz invariance [14] identities, solved with the Laporta algorithm [15]. The master integrals [16] were computed from their differential equations [14] and expressed analytically in terms of one- and two-dimensional harmonic polylogarithms [17].

The one-loop four-parton matrix elements relevant here [11] were originally derived in the context of NLO corrections to four-jet production and related event shapes [18,19]. One of these four-jet parton-level event generator programs [19] is the starting point for our calculation, since it

already contains all relevant four-parton and five-parton matrix elements.

The four-parton and five-parton contributions to three-jet-like final states at NNLO contain infrared real radiation singularities, which have to be extracted and combined with the infrared singularities [20] present in the virtual three-parton and four-parton contributions to yield a finite result. In our case, this is accomplished by introducing subtraction functions, which account for the infrared real radiation singularities, and are sufficiently simple to be integrated analytically. Schematically, this subtraction reads:

$$\begin{aligned}
d\sigma_{NNLO} = & \int_{d\Phi_5} (d\sigma_{NNLO}^R - d\sigma_{NNLO}^S) \\
& + \int_{d\Phi_4} (d\sigma_{NNLO}^{V,1} - d\sigma_{NNLO}^{VS,1}) \\
& + \int_{d\Phi_5} d\sigma_{NNLO}^S + \int_{d\Phi_4} d\sigma_{NNLO}^{VS,1} + \int_{d\Phi_3} d\sigma_{NNLO}^{V,2} ,
\end{aligned}$$

where $d\sigma_{NNLO}^S$ denotes the real radiation subtraction term coinciding with the five-parton

tree level cross section $d\sigma_{NNLO}^R$ in all singular limits [21]. Likewise, $d\sigma_{NNLO}^{VS,1}$ is the one-loop virtual subtraction term coinciding with the one-loop four-parton cross section $d\sigma_{NNLO}^{V,1}$ in all singular limits [22]. Finally, the two-loop correction to the three-parton cross section is denoted by $d\sigma_{NNLO}^{V,2}$. With these, each line in the above equation is individually infrared finite, and can be integrated numerically.

Systematic methods to derive and integrate subtraction terms were available in the literature only to NLO [23, 24], with extension to NNLO in special cases [25]. In the context of this project, we fully developed an NNLO subtraction formalism [26], based on the antenna subtraction method originally proposed at NLO [19, 24]. The basic idea of the antenna subtraction approach is to construct the subtraction terms from antenna functions. Each antenna function encapsulates all singular limits due to the emission of one or two unresolved partons between two colour-connected hard partons. This construction exploits the universal factorisation of phase space and squared matrix elements in all unresolved limits. The individual antenna functions are obtained by normalising three-parton and four-parton tree-level matrix elements and three-parton one-loop matrix elements to the corresponding two-parton tree-level matrix elements. Three different types of antenna functions are required, corresponding to the different pairs of hard partons forming the antenna: quark-antiquark, quark-gluon and gluon-gluon antenna functions. All these can be derived systematically from matrix elements [27] for physical processes.

The factorisation of the final state phase space into antenna phase space and hard phase space requires a mapping of the antenna momenta onto reduced hard momenta. We use the mapping derived in [28] for the three-parton and four-parton antenna functions. To extract the infrared poles of the subtraction terms, the antenna functions must be integrated analytically over the appropriate antenna phase spaces, which is done by reduction [29] to known phase space master integrals [30].

A detailed description of the calculation will be given elsewhere [31].

3 Results

The resulting numerical programme, EERAD3, yields the full kinematical information on a given multi-parton final state. It can thus be used to compute any infrared-safe observable in e^+e^- annihilation related to three-particle final states at $\mathcal{O}(\alpha_s^3)$. As a first application, we derived results for the NNLO corrections to the thrust distribution [32].

In the numerical evaluation, we use $M_Z = 91.1876$ GeV and $\alpha_s(M_Z) = 0.1189$ [8]. Figure 1 displays the perturbative expression for the thrust distribution at LO, NLO and NNLO, evaluated for LEP and ILC energies. The error band indicates the variation of the prediction under shifts of the renormalisation scale in the range $\mu \in [Q/2; 2Q]$ around the e^+e^- centre-of-mass energy Q .

It can be seen that even at linear collider energies, inclusion of the NNLO corrections enhances the thrust distribution by around 10% over the range $0.03 < (1 - T) < 0.33$, where relative scale uncertainty is reduced by about 30% between NLO and NNLO. Outside this range, one does not expect the perturbative fixed-order prediction to yield reliable results. For $(1 - T) \rightarrow 0$, the convergence of the perturbative series is spoilt by powers of logarithms $\ln(1 - T)$ appearing in higher perturbative orders, thus necessitating an all-order resummation of these logarithmic terms [3, 4], and a matching of fixed-order and resummed predictions [33].

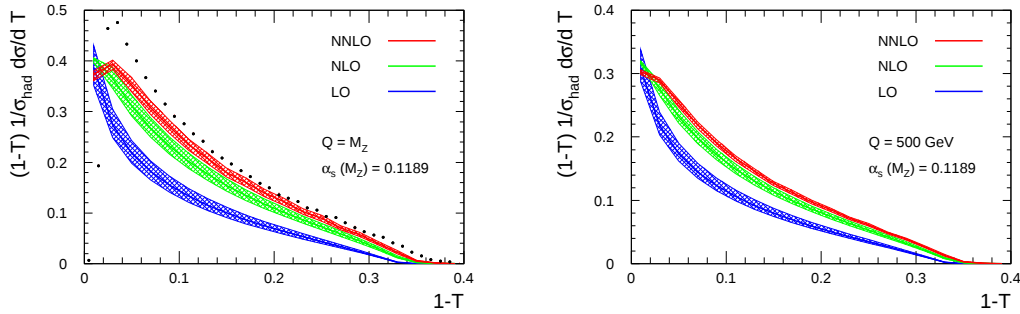


Figure 1: Thrust distribution at LEP and at the ILC with $Q = 500$ GeV.

The perturbative parton-level prediction is compared with the hadron-level data from the ALEPH collaboration [34] in Figure 1. Similar data are also available from the other LEP experiments [35]. The shape and normalisation of the parton-level NNLO prediction agrees better with the data than at NLO. We also see that the NNLO corrections account for approximately half of the difference between the parton-level NLO prediction and the hadron-level data. A full study including resummation of infrared logarithms and hadronisation corrections is underway.

4 Conclusions

We developed a numerical programme which can compute any infrared-safe observable through to $\mathcal{O}(\alpha_s^3)$, which we applied here to determine the NNLO corrections to the thrust distribution. These corrections are moderate, indicating the convergence of the perturbative expansion. Their inclusion results in a considerable reduction of the theoretical error on the thrust distribution. Our results will allow a significantly improved determination of the strong coupling constant from jet observables from existing LEP data as well as from future ILC data.

Acknowledgements

This research was supported in part by the Swiss National Science Foundation (SNF) under contract 200020-109162, by the UK Science and Technology Facilities Council and by the European Commission under contract MRTN-2006-035505 (Heptools).

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