New results for 5-point functions

J. Gluza[1] and T. Riemann[2*]

1 – Institute of Physics, Univ. of Silesia, Universytecka 4, 40007 Katowice, Poland

2 – Deutsches Elektronen-Synchrotron DESY
Platanenallee 6, D–15738 Zeuthen, Germany

Bhabha scattering is one of the processes at the ILC where high precision data will be expected. The complete NNLO corrections include radiative loop corrections, with contributions from Feynman diagrams with five external legs. We take these diagrams as an example and discuss several features of the evaluation of pentagon diagrams. The tensor functions are usually reduced to simpler scalar functions. Here we study, as an alternative, the application of Mellin-Barnes representations to 5-point functions. There is no evidence for an improved numerical evaluation of their finite, physical parts. However, the approach gives interesting insights into the treatment of the IR-singularities.

1 Introduction

Bhabha scattering,

$$e^+ + e^- \to e^+ + e^-, \tag{1}$$

is one of the most important reactions at e^+e^- colliders.[a] At ILC energies, small angle Bhabha scattering is dominated by pure photonic contributions and is foreseen as a luminosity monitor, and large angle Bhabha scattering is also one of the reactions with an expected jigh event statistics and with a very clean theoretical Standard Model prediction. For these reasons, a NNLO (next-to-next-to leading order) prediction of the complete QED contributions and a NNLLO (next-to-next-to leading logarithmic order) prediction in the Standard Model are needed. The virtual QED corrections at NNLO accuracy have been determined in a series of articles quite recently [3–11]. A complete evaluation of the photonic corrections covers additionally the real photon emission contributions and fermion pair production.

In this talk, we discuss one class of Feynman diagrams for real photon emission, namely radiative loop corrections,

$$e^+ + e^- \to e^+ + e^- + \gamma, \tag{2}$$

which are contributing at NNLO to reaction (1). Their evaluation includes 5-point functions. Usually, the scalar, vector, and tensor functions of this type will be reduced to simpler one-loop functions. We also discuss an alternative approach, based on Mellin-Barnes representations of Feynman parameter integrals.

*Presented by T.R.

[a]A link to the slides of this contribution is [1]. See also [2].

2 Reduction of 5-point functions

A critical point in an algebraic reduction of vector or higher tensor 5-point functions to scalar 2-point, 3-point, and 4-point functions (in four dimensions) is the appearance of inverse Gram determinants. It is known that these inverse Gram determinants are spurious [12–14] and that they may be canceled out in the final analytical expressions. For the approach proposed in [15], we have demonstrated this cancellation quite recently. Because that part of the presentation was decribed in some detail in other contexts [16, 17], we don't repeat the material in these proceedings again.

The focus will be on two questions:

- Is the MB-approach useful for the numerical evaluation of the finite parts of scalar, vector, and tensor 5-point functions?
- How to treat the infrared divergencies of these functions?

3 Mellin-Barnes representation for massive 5-point functions

The use of Mellin-Barnes (MB) integrals for the representation and evaluation of Feynman integrals has a long history, although a systematic use of it became possible quite recently. The replacement of massive propagators by MB-integrals was proposed in [18] for a finite 3-point function. It was worked out for one-loop n-point functions with arbitrary indices (powers of propagators) in $d = 4 - 2\epsilon$ dimensions in [19–21], where also some of the related earlier literature is discussed, as well as the applicability to tensor integrals and to multi-loop problems. The aim was a replacement of massive by massless propagators. In [22], the Feynman parameter representation (or α-parameter representation, the difference plays no role here) was derived and then for the characteristic function of the diagrams an MB-representation was applied. Along this line, a systematic approach to MB-presentations for divergent multi-loop integrals was derived and solved for non-trivial massless and massive cases [23–29]. Since software packages like AMBRE.m [30] (in Mathematica, for the derivation of MB-representations), MB.m [31] (in Mathematica, for their analytical and series expansion in ϵ), and XSUMMER [32] (in FORM [33], for taking sums of their residues) became publicly available, quite involved integrals may be treated, see e.g. [34].

Of course, such a complicated task like the evaluation of – ideally – arbitrary Feynman integrals will not be finally solved with using one or the other method. In fact, already quite simple problems may be used to demonstrate the limitations of some approach. We will study here, with MB-integrals, some one-loop functions of massive QED as occurring in Bhabha scattering, with focussing on the 5-point function shown in Figure 1.

We define

$$I_5[A(q)] \quad = \quad e^{\epsilon\gamma_E} \int \frac{d^d q}{i\pi^{d/2}} \frac{A(q)}{d_1 d_2 d_3 d_4 d_5}, \tag{3}$$

with the chords Q_i,

$$d_i = (q - Q_i)^2 - m_i^2. \tag{4}$$

This representation becomes unique after choosing one of the chords (and the direction of the loop momentum), e.g.:

$$Q_5^\mu = 0, \quad Q_1^\mu = p_1^\mu. \tag{5}$$

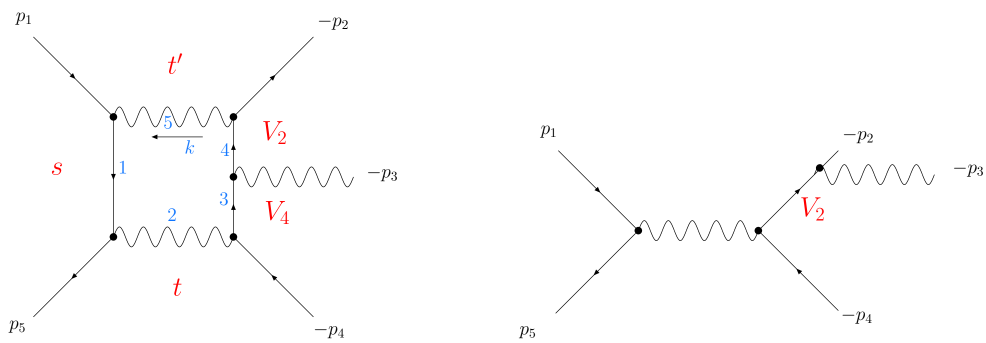

Figure 1: A pentagon topology and a Born topology

The numerator $A(q)$ contains the tensor structure,

$$A(q) = \{1, q^\mu, q^\mu q^\nu, q^\mu q^\nu q^\rho, \cdots\}, \tag{6}$$

or may be used to define pinched diagrams; e.g. a shrinking of line 5 leads to a box diagram corresponding to

$$I_5[d_5] = e^{\epsilon\gamma_E} \int \frac{d^d q}{i\pi^{d/2}} \frac{1}{d_1 d_2 d_3 d_4}. \tag{7}$$

For details of the derivation of Feynman parameter integrals we refer to any textbook on perturbative quantum field theory or to [35]. A Feynman parameter representation for Fig. 1 is:

$$I_5[A(q)] \quad = \quad -e^{\epsilon\gamma_E} \int_0^1 \prod_{j=1}^{5} dx_j \; \delta\left(1 - \sum_{i=1}^{5} x_i\right) \frac{\Gamma\left(3+\epsilon\right)}{F(x)^{3+\epsilon}} \; B(q), \tag{8}$$

with $B(1) = 1, B(q^\mu) = Q^\mu, B(q^\mu q^\nu) = Q^\mu Q^\nu - \frac{1}{2} g^{\mu\nu} F(x)/(2+\epsilon)$, and $Q^\mu = \sum x_i Q_i^\mu$. The diagram depends on five kinematical invariants and the F-form in (8) is:

$$F(x) \quad = \quad m_e^2(x_2+x_4+x_5)^2 + [-s]x_1x_3 + [-V_4]x_3x_5 + [-t]x_2x_4 + [-t']x_2x_5 + [-V_2]x_1x_4. \tag{9}$$

Henceforth, $m_e = 1$. It is evident that the F-form cannot be made more compact. After the introduction of seven subsequent Mellin-Barnes representations,

$$\frac{1}{[A(x) + Bx_ix_j]^R} \quad = \quad \frac{1}{2\pi i} \int_{\mathcal{C}} dz [A(x)]^z [Bx_ix_j]^{-R-z} \frac{\Gamma(R+z)\Gamma(-z)}{\Gamma(R)}, \tag{10}$$

one for each additive term in F, we may perform the x-integrations using a generalization of the integral representation of the Beta function:

$$\int_0^1 \prod_{j=1}^{N} dx_j \; x_j^{\alpha_j - 1} \; \delta\left(1 - x_1 - \cdots - x_N\right) \quad = \quad \frac{\Gamma(\alpha_1)\cdots\Gamma(\alpha_N)}{\Gamma\left(\alpha_1 + \cdots + \alpha_N\right)}. \tag{11}$$

The final MB-integral may be easily derived using our Mathematica package AMBRE.m [30] (see also example1.nb and example2.nb of the package). The representation is five-dimensional after twice applying Barnes' first lemma. The integrals are well-defined on integration strips parallel to the imaginary axis, for a finite value of $\epsilon = d/2 - 2$. After an analytical continuation in ϵ, preferrably done by MB.m [31], one gets a sequence of finite, multi-dimensional MB-integrals. We performed these steps and met the following situation for the terms proportional to $1/\epsilon$ and $O(1)$:

- scalar integrals: the MB-integrals are up to three-fold;
- vector integrals: the MB-integrals are up to three-fold;
- tensor integrals: the MB-integrals are up to five-fold.

We performed some experimental calculations, but there is no need to go into more detail: A numerical evaluation of these integrals, especially of the five-dimensional ones, in the Minkowskian region, is not competative to the old-fashioned numerical packages like FF [36], or LoopTools [36–38], which rely on the preceding algebraic reduction of all the 5-point functions to well-known scalar 2- to 4-point functions.

For this reason, we restrict the discussion now to the infrared divergent parts only. As is well-known, they have a lower dimensionality, and here the MB-presentations are well-suited. As examples we will use the scalar and vector 5-point functions.

4 Infrared singularities

Let us consider first the scalar function. A set of five independent invariants may be read off from (9):

$$\begin{aligned} s &= (p_1 + p_5)^2, \\ t &= (p_4 + p_5)^2, \\ t' &= (p_1 + p_2)^2, \\ V_2 &= 2p_2p_3 \quad \sim E_3, \\ V_4 &= 2p_4p_3 \quad \sim E_3, \end{aligned}$$

and the two massless propagators are $d_5 = q^2$ and $d_2 = (q + p_1 + p_5)^2$. In the IR-limit, where $E_3 \to 0$, it will be $t' \approx t$ and $0 \le V_2, V_4 << s, |t|$. The leading IR-singularities are easily found algebraically from the following decomposition:

$$\begin{aligned} \frac{1}{d_1d_2d_3d_4d_5} &= \frac{-1}{s}\left[\frac{2(q-Q_5)(q-Q_2)}{d_1d_2d_3d_4d_5} + \frac{1}{V_2}\left(\frac{2(q-Q_5)(q-Q_3)}{d_1d_3d_4d_5} - \frac{1}{d_1d_3d_4} - \frac{1}{d_1d_4d_5}\right)\right. \\ &\left. + \frac{1}{V_4}\left(\frac{2(q-Q_2)(q-Q_4)}{d_1d_2d_3d_4} - \frac{1}{d_1d_2d_3} - \frac{1}{d_1d_3d_4}\right)\right]. \end{aligned} \tag{12}$$

The 4-point functions depend on the variables (t, t', V_2) and (t, t', V_4), respectively, and the leading IR-singularities of I_5 trace back, by construction, to the two IR-divergent 3-point functions:

$$\int \frac{d^dk}{d_1d_2d_3d_4d_5} = \frac{1}{sV_2}\int \frac{d^dk}{d_1d_4d_5} + \frac{1}{sV_4}\int \frac{d^dk}{d_1d_2d_3} + \cdots = \frac{1}{\epsilon}\left[\frac{F(t')}{sV_2} + \frac{F(t)}{sV_4}\right] + \cdots \tag{13}$$

The integrals with numerators are constructed such that they are free of IR-singularities arising from the virtual photon lines. It is of importance here to observe that the denominators V_2 and V_4 are proportional to the photon energy E_3 and thus give rise to additional IR-problems, stemming from the photon phase space integral over the squared sum of matrix elements; e.g.:

$$\int \frac{d^3p_3}{2E_3}\,\frac{A}{E_3}\,\frac{B(E_3)}{E_3} \to \int_0^\omega \frac{dE_3}{E_3} = \ln(E_3)|_0^\omega. \tag{14}$$

Here, one term (A/E_3) comes from the real photon emission Born diagram, and the other one $(B(E_3)/E_3)$ from our pentagon diagram. After dimensional regularisation, this becomes evaluable and contributes also to the Laurent series in ϵ. We learn from (14) that a complete treatment of the IR-problem includes a careful control of the subleading (and in 4 dimensions non-integrable) terms like $1/V_i$ and $\ln(V_i)/V_j$. This leads to phase space integrals with a behaviour like:

$$\begin{aligned}\int_0^\omega \frac{dE_3}{E_3^{5-d}}\left(\frac{a}{\epsilon} + b\ln(E_3) + c\right) &= -\frac{2a+b}{4\epsilon^2} - \frac{c - 2a\ln(\omega)}{2\epsilon} \\ &\quad + c\ln(\omega) + \frac{1}{2}(2a+b)\ln^2(\omega) + O(\epsilon).\end{aligned} \tag{15}$$

Evidently, one separates with the 3-point functions in (13) only a leading singularity, while we expect expressions like

$$\int \frac{d^dk}{d_1d_2d_3d_4d_5} = \frac{A_2}{sV_2\epsilon} + \frac{A_4}{sV_4\epsilon} + \frac{B_2}{sV_2}\ln(V_2) + \frac{B_4}{sV_4}\ln(V_4) + \frac{C_2}{sV_2} + \frac{C_4}{sV_4} + \cdots \tag{16}$$

Subleading singuarities may arise from the ϵ-finite 4- and 3-point functions with pre-factors $1/V_i$.

It is also evident that the whole above discussion immediately transfers over to vector and tensor integrals.

Concentrating now on the IR-divergent parts, we may safely assume now the validity of the Born kinematics, including

$$t' = t, \tag{17}$$

which is justified bcause of the vanishing photon momentum in this limit. This 'eats' another MB-integration (in the F-form (9) one additive term vanishes), and the starting point of further discussions are four-dimensional MB-integrals. For the scalar pentagon:

$$I_5 = \frac{-e^{\epsilon\gamma_E}}{(2\pi i)^4}\prod_{i=1}^{4}\int\limits_{-i\infty+u_i}^{+i\infty+u_i} dz_i(-s)^{z_2}(-t)^{z_4}(-V_2)^{z_3}(-V_4)^{-3-\epsilon-z_1-z_2-z_3-z_4}\frac{\prod\limits_{j=1}^{12}\Gamma_j}{\Gamma_0\Gamma_{13}\Gamma_{14}}, \tag{18}$$

with a normalization $\Gamma_0 = \Gamma[-1-2\epsilon]$, and the other Γ-functions are:

$$\begin{aligned}
\Gamma_1 &= \Gamma[-z_1], \;\; \Gamma_2 = \Gamma[-z_2], \;\; \Gamma_3 = \Gamma[-z_3], \;\; \Gamma_4 = \Gamma[1+z_3], \\
\Gamma_5 &= \Gamma[1+z_2+z_3], \;\; \Gamma_6 = \Gamma[-z_4], \;\; \Gamma_7 = \Gamma[1+z_4], \;\; \Gamma_8 = \Gamma[-1-\epsilon-z_1-z_2], \\
\Gamma_9 &= \Gamma[-2-\epsilon-z_1-z_2-z_3-z_4], \;\; \Gamma_{10} = \Gamma[-2-\epsilon-z_1-z_3-z_4], \\
\Gamma_{11} &= \Gamma[-\epsilon+z_1-z_2+z_4], \; \Gamma_{12} = \Gamma[3+\epsilon+z_1+z_2+z_3+z_4],
\end{aligned}$$

and, in the denominator:

$$\Gamma_{13} = \Gamma[-1-\epsilon-z_1-z_2-z_4], \;\; \Gamma_{14} = \Gamma[-\epsilon-z_1-z_2+z_4]. \tag{19}$$

The I_5 is finite if all Γ-functions in the numerator have positive real parts of the arguments; this may be fulfilled for finite ϵ (here we follow the method invented in [25]):

$$\epsilon = -\frac{3}{4}. \tag{20}$$

The real shifts u_i of the integration strips r_i may be chosen to be:

$$\begin{aligned}
u_1 &= -5/8, \\
u_2 &= -7/8, \\
u_3 &= -1/16, \\
u_4 &= -5/8, \\
u_5 &= -1/32.
\end{aligned} \tag{21}$$

The further discussion of the scalar case is very similar to that of the QED vertex function given in [39], so we may concentrate here on the results for the IR-divergent part:

$$I_5^{IR} = I_5^{IR}(V_2) + I_5^{IR}(V_4), \tag{22}$$

$$I_5^{IR}(V_i) = \frac{I_{-1}^s(V_i)}{\epsilon} + I_0^s(V_i). \tag{23}$$

The explicit expressions for the inverse binomial sums solving the MB-integrals are obtained by applying the residue theorem (closing the integration contours to the left):

$$\frac{I_{-1}(V_i)^s}{\epsilon} = \frac{1}{2sV_i\epsilon}\sum_{n=0}^{\infty}\frac{(t)^n}{\binom{2n}{n}(2n+1)}, \tag{24}$$

with I_{-1}^s being in accordance with (13), and:

$$I_0(V_i)^s = \frac{1}{2sV_i}\sum_{n=0}^{\infty}\frac{(t)^n}{\binom{2n}{n}(2n+1)}\left[-2\ln(-V_i) - 3S_1(n) + 2S_1(2n+1)\right], \tag{25}$$

where we introduce the harmonic numbers $S_k(n) = \sum_{i=1}^{n} 1/i^k$, and have to understand $\ln(-V_i) = \ln(V_i/s) + \ln[-(s+i\delta)/m_e^2]$.

The series may be summed up in terms of polylogarithmic functions with the aid of Table 1 of Appendix D of [40]:

$$\sum_{n=0}^{\infty} \frac{t^n}{\binom{2n}{n}(2n+1)} = \frac{y}{y^2-1} 2\ln(y), \tag{26}$$

$$\sum_{n=0}^{\infty} \frac{t^n}{\binom{2n}{n}(2n+1)} S_1(n) = \frac{y}{y^2-1}\left[-4\mathrm{Li}_2(-y) - 4\ln(y)\ln(1+y) + \ln^2(y) - 2\zeta_2\right], \tag{27}$$

$$\sum_{n=0}^{\infty} \frac{t^n}{\binom{2n}{n}(2n+1)} S_1(2n+1) = \frac{y}{y^2-1}\Bigg[2\mathrm{Li}_2(y) - 4\mathrm{Li}_2(-y) - 4\ln(y)\ln(1+y) +2\ln(y)\ln(1-y) + \frac{1}{2}\ln^2(y) - 4\zeta_2\Bigg], \tag{28}$$

with

$$y \equiv y(t) = \frac{\sqrt{1-4/t}-1}{\sqrt{1-4/t}+1}. \tag{29}$$

For the vector and higher tensor 5-point functions one gets quite similar results. The IR-divergent pieces arise only from those contributions, which are proportional to the chords Q_2 and Q_5 of the massless internal lines (one of them is set to zero here, $Q_5 = 0$):

$$I_5^{IR}[q^\mu] = Q_2^\mu \left(\frac{I_{-1}^v(V_2, V_4)}{\epsilon} + I_0^v(V_2, V_4) \right). \tag{30}$$

The MB-integrals introduced in (8) will not get modified by the additional factors $B(q^\mu)$ etc., but the subsequent x-integrations will. For the vector integrals, we obtain:

$$I_5[q^\mu]|_{t'=t} = \sum_{i=1}^{5} Q_i^\mu I_5(i), \tag{31}$$

and

$$I_5(2) = \frac{-e^{\epsilon\gamma_E}}{(2\pi i)^4} \prod_{i=1}^{4} \int_{-i\infty+u_i}^{+i\infty+u_i} dz_i (-s)^{z_2}(-t)^{z_4}(-V_2)^{z_3}(-V_4)^{-3-\epsilon-z_1-z_2-z_3-z_4} \frac{\prod_{j=1}^{12}\Gamma_j^v}{\Gamma_0^v\Gamma_{13}^v\Gamma_{14}^v}, \tag{32}$$

where it is $\Gamma_j^v = \Gamma_j$ with two exceptions:

$$\begin{aligned} \Gamma_{10}^v &= \Gamma[-1-\epsilon-z_1-z_3-z_4], \\ \Gamma_0^v &= \Gamma[-2\epsilon]. \end{aligned} \tag{33}$$

After similar manipulations as described above, we obtain finally for the IR-divergent part of the vector pentagon (and, not discussed at all, the tensor pentagon):

$$I_5^{IR}[q^\mu] \quad = \quad Q_2^\mu \; I_5^{IR}(V_4) + Q_5^\mu \; I_5^{IR}(V_2), \tag{34}$$

and

$$I_5^{IR}[q^{\mu\nu}] \quad = \quad Q_2^\mu Q_2^\nu \; I_5^{IR}(V_4) + Q_5^\mu Q_5^\nu \; I_5^{IR}(V_2). \tag{35}$$

In the above derivations, we chose arbitrarily $Q_5 = 0$. The leading and non-leading IR-divergent parts of the tensor functions are contained in those terms of the tensor decomposition, which are proportional to the chords of the massless internal lines, and they agree with the corresponding scalar functions.

In conclusion, we have demonstrated, by analysing the loop functions without squaring matrix elements, that IR divergencies of scalar and tensor one-loop pentagon diagrams can be treated in a systematic, efficient way by using Mellin-Barnes representations. The leading singularities of the ϵ expansion of MB-integrals are obtained straightforwardly and have the same IR-structure as the vertex functions obtained by quenching. Both the leading and non-leading singular parts (the latter being kinematical end point singularities) can be expressed by a few well-known inverse binomial sums or, equivalently, polylogarithmic functions. The IR-structure of vector and tensor functions is completely reducible to that of the scalar function.

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