

Two-Loop Electroweak NLL Corrections: from Massless to Massive Fermions

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Recently the two-loop next-to-leading logarithmic (NLL) virtual corrections to arbitrary processes with massless external fermions have been calculated. Within the spontaneously broken electroweak theory the one- and two-loop mass singularities have been derived to NLL accuracy and expressed as universal correction factors depending only on the quantum numbers of the external particles. This talk summarizes the results for massless fermionic processes and presents new aspects arising in the extension of the corresponding loop calculations to massive external fermions. As a preliminary result, the Abelian form factor for massive fermions is given.

1 Electroweak corrections at high energies

Past and present collider experiments have explored high-energy processes at energy scales at the order of or below the masses M_W and M_Z of the weak gauge bosons. But the Large Hadron Collider (LHC) and the proposed International Linear Collider (ILC) will reach scattering energies in the TeV regime. For the first time, the characteristic energy Q of the reactions will be very large compared to M_W . At these high energies $Q \gg M_W$, electroweak radiative corrections are enhanced by large logarithms $\ln(Q^2/M_W^2)$, which start to be sizable at energies of a few hundred GeV and increase with energy. At LHC and ILC, logarithmic electroweak effects can amount to tens of per cent at one loop and several per cent at two loops. In view of the expected experimental precision especially at ILC, theoretical predictions with an accuracy of about 1% are required, so the two-loop corrections are crucial.

For sufficiently high Q , mass-suppressed terms of $\mathcal{O}(M_W^2/Q^2)$ become negligible and the electroweak corrections assume the form of a tower of logarithms with terms $\alpha^l \ln^j(Q^2/M_W^2)$, $0 \leq j \leq 2l$, at l loops. The leading logarithms (LLs) with power $j = 2l$ are known as Sudakov logarithms [2]. The subleading logarithms with $j = 2l - 1, 2l - 2, \dots$ are denoted as next-to-leading logarithmic (NLL), next-to-next-to-leading logarithmic (N²LL) terms, and so on. The experience with four-fermion processes [3, 4] shows that the subleading logarithmic contributions may be of the same size as the leading ones. In addition, large cancellations occur between the individual logarithmic terms, so the restriction to the LL approximation is not sufficient, and the NLL corrections or even further subleading terms are required.

1.1 Origin of electroweak logarithms

Logarithms $\ln(Q^2/M_W^2)$ arise from mass singularities, when a virtual gauge boson (photon γ , Z or W^\pm boson) couples to an on-shell external leg and to any other (internal or external) line of the diagram. The region where the gauge boson momentum is collinear to the

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momentum of the external particle yields a single-logarithmic one-loop contribution. In the special case that the gauge boson is exchanged between two different external legs, a double-logarithmic contribution arises from the regions where the gauge boson momentum is soft and collinear to one of the external momenta. In addition, ultraviolet (UV) singularities lead to single-logarithmic contributions.

In the case of photons, the mass singularities are not regulated by a finite gauge boson mass. In $D = 4 - 2\epsilon$ space-time dimensions, the singularities appear as poles $1/\epsilon$ and $1/\epsilon^2$ per loop. For a consistent treatment of leading and subleading logarithmic contributions, each pole in ϵ has to be counted like a logarithm $\ln(Q^2/M_W^2)$. Finite masses of the external particles regularize the collinear singularities and lead to logarithms involving these masses.

It has been shown at one loop for arbitrary processes [5] and at two loops for massless fermionic processes [6] that the electroweak LL and NLL corrections are universal: they depend only on the quantum numbers of the external particles and can be written in terms of universal correction factors which factorize from the Born matrix element.

1.2 Approaches for virtual two-loop electroweak corrections at high energies

Two-loop electroweak corrections at high energies have been studied in recent years with two complementary approaches. On the one hand, evolution equations known from QCD have been applied to the electroweak theory by splitting the latter into a symmetric $SU(2) \times U(1)$ regime above the weak scale M_W and a QED regime below the weak scale. Then the evolution equations permit to resum the one-loop result to all orders in perturbation theory. From this approach the LL [7] and NLL [8] corrections for arbitrary processes as well as the N²LL approximation for massless four-fermion processes $ff \rightarrow f'f'$ [9] are known, where the NLL and N²LL terms are valid in the equal mass approximation $M_Z = M_W$.

On the other hand, various calculations have checked and extended the resummation predictions by explicit diagrammatic two-loop calculations. At first, the LLs for the fermionic form factor [10] were obtained, then the LLs for arbitrary processes [11], the angular-dependent NLLs for arbitrary processes [12] and the complete NLLs for the massless fermionic form factor [13]. Finally, the N³LL approximation for the massless fermionic form factor was calculated for $M_Z = M_W$ and combined with the evolution equations, yielding the N³LL corrections for massless neutral-current four-fermion processes in an expansion $M_Z \approx M_W$ around the equal mass case [3, 4].

2 Two-loop next-to-leading logarithmic corrections

In order to complete the missing diagrammatic NLL calculations, the goal of this project is to derive virtual two-loop electroweak corrections for arbitrary processes in NLL accuracy. In contrast to the resummation approaches, we rely on the complete spontaneously broken electroweak theory. We consider processes with external momenta p_i , where all kinematical invariants, $r_{ij} = (p_i + p_j)^2$, are of the order of the large scale $Q^2 \gg M_W^2$. We implement the particle masses M_W , M_Z , m_t and M_{Higgs} , which are different, but of the same order. In particular, we consider a massive top quark and neglect the masses of the other fermions. We thus get combinations of large logarithms $L = \ln(Q^2/M_W^2)$ and poles in ϵ from virtual photons. At l loops, terms $\alpha^l L^n \epsilon^{-j+n}$ are LLs if $j = 2l$, and NLLs if $j = 2l - 1$ ($n = 0, 1, \dots$). The NLL coefficients involve angular-dependent logarithms, $\ln(-r_{ij}/Q^2)$, and logarithms of mass ratios, $\ln(M_Z^2/M_W^2)$ and $\ln(m_t^2/M_W^2)$.

We have completed the calculation for processes with massless external fermions [6] and are about to extend our results to massive fermionic processes.

2.1 Extraction of NLL mass singularities

In order to extract the mass singularities from the loop diagrams, we first isolate the so-called factorizable contributions: These are diagrams where the gauge bosons couple only to external legs, not to internal legs of the tree subdiagram, and where the gauge boson momenta have been set to zero in the tree subdiagram. For these factorizable contributions we use a soft-collinear approximation which eliminates the Dirac structure of the loop corrections and factorizes the loop integrals from the Born matrix element. This approximation is an extension of the eikonal approximation and reproduces the correct NLL result not only for soft, but also for collinear gauge bosons.

The remaining non-factorizable contributions are obtained by subtracting from all diagrams yielding mass singularities the factorizable contributions. We have shown that the non-factorizable contributions vanish due to the collinear Ward identities proven in [5].

Therefore only the factorizable contributions need to be evaluated explicitly. For the LL and NLL terms at two loops, we need a double-logarithmic contribution from a soft and collinear gauge boson which is exchanged between two different external legs, and another, at least single-logarithmic, loop correction. The two-loop factorizable contributions in the case of massless external fermions are depicted in Figure 1.

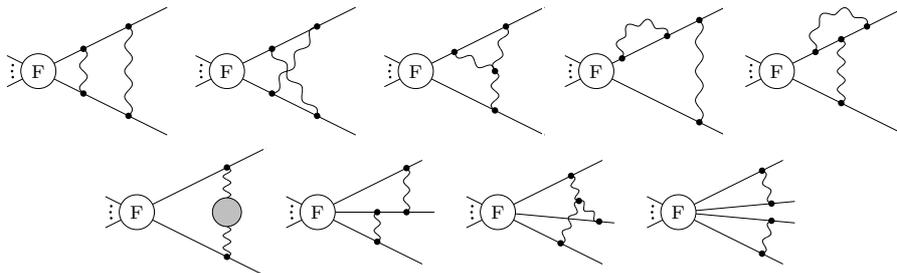


Figure 1: Two-loop factorizable contributions for massless external fermions. “F” denotes the factorized tree subdiagram, in which the gauge boson momenta are set to zero. The grey blob in the gauge boson propagator stands for all possible self-energy insertions.

The factorizable diagrams also include NLL contributions from UV momentum regions. When a subdiagram with a small characteristic scale of the order M_W^2 yields UV singularities which are renormalized at the scale Q^2 , large logarithms $\ln(Q^2/M_W^2)$ arise. The soft-collinear approximation mentioned above is not valid for UV momenta, so we cannot use it for subdiagrams of this type and employ projection techniques instead.

2.2 Results for massless fermionic processes

We have evaluated the loop integrals of the factorizable contributions with two independent methods: An automatized algorithm which is based on the sector decomposition technique [14], and the method of expansion by regions combined with Mellin-Barnes representations (see [4] and references therein). The NLL result for massless fermionic processes

$f_1 f_2 \rightarrow f_3 \cdots f_n$ has been published in [6]. It allows to write the combined one- and two-loop result in the factorized form $\mathcal{M} = \mathcal{M}_0 F^{\text{sew}} F^Z F^{\text{em}}$, where \mathcal{M}_0 is the Born matrix element, and the correction terms read $F^{\text{sew}} = \exp[\frac{\alpha}{4\pi} F_1^{\text{sew}} + (\frac{\alpha}{4\pi})^2 G_2^{\text{sew}}]$, $F^Z = 1 + \frac{\alpha}{4\pi} \Delta F_1^Z$ and $F^{\text{em}} = \exp[\frac{\alpha}{4\pi} \Delta F_1^{\text{em}} + (\frac{\alpha}{4\pi})^2 \Delta G_2^{\text{em}}]$. The symmetric-electroweak factor F^{sew} equals the result from a symmetric $SU(2) \times U(1)$ theory where all gauge boson masses are equal to M_W . The factor F^Z incorporates the terms from the mass difference $M_Z \neq M_W$. And the electromagnetic terms in F^{em} factorize and exponentiate separately, such that a separation of the singularities due to the massless photon is possible. The one-loop terms F_1^{sew} and ΔF_1^{em} get exponentiated, and the additional two-loop terms G_2^{sew} and ΔG_2^{em} are proportional to β -function coefficients. For details of the correction terms, we refer to [6].

Our results confirm the resummation predictions based on the evolution equations. By applying our general correction factors to the case of massless four-fermion scattering, we have found agreement with the neutral-current results in [3, 9], and we have obtained a new NLL result for the charged-current processes.

3 From massless to massive fermions

For massive external fermions, the diagrams from the factorizable contributions have to be reevaluated, additional diagrams with Yukawa interactions have to be considered and the cancellation of the non-factorizable contributions must be verified. This section deals with new complications which arise from massive external fermions in the loop integrals.

3.1 Expansion by regions with massive external particles

Expansion by regions [15, 16] is a powerful method for the asymptotic expansion of loop integrals. It is based on the following recipe: Divide the integration domain of the loop momenta into regions corresponding to the asymptotic limit considered. In every region, expand the integrand appropriately. Integrate each of the expanded terms over the whole integration domain.

The integrand is expanded before integration, and each expanded term has a unique order in powers of the large scale Q and the small scale M_W . But on-shell momenta p_i of massive external particles involve two scales, as their momentum squared is $p_i^2 = m_i^2 \sim M_W^2$ and their combinations with other external momenta are $r_{ij} = (p_i + p_j)^2 \sim Q^2$. In order to separate these two scales, the external momenta are reparametrized in terms of light-like momenta \tilde{p}_i as $p_i = \tilde{p}_i + (p_i^2/\tilde{r}_{ij})\tilde{p}_j$, with some other external leg $j \neq i$ and $\tilde{p}_i^2 = \tilde{p}_j^2 = 0$, $\tilde{r}_{ij} = 2\tilde{p}_i\tilde{p}_j$ [16]. Through this shift, all contractions of external momenta with loop momenta can now be divided into parts of distinct scales, and the expansion is done in inverse powers of the new large scales $\tilde{r}_{ij} = r_{ij} + \mathcal{O}(M_W^2)$.

With respect to any pair of external light-like momenta \tilde{p}_i, \tilde{p}_j , the loop momenta can be expressed in Sudakov components parallel and perpendicular to these external momenta: $k = k_i^{(i,j)}\tilde{p}_i/Q + k_j^{(i,j)}\tilde{p}_j/Q + k_\perp^{(i,j)}$, with $k_i^{(i,j)} = 2\tilde{p}_j k Q/\tilde{r}_{ij}$, $k_j^{(i,j)} = 2\tilde{p}_i k Q/\tilde{r}_{ij}$ and $\tilde{p}_i k_\perp^{(i,j)} = \tilde{p}_j k_\perp^{(i,j)} = 0$. In each region, the components of the loop momenta are assigned specific sizes in powers of Q and M_W . Typical regions are listed in Table 1. While the hard, soft, collinear and ultrasoft regions are already present for massless external particles, the two ultracollinear regions are only relevant for massive external particles.

Region	hard	soft	i -collinear	j -collinear	ultrasoft	i -ultracoll.	j -ultracoll.
$k_i^{(i,j)}$	Q	M_W	Q	M_W^2/Q	M_W^2/Q	M_W^2/Q	M_W^4/Q^3
$k_j^{(i,j)}$	Q	M_W	M_W^2/Q	Q	M_W^2/Q	M_W^4/Q^3	M_W^2/Q
$k_1^{(i,j)}$	Q	M_W	M_W	M_W	M_W^2/Q	M_W^3/Q^2	M_W^3/Q^2

Table 1: Typical regions with the corresponding sizes of loop momentum components

3.2 Power singularities and fermion masses

Asymptotic expansions with small masses and large kinematical scales not only produce logarithmic mass singularities, but also power singularities $Q^2/M_{W,Z}^2$ and Q^2/m_i^2 . These are generated at two loops by subdiagrams with a small scale of the order M_W^2 . The method of expansion by regions predicts, for the contribution of each region, where power singularities can appear, by means of a simple power counting in the expanded integrals.

When complete Feynman diagrams are considered, the terms in the numerator ensure the cancellation of the power singularities. In diagrams where power singularities are present for individual scalar integrals, care must be taken to keep all the mass factors in the numerator which ensure the cancellations. In particular, the masses in the numerator of fermion propagators and in the Dirac equation of the spinors may not be neglected. Therefore we are not allowed to use the soft–collinear approximation for small-scale subdiagrams. However, these are exactly the same diagrams where we have employed alternative projection techniques already in the massless case in order to get the UV contributions right.

Additional complications originate from fermion masses in the numerator due to the chiral structure of the electroweak theory. With each mass factor along a fermion line, the chirality of the fermion in its interactions with the weak gauge bosons changes. We have found, though, that fermion masses in the numerator are relevant exclusively in pure QED diagrams where the chirality changes do not matter.

3.3 Preliminary results

We have completed the calculation of all factorizable contributions involving two massive or massless external fermion legs. This permits to determine the two-loop form factor in an Abelian model with both a massive gauge boson (mass M_W , coupling α) and a massless one (coupling α'). The one-loop form factor as a function of the two external fermion masses is given by $F_1(m_1, m_2) = \frac{\alpha}{4\pi} F_1^M + \frac{\alpha'}{4\pi} [F_1^0(0, 0) + \Delta F_1^0(m_1) + \Delta F_1^0(m_2)]$. The NLL contribution (up to the order ϵ^2) from the massive gauge boson is independent of the fermion masses,

$$F_1^M = -L^2 - \frac{2}{3}L^3\epsilon - \frac{1}{4}L^4\epsilon^2 + 3L + \frac{3}{2}L^2\epsilon + \frac{1}{2}L^3\epsilon^2, \quad (1)$$

with $L = \ln(Q^2/M_W^2)$, while the contribution from the massless gauge boson is split into a completely massless part and corrections for each of the fermion masses:

$$F_1^0(0, 0) = -2\epsilon^{-2} - 3\epsilon^{-1}, \quad \Delta F_1^0(0) = 0, \\ \Delta F_1^0(m_i) = \epsilon^{-2} + L_i\epsilon^{-1} + \frac{1}{2}L_i^2 + \frac{1}{6}L_i^3\epsilon + \frac{1}{24}L_i^4\epsilon^2 + \frac{1}{2}\epsilon^{-1} + \frac{1}{2}L_i + \frac{1}{4}L_i^2\epsilon + \frac{1}{12}L_i^3\epsilon^2, \quad (2)$$

with $L_i = \ln(Q^2/m_i^2)$. We have found that the NLL two-loop form factor (without closed fermion loops) simply exponentiates the one-loop result, $F_2(m_1, m_2) = \frac{1}{2}[F_1(m_1, m_2)]^2$.

4 Conclusions

We evaluate two-loop electroweak corrections in NLL accuracy for arbitrary processes with massive and massless external fermions. The methods which we have successfully applied for massless fermions work well also in the massive case, and the complications arising from fermion masses are under control. Preliminary results are already available for the form factor, they factorize and exponentiate like in the massless case. The calculation for processes with external fermions will soon be completed, and our method can be extended to arbitrary processes involving external gauge bosons or scalar particles.

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