

\mathcal{CP} -violating Loop Effects in the Higgs Sector of the MSSM

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\mathcal{CP} -violating effects in the Higgs sector of the Minimal Supersymmetric Standard Model with complex parameters (cMSSM) are induced by potentially large higher-order corrections. As a consequence, all three neutral Higgs bosons can mix with each other. Recent results for loop corrections in the Higgs sector of the cMSSM are reviewed [1]. Results for propagator-type corrections of $\mathcal{O}(\alpha_t \alpha_s)$ and complete one-loop results for Higgs cascade decays of the kind $h_a \rightarrow h_b h_c$ are summarised, and the proper treatment of external Higgs bosons in Higgs-boson production and decay processes is discussed.

1 Introduction

A striking prediction of models of supersymmetry (SUSY) is a Higgs sector with at least one relatively light Higgs boson. In the Minimal Supersymmetric extension of the Standard Model (MSSM) two Higgs doublets are required, resulting in five physical Higgs bosons. In lowest order these are the light and heavy \mathcal{CP} -even h and H , the \mathcal{CP} -odd A , and the charged Higgs bosons H^\pm . The Higgs sector of the MSSM can be characterised at lowest order by the two parameters (besides the gauge couplings) M_{H^\pm} and $\tan \beta \equiv v_2/v_1$, the ratio of the two vacuum expectation values. All other masses and mixing angles can be predicted in terms of these parameters. Higher-order contributions yield large corrections to the tree-level relations and, via complex phases, induce \mathcal{CP} -violating effects. In the MSSM with complex parameters (cMSSM) therefore all three neutral Higgs bosons can mix with each other. The corresponding mass eigenstates are denoted as h_1, h_2, h_3 . If the mixing between the three neutral mass eigenstates is such that the coupling of the lightest Higgs boson to gauge bosons is significantly suppressed, this state can be very light without being in conflict with the exclusion bounds from the LEP Higgs searches [2, 3]. In this case the second-lightest Higgs boson, h_2 , may predominantly decay into a pair of light Higgs bosons, $h_2 \rightarrow h_1 h_1$.

We report in this paper on recent progress on higher-order corrections in the Higgs sector of the cMSSM^a. We briefly discuss propagator-type corrections of $\mathcal{O}(\alpha_t \alpha_s)$ [8] and complete one-loop results for Higgs cascade decays of the kind $h_a \rightarrow h_b h_c$ ($a, b, c = 1, 2, 3$) [9]. In this context we put a particular emphasis on the treatment of external Higgs states in Higgs-boson production and decay process in the presence of \mathcal{CP} -violating mixing among all three neutral Higgs bosons.

^aSee e.g. Refs. [4–7] for recent reviews of the present status of higher-order corrections in the Higgs sector of the MSSM with and without complex phases.

2 External on-shell Higgs-bosons

The propagator matrix of the neutral Higgs bosons h, H, A can be written as a 3×3 matrix, $\Delta_{hHA}(p^2)$ (we neglect mixing with the Goldstone boson G and the Z boson in the propagator matrix since the corresponding contributions are of sub-leading two-loop order, see the discussion in Ref. [10]). This propagator matrix is related to the 3×3 matrix of the irreducible vertex functions by

$$\Delta_{hHA}(p^2) = - \left(\hat{\Gamma}_{hHA}(p^2) \right)^{-1}, \quad (1)$$

where $\hat{\Gamma}_{hHA}(p^2) = i [p^2 \mathbb{1} - \mathbf{M}_n(p^2)]$,

$$\mathbf{M}_n(p^2) = \begin{pmatrix} m_h^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_H^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_A^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}. \quad (2)$$

Here m_i ($i = h, H, A$) denote the tree-level Higgs-boson masses, and $\hat{\Sigma}_{ij}$ are the renormalised self-energies. Inversion of $\hat{\Gamma}_{hHA}(p^2)$ yields for the diagonal Higgs propagators ($i = h, H, A$)

$$\Delta_{ii}(p^2) = \frac{i}{p^2 - m_i^2 + \hat{\Sigma}_{ii}^{\text{eff}}(p^2)}, \quad (3)$$

where $\Delta_{hh}(p^2)$, $\Delta_{HH}(p^2)$, $\Delta_{AA}(p^2)$ are the (11), (22), (33) elements of the 3×3 matrix $\Delta_{hHA}(p^2)$, respectively. The structure of eq. (3) is formally the same as for the case without mixing, but the usual self-energy is replaced by the effective quantity $\hat{\Sigma}_{ii}^{\text{eff}}(p^2)$ which contains mixing contributions of the three Higgs bosons. It reads (no summation over i, j, k)

$$\hat{\Sigma}_{ii}^{\text{eff}}(p^2) = \hat{\Sigma}_{ii}(p^2) - i \frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}_{ki}^2(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}_{ij}^2(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}_{jk}^2(p^2)}, \quad (4)$$

where the $\hat{\Gamma}_{ij}(p^2)$ are the elements of the 3×3 matrix $\hat{\Gamma}_{hHA}(p^2)$ as specified above. The expressions for the off-diagonal Higgs propagators read (i, j, k all different, no summation over i, j, k)

$$\Delta_{ij}(p^2) = \frac{\hat{\Gamma}_{ij}\hat{\Gamma}_{kk} - \hat{\Gamma}_{jk}\hat{\Gamma}_{ki}}{\hat{\Gamma}_{ii}\hat{\Gamma}_{jj}\hat{\Gamma}_{kk} + 2\hat{\Gamma}_{ij}\hat{\Gamma}_{jk}\hat{\Gamma}_{ki} - \hat{\Gamma}_{ii}\hat{\Gamma}_{jk}^2 - \hat{\Gamma}_{jj}\hat{\Gamma}_{ki}^2 - \hat{\Gamma}_{kk}\hat{\Gamma}_{ij}^2}, \quad (5)$$

where we have dropped the argument p^2 of the $\hat{\Gamma}_{ij}(p^2)$ appearing on the right-hand side for ease of notation. The three complex poles \mathcal{M}^2 of Δ_{hHA} , eq. (1), are defined as the solutions of

$$\mathcal{M}_i^2 - m_i^2 + \hat{\Sigma}_{ii}^{\text{eff}}(\mathcal{M}_i^2) = 0, \quad i = h, H, A, \quad (6)$$

with a decomposition of the complex pole as $\mathcal{M}^2 = M^2 - iM\Gamma$, where M is the mass of the particle and Γ its width. We define the loop-corrected mass eigenvalues according to $M_{h_1} \leq M_{h_2} \leq M_{h_3}$.

We now turn to the on-shell properties of an in- or out-going Higgs boson. In order to ensure the correct on-shell properties of S-matrix elements involving external Higgs it is

convenient to introduce finite wave function normalisation factors \hat{Z}_i, \hat{Z}_{ij} (“Z-factors”). A vertex with an external Higgs boson, i , can be written as (with i, j, k all different, $i, j, k = h, H, A$, and no summation over indices)

$$\sqrt{\hat{Z}_i} \left(\Gamma_i + \hat{Z}_{ij} \Gamma_j + \hat{Z}_{ik} \Gamma_k + \dots \right), \quad (7)$$

where the ellipsis represents contributions from the mixing with the Goldstone boson and the Z boson, see Refs. [9, 10]. The Z-factors are given by:

$$\hat{Z}_i = \frac{1}{1 + \left(\hat{\Sigma}_{ii}^{\text{eff}} \right)'(\mathcal{M}_i^2)}, \quad \hat{Z}_{ij} = \frac{\Delta_{ij}(p^2)}{\Delta_{ii}(p^2)} \Big|_{p^2=\mathcal{M}_i^2} \quad (8)$$

where the propagators $\Delta_{ii}(p^2), \Delta_{ij}(p^2)$ have been given in eqs. (3) and (5), respectively. The Z-factors can be expressed in terms of a (non-unitary) matrix $\hat{\mathbf{Z}}$, whose elements take the form (with $\hat{Z}_{ii} = 1, i, j = h, H, A$, and no summation over i)

$$(\hat{\mathbf{Z}})_{ij} := \sqrt{\hat{Z}_i} \hat{Z}_{ij}. \quad (9)$$

A vertex with one external Higgs boson h_1 , for instance, is then given by

$$(\hat{\mathbf{Z}})_{hh} \Gamma_h + (\hat{\mathbf{Z}})_{hH} \Gamma_H + (\hat{\mathbf{Z}})_{hA} \Gamma_A + \dots, \quad (10)$$

where the ellipsis again represents contributions from the mixing with the Goldstone boson and the Z boson.

It should be noted that the definition of the Z-factors used here and in Ref. [9] differs slightly from the one in Ref. [10]. The Higgs-boson self-energies in eq. (8) are evaluated at the complex pole, whereas in Ref. [10] the real part of the complex pole had been used. Furthermore, in the definition of \hat{Z}_i in eq. (8) $\hat{\Sigma}_{ii}^{\text{eff}}$ appears, as compared to $\text{Re } \hat{\Sigma}_{ii}^{\text{eff}}$ in Ref. [10]. While the contributions of the imaginary parts in eq. (8) to Higgs-boson production and decay processes are formally of sub-leading two-loop order, it turns out that their inclusion in general improves the numerical stability of the results.

3 Propagator-type corrections of $\mathcal{O}(\alpha_t \alpha_s)$

The leading two-loop corrections of $\mathcal{O}(\alpha_t \alpha_s)$ have been recently been obtained [8] in the Feynman-diagrammatic approach for propagator-type corrections, which contribute to the predictions for the Higgs-boson masses, to wave function normalisation factors of external Higgs bosons and to effective couplings. The results are valid for arbitrary values of the complex parameters. The impact of the complex phases of the trilinear coupling A_t and the gluino mass parameter M_3 at the two-loop level turns out to be numerically sizable. As an example, in Fig. 1 the lightest Higgs-boson mass, M_{h_1} , is shown as a function of the phase φ_{A_t} of the trilinear coupling A_t . The one-loop result (dotted line) is compared with the new result that includes the $\mathcal{O}(\alpha_t \alpha_s)$ contributions (solid line). The dependence on the complex phase φ_{A_t} is much more pronounced in the two-loop result than in the one-loop case, which can easily be understood from the analytical structure of the corrections [8]. Thus, varying φ_{A_t} can give rise to shifts in the prediction for M_{h_1} of more than ± 5 GeV even in cases where the dependence on the complex phases in the one-loop result is very small. The new corrections have recently been implemented into the program **FeynHiggs** [10–12].

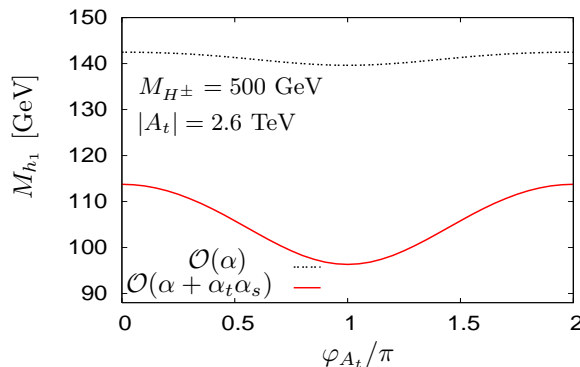


Figure 1: The lightest Higgs-boson mass, M_{h_1} , as a function of φ_{A_t} for $|A_t| = 2.6$ TeV and $M_{H^\pm} = 500$ GeV. The one-loop result (dashed line) is compared with the result including the $\mathcal{O}(\alpha_t \alpha_s)$ corrections (solid line). The other parameters are $M_{\text{SUSY}} = 1000$ GeV, $\mu = 1000$ GeV, $M_2 = 500$ GeV, $m_{\tilde{g}} = 1000$ GeV, $\tan \beta = 10$.

4 Complete one-loop results for Higgs cascade decays

For Higgs cascade decays of the kind $h_a \rightarrow h_b h_c$, where $a, b, c = 1, 2, 3$, recently complete one-loop results have been obtained in the cMSSM [9]. They have been supplemented with the state-of-the-art propagator-type corrections (see above), yielding the currently most precise prediction for this class of processes. The genuine vertex corrections turn out to be very important, yielding a large increase of the decay width compared to a prediction based on only the tree-level vertex dressed with propagator-type corrections. This is demonstrated in Fig. 2, where the full result for $\Gamma(h_2 \rightarrow h_1 h_1)$ as a function of M_{h_1} in the CPX scenario [13] is compared with results based on various approximations for the genuine contributions to the $h_2 h_1 h_1$ vertex. The complete result (denoted as ‘Full’) differs by more than a factor of six in this example (for values of M_{h_1} sufficiently below the kinematic limit of $M_{h_1} = 0.5 M_{h_2}$ where the decay width goes to zero) from the result for the case where only wave-function normalisation factors but no genuine one-loop vertex contributions are taken into account (‘Tree’). See Ref. [9] for a discussion of the other approximations shown in Fig. 2.

The new results for the Higgs cascade decays [9] have been used to analyse the impact of the limits on topological cross sections obtained from the LEP Higgs searches on the parameter space with a very light Higgs boson within the cMSSM. It has been found for the example of the CPX scenario [13] that, over a large part of the parameter space where the decay $h_2 \rightarrow h_1 h_1$ is kinematically possible, it is the dominant decay channel. A parameter region with $M_{h_1} \approx 45$ GeV and $\tan \beta \approx 6$ remains unexcluded by the limits on topological cross sections obtained from the LEP Higgs searches, confirming the results of the four LEP collaborations achieved in a dedicated analysis of the CPX benchmark scenario. The results of Ref. [9] will be incorporated into the public code **FeynHiggs**.

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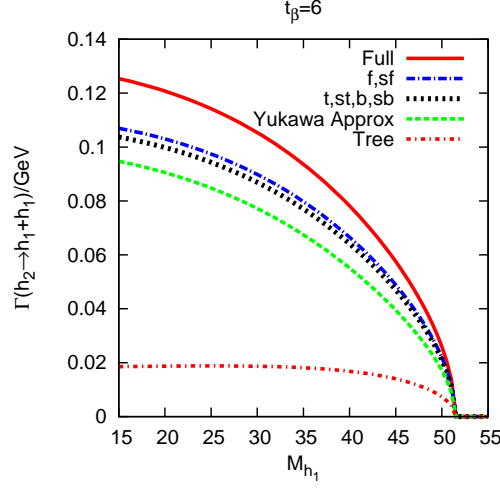


Figure 2: The full result for $\Gamma(h_2 \rightarrow h_1 h_1)$ as a function of M_{h_1} in the CPX scenario [13] for $\tan \beta = 6$ (M_{H^\pm} is varied) is compared with various approximations, see text.

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