The charged Higgs boson mass in the 2HDM: decoupling and CP violation

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Mass range of the charged Higgs boson in the 2HDM with explicit and spontaneous CP violation is discussed. Constraints on $M_{H^\pm}$ in the CP conserving 2HDM(II) are shown.

1 The 2HDM potential and spontaneous symmetries breaking

The most general, invariant under gauge group $SU(2)_L \times U(1)_Y$ and renormalizable potential of the Two Higgs Doublet Model (2HDM) [2, 3, 4] is given by

$$\begin{aligned} V = &\frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\ &+ \left[\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_6\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + h.c.\right] \\ &-\frac{1}{2}m_{11}^2\left(\Phi_1^\dagger\Phi_1\right) - \frac{1}{2}m_{22}^2\left(\Phi_2^\dagger\Phi_2\right) - \left[\frac{1}{2}m_{12}^2\left(\Phi_1^\dagger\Phi_2\right) + h.c.\right], \end{aligned} \tag{1}$$

where $\lambda_{1-4}, m_{11}^2, m_{22}^2 \in \mathbb{R}$ (by the hermicity of the potential), while in general $\lambda_{5-7}, m_{12}^2 \in \mathbb{C}$. In the most general CP breaking form it has 14 parameters, however only 11 are independent, see e.g. [5, 6]. In the model there are five Higgs particles: three neutral h_1, h_2, h_3 (for CP conservation - two CP-even h, H and one CP-odd A) and two charged Higgs bosons $H^\pm$.

1.1 Z_2 and CP symmetries

The Z_2 symmetry of the potential (1) is defined as the invariance of V under the following transformation of doublets: $\Phi_1 \to -\Phi_1, \Phi_2 \to \Phi_2$ or $\Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2$. If Z_2 (in either form) is a symmetry of the potential, then $m_{12}^2 = \lambda_6 = \lambda_7 = 0$. The Z_2 symmetry is *softly* broken by the terms proportional to m_{12}^2.

General 2HDM allows for CP violation both explicitly and spontaneously [7, 8, 2]. The CP violation can be naturally suppressed by imposing a Z_2 symmetry on the Higgs potential.

1.2 Reparametrization transformation

A global unitary transformation which mix two doublets and change their relative phase does not change the physical content of 2HDM as discussed recently in [9], see also [3, 4, 2]. It is given by

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \mathcal{F}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \qquad \mathcal{F} = e^{-i\rho_0}\begin{pmatrix} \cos\theta e^{i\rho/2} & \sin\theta e^{i(\tau-\rho/2)} \\ -\sin\theta e^{-i(\tau-\rho/2)} & \cos\theta e^{-i\rho/2} \end{pmatrix}. \tag{2}$$

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There are three *reparametrization* parameters - ρ, θ, τ, and in addition ρ_0 parameter as an overall phase. If $\theta = 0$ there is no mixing of two dublets and the transformation becomes a global transformation of doublets with an independent phase rotations (*rephasing*):

$$k = 1, 2 : \Phi_k \to e^{-i\rho_i}\Phi_k, \quad \rho_1 = \rho_0 - \frac{\rho}{2}, \quad \rho_2 = \rho_0 + \frac{\rho}{2}, \quad \rho = \rho_2 - \rho_1. \tag{3}$$

The original form of the potential is recovered by the appropriate changes of phases of the following coefficients:

1.3 Explicit and spontaneous CP violation in 2HDM

CP violation may occur in 2HDM only if Z_2 symmetry is broken [8, 2, 3, 4, 9]. A necessary condition for an *explicit CP violation* in the Higgs potential V is an existence of complex parameters. However, if there exists a reparametrization leading to V with only real parameters (*real basis*), then there is no explicit CP violation in V. A spontaneous CP breaking, by the vacuum state, is still possible [7, 8, 2].

In the simply analysis [14], which results we present here, only the potential with exact and softly broken Z_2 symmetry was considered, i.e. $\lambda_{6,7} = 0$. In studying 2HDM with an *explicit* CP conservation or violation the *real vacuum representation* [4] was applied. A spontaneous CP violation was discussed assuming the *explicitly CP conserving V*.

1.4 Vacuum expectation values

The most general vacuum (extremum) state can be described by [8, 11, 12, 13, 14]

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 e^{i\xi} \end{pmatrix}, \tag{4}$$

where $v_1, v_2, \xi, u \in \mathbb{R}$. By gauge transformation one can always make $v_1 > 0$. Below we will assume that $v_2 \neq 0$, with $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$, and $0 \leq \xi < 2\pi$.

For vacuum with $u \neq 0$ the electric charge is not conserved and the photon becomes a massive particle ("charged vacuum"). If $u = 0$ then a "neutral vacuum" are possible. Depending on the value of ξ there may or may not be a spontaneous CP violation [8, 3, 12, 13]. The useful quantity is $\nu = \frac{m_{12}^2}{2v_1 v_2}$ (or $\nu = \frac{\Re m_{12}^2}{2v_1 v_2}$) [4], which here is taken to be positive.

1.5 Extremum conditions

For the extremum states (4) the first derivatives of the considered potential lead to the following set of extremum conditions:

$$0 = u\left[v_1 v_2 \cos\xi\,(\lambda_4 + \lambda_5) - m_{12}^2\right], \; 0 = u\left[\lambda_2\left(u^2 + v_2^2\right) + \lambda_3 v_1^2 - m_{22}^2\right] \tag{5}$$

$$0 = v_2 \sin\xi\left[2\lambda_5 v_1 v_2 \cos\xi - m_{12}^2\right], \; 0 = v_2 \sin\xi\left[v_1^2\,(\lambda_3 + \lambda_4 - \lambda_5) + \lambda_2\left(u^2 + v_2^2\right) - m_{22}^2\right] \tag{6}$$

$$0 = v_1\left[v_2^2\left(\lambda_5 \cos^2 2\xi + \lambda_4\right) + \lambda_1 v_1^2 + \lambda_3\left(u^2 + v_2^2\right) - m_{11}^2\right] - m_{12}^2 v_2 \cos\xi \tag{7}$$

$$0 = u v_1 v_2 \sin\xi\,(\lambda_4 - \lambda_5), \; 0 = v_2 \cos\xi\left[v_1^2\,(\lambda_3 + \lambda_4 + \lambda_5) + \lambda_2\left(u^2 + v_2^2\right) - m_{22}^2\right] - m_{12}^2 v_1 \tag{8}$$

If $u = 0$ then above conditions are satisfied for an exact Z_2 symmetry ($m_{12}^2 = 0$) when the only possible neutral vacuum state is the one which respects CP, i.e. with $\sin\xi = 0$, and for a broken Z_2 symmetry. In the latter case two neutral vacuum states are possible - without and with CP violation, for $\sin\xi = 0$ and $\sin\xi \neq 0$, respectively. To get a real minimum of the potential the eigenvalues of the squared mass matrix have to be positive. We will assume in addition that positivity constraints hold guaranteeing stability of the vacuum [10].

1.6 Physical regions for CP conserving 2HDM

Expressions for masses of $H^\pm$ and A for 2HDM with an explicit or a spontaneous CP *conservation* are as follows.

Z_2 symmetry broken If Z_2 symmetry is softly broken ($\nu \neq 0$), then the masses squared of $H^\pm$ and A are given by:

$$M_{H^\pm}^2 = v^2\left(\nu - \frac{1}{2}(\lambda_4 + \lambda_5)\right), \quad M_A^2 = v^2(\nu - \lambda_5). \tag{9}$$

In order to have positive $M_{H^\pm}^2$ and M_A^2 inequalities $\lambda_5 + \lambda_4 < 2\nu$ and $\lambda_5 < \nu$ should hold.

Large masses for $H^\pm$ and A (9) can arise from large ν. In the limit $\nu \to \infty$ the decoupling is realized - h is like the Higgs boson in the Standard Model, while $H^\pm, A, H$ are heavy and almost degenerate [3, 4].

Exact Z_2 symmetry The results for an exact Z_2 symmetry can be obtained from above expressions in the limit $\nu \to 0$. Then $\lambda_5 < 0$. Masses cannot be too large, as here they can arise only due to $\lambda's$. However, large $\lambda's$ may violate tree-level unitarity constraints [15].

1.7 Physical regions for CP violating 2HDM

As it was mentioned above if the 2HDM potential breaks Z_2 symmetry then CP violation may be realized in the model. Note, that if CP is violated physical neutral Higgs states are h_1, h_2, h_3, without definite CP properties, while h, H, A are useful but only auxiliary states.

Explicit CP violation If there is explicit CP violation all formulae derived for the CP conservation case (9 and beyond) hold after the replacements: $\lambda_5 \to \Re\lambda_5$ and $m_{12}^2 \to \Re m_{12}^2$. Note, that the decoupling can be realized here as well, with large $M_{H^\pm}^2$ arising from large ν.

Spontaneous CP violation Spontaneous CP violation may appear if there is a CP breaking phase of the VEV, so $\sin\xi \neq 0$. From the extremum condition one gets that:

$$\cos\xi = \frac{m_{12}^2}{\lambda_5 2 v_1 v_2} = \frac{\nu}{\lambda_5}, \tag{10}$$

from which it follows that $|\nu/\lambda_5| < 1$. The squared masses for $H^\pm$ and A are given by the following expressions, see also [13]:

$$M_{H^\pm}^2 = \frac{v^2}{2}(\lambda_5 - \lambda_4), \quad M_A^2 = \frac{v^2}{\lambda_5}\left(\lambda_5^2 - \nu^2\right) = v^2\lambda_5\sin^2\xi. \tag{11}$$

We see that they are quite different from the formulae for $M_{H^\pm}^2$ and M_A^2 discussed above. (Note, that although A is no longer a physical state, positivity of M_A^2 still provides a good constraint since it gives at the same time a condition for positivity of squared masses of physical particles.) From the last expression for M_A^2 (11) it is easy to see that λ_5 have to be positive. Furthermore, squared masses (11) are positive if $\lambda_5 > \lambda_4$ and $\lambda_5 > \nu > 0$.

It is worth mentioning that the squared mass of $H^\pm$ does not depend on ν at all. Therefore, $M_{H^\pm}$ cannot be too large in 2HDM with CP violated spontaneously, for the same reason as in the discussed above case of exact Z_2 symmetry.

1.8 Conclusion on possible vacuum states in 2HDM

Regions where various vacuum states (conserving or spontaneously violating CP) can be realized in 2HDM are mutually exclusive [10, 12, 13, 14]. The mass of charged Higgs boson may serve as a guide over various regimes of the 2HDM. Existence of heavy charged Higgs boson, with mass above 600-700 GeV [4, 14], would be a signal that in 2HDM Z_2 symmetry is violated, and CP can be violated only explicitly.

2 Experimental constraints on the 2HDM(II) with CP conservation

Here we consider the CP conserving 2HDM, assuming that Z_2 symmetry is extended also on the Yukawa interaction, which allows to suppress the FCNC [16]. We limit ourself to constraints on the Model II of the Yukawa interaction, as in MSSM, see e.g. [17]. There are 7 parameters for the potential with softly breaking Z_2 symmetry: masses $M_h, M_H, M_A, M_{H^\pm}$, mixing angles α and $\tan\beta = v_2/v_1$, and parameter ν.

Couplings (relative to the corresponding couplings of the SM Higgs) are as follows:

	h	A
to W/Z:	$\chi_V = \sin(\beta-\alpha)$	0
to down quarks/charged leptons:	$\chi_d = \chi_V - \sqrt{1-\chi_V^2}\tan\beta$	$-i\gamma_5\tan\beta$
to up quarks:	$\chi_u = \chi_V + \sqrt{1-\chi_V^2}/\tan\beta$	$-i\gamma_5/\tan\beta$

H couples like h with following replacements: $\sin(\beta-\alpha) \to \cos(\beta-\alpha)$ and $\tan\beta \to -\tan\beta$. For large $\tan\beta$ there are enhanced couplings to $d-$type fermions. Note, that coupling $\chi^h_{VH^+} = \cos(\beta-\alpha)$ is complementary to the χ^h_V.

Important constraints on mass of charged Higgs boson in 2HDM (II) are coming from the $b \to s\gamma$ and $B \to \tau\nu$ decays. The rate for the first process calculated at the NNLO accuracy in the SM [18], after a comparison with the precise data from BaBar and Belle, leads to the constraint: $M_{H^\pm} > 295$ GeV at 95 % CL for $\tan\beta > 2$. This limit together with the constraints from the tree-level analysis of $B \to \tau\nu$ [19] is presented in Fig.1 (Left).

The 2HDM analysis has been performed at the one-loop level for the leptonic tau decays [20]. The constraints are shown in Fig.1 (Right). Not only lower, but also in the non-decoupling scenario upper limits can be derived here. In contrast to the mentioned results from b decays here the (one-loop) constraints depend on masses of neutral Higgs bosons.

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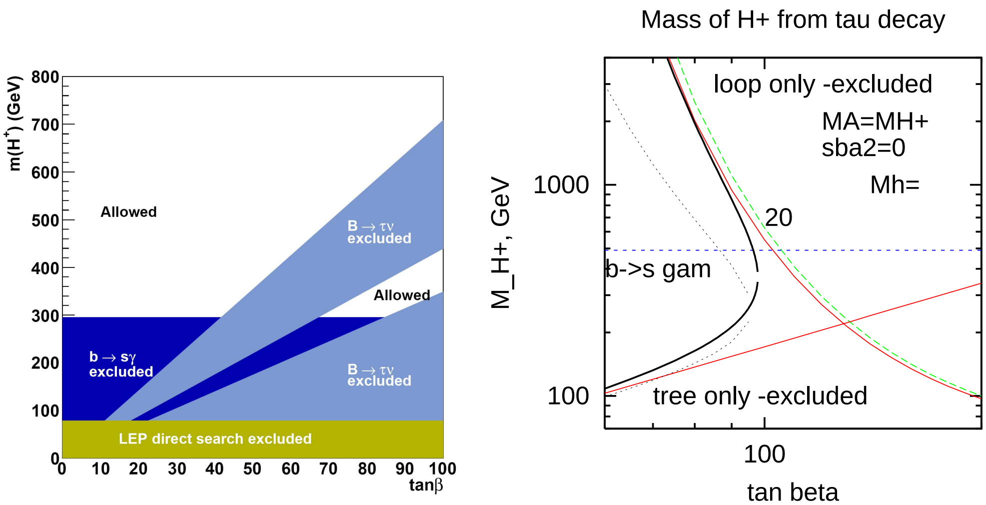

Figure 1: Left: Constraints from $B \to \tau\nu_\tau$ and $b \to s\gamma$ data on the charged Higgs boson mass as a function of $\tan\beta$ in 2HDM (II) [19]; Right: Limits from the leptonic τ decay for $M_h = 20$ GeV and $\chi_V^h = 0$ in 2HDM(II): tree-level exclusion of a region below the straight line $M_{H^\pm} \geq 1.71 \tan\beta$ GeV and one-loop exclusion of the region above the curve $\Delta \sim \tan\beta^2 \, [\ln \frac{M_h}{M_{H+}} + 1]$. The excluded region lies on the right on the curves: bold for $M_A = M_{H^\pm}$, dotted for $M_A = 100$ GeV. Exclusion from $\tau \to e\nu_\tau\bar{\nu}_e$ is represented by dashed line [20].

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