Fast Beam Diagnostics through Beamstrahlung at TESLA

Marko Ternick
BTU Cottbus
Email: markoternick@gmx.de

27. September 2002

To adjust the beams of the TESLA linear collider at high luminosity, fast beam diagnostics are needed. In other words, one has to measure the beam parameters within a few bunch crossings of a few hundred nanoseconds. We use energy distributions as well as angular distributions of e^+e^- *pairs created by beamstrahlung. These distributions depend strongly on the beam parameters as for example horizontal and vertical beam sizes and bunch length. A method to reconstruct these beam parameters is shown here. Numerical examples are presented for TESLA but the technique is equally valid for any linear collider.*
This report summarizes my work as a summer student at DESY Zeuthen.

1 Introduction

At circular e^+e^- colliders one of the main problems is synchrotron radiation, because it is the main source of energy loss. To avoid this, linear accelerators are planned, such as JLC in Japan, NLC in the U.S. and TESLA in Germany. Linear colliders have another big drawback - one can use the beams only once. Therefore, one has to increase the luminosity to get reasonable high event rates. The luminosity is defined as [1]

$$\mathcal{L} = \frac{1}{4\pi} \frac{f_{rep} n_b N_e^2}{\sigma_x \sigma_y} \times H_D, \qquad (1)$$

where

- f_{rep} is the pulse repetition frequency,
- n_b is the number of bunches per pulse,
- N_e is the number of electrons (positrons) per bunch,
- $\sigma_{x,y}$ is the horizontal (vertical) beam size at the interaction point and
- H_D is the disruption enhancement factor (typically $H_D \approx 2$).

To increase the luminosity one can for example increase the repetition frequency, the number of bunches or the number of particles within one bunch, however there are technical limitations. Another way is to decrease the beamsize σ_x or σ_y. But for very small bunches the electric field becomes so large that the interaction between the electron and positron bunches cannot be neglected. In this way electrons are accelerated towards the center of the positron bunch, and vice versa. This is known as the pinch effect. On the one hand this is good because it increases the luminosity by a factor H_D of approximately two. This effect is often called disruption enhancement. On the other hand, accelerated charges emit photons. This radiation is called beamstrahlung and was first observed at SLAC [4]. The average fractional beam energy loss due to beamstrahlung is [1]

$$\delta_E \approx 0.86 \frac{r_e^3 {N_e}^2 \gamma}{\sigma_z (\sigma_x + \sigma_y)^2}, \qquad (2)$$

where
r_e is the classical electron radius and
γ is the relativistic factor E_{beam}/m_e.

By choosing $\sigma_x \gg \sigma_y$, the energy loss δ_E becomes independent of the vertical beamsize σ_y. So we can increase the luminosity by decreasing the vertical beam size without increasing the energy loss at the same time. The relation $\sigma_x \gg \sigma_y$ means we use flat beams. This is planned at all future linear colliders with a ratio $R = \frac{\sigma_x}{\sigma_y} \approx 100$ [1, 2, 3].

2 LCAL at TESLA Detector

Beamstrahlung follows almost completely the direction of the beams. The angles with respect to the beam axis are typically only a few mrad [5]. Therefore, most of the photons vanish through the beampipe and it is not easy to detect them. But if the photons are highly energetic, i.e. photons with an energy above one MeV, they can create e^+ e^- pairs in the field of the bunch. Due to electromagnetic repulsion electrons (positrons) are deflected to larger angles in direction of flight of electrons (positrons). In the TESLA-Detector these particles can be studied with the so-called LCAL (Luminosity CALorimeter). It will cover an angular range from 6 to 28 mrad and will be located at a distance of 220 cm on both sides of the interaction point [6].

Figures 1 to 6 show the normalized energy distribution for different beam parameters.[1] The circle in the figures is the outer radius of the beampipe. One can see the energy distribution for the standard TESLA setup and energy distributions for different beam parameters. Obviously, with modified beam parameters the distributions change. For example a somewhat longer bunch will result in a smaller maximal radius (figure 3) and a bunch with an angle in the x-z plane will result in a different angular distribution (figure 4). In this case the z axis is the direction of flight, x and y are the horizontal and vertical direction, respectively. An tilt angle is a rotation around the z axis.

To characterize the distribution at LCAL we have defined special quantities: the average radius, energy and arrival time which will referred to hereafter as observables.

The average radius is defined as:

$$\bar{r} = \sum_i r_i E_i, \tag{3}$$

where the sum runs over all bins in figure 1 with a radius r_i larger than the beampipe radius of 12 mm. These bins represent calorimeter cells (1mm $\times$ 1mm). The energy within one cell is E_i

The average energy is:

$$\bar{E} = \frac{E}{N}, \tag{4}$$

where
E is the total energy and
N is the number of particles.

[1] The simulations are done with Daniel Schulte's GUINEA PIG [7].

The arrival time is:

$$\bar{t} = \frac{1}{0.9N} \sum_i t_i, \tag{5}$$

where the sum runs over the central 90% of the distribution. We have chosen to use a truncated mean because a very small fraction of extremely late particles would shift the average value by a factor of order ten.[2]

3 The Mathematics

Now we can vary some beam parameters and see how the observables change. In figure 13 to 18 the change of the observables $\bar{r}$, $\bar{E}$ and $\bar{t}$ is shown as a function of the parameters σ_x and σ_z. The vertical line corresponds to the TESLA standard value. In the diagrams, you can see curves $\bar{r}(\sigma_x, \sigma_z)$, $\bar{E}(\sigma_x, \sigma_z)$ and $\bar{t}(\sigma_x, \sigma_z)$ fitted to the data. You can also see that these curves are different for example for the energy. We can expand the functions in a Taylor series:

$$\begin{aligned}\bar{E}(\sigma_x, \sigma_z) = \bar{E}(\sigma_{x_0}, \sigma_{z_0}) \\ + \frac{\partial \bar{E}}{\partial \sigma_x}(\sigma_x - \sigma_{x_0}) + \frac{\partial \bar{E}}{\partial \sigma_z}(\sigma_z - \sigma_{z_0}) \\ + \dots,\end{aligned} \tag{6}$$

where
$\bar{E}(\sigma_{x_0}, \sigma_{z_0}) = \bar{E}_0$ is the energy average for standard TESLA parameters. The partial derivatives are obtained from the slopes of the fitted functions at the TESLA standard parameter values.

We can do a similar expansion for $\bar{t}$ and $\bar{r}$ to get the matrix equation (to first order)

$$\begin{pmatrix} \bar{E} - \bar{E}_0 \\ \bar{t} - \bar{t}_0 \\ \bar{r} - \bar{r}_0 \end{pmatrix} = \begin{pmatrix} \frac{\partial \bar{E}}{\partial \sigma_x} & \frac{\partial \bar{E}}{\partial \sigma_z} \\ \frac{\partial \bar{t}}{\partial \sigma_x} & \frac{\partial \bar{t}}{\partial \sigma_z} \\ \frac{\partial \bar{r}}{\partial \sigma_x} & \frac{\partial \bar{r}}{\partial \sigma_z} \end{pmatrix} \begin{pmatrix} \sigma_x - \sigma_{x_0} \\ \sigma_z - \sigma_{z_0} \end{pmatrix} \tag{7}$$

or

$$b = Ax \tag{8}$$

This overdetermined system is not trivial to solve because a rectangular matrix can not be inverted in the usual way. So we use the generalized matrix inverse of Moore and Penrose [8]

[2] The number of particles N is not measurable (since there are in the order of 10^4). This is only to show how the basic technique works.

which has a very nice property: for a given system $b = Ax$, the shortest length least squares solution is $x = A^+b$. We can calculate the inverse in the following ways

$$A^+ = (A^T A)^{-1} A^T \tag{9}$$

if $A^T A$ is invertible or

$$A^+ = A^T (AA^T)^{-1} \tag{10}$$

if AA^T is invertible.

To get the beam parameters one only has to multiply a known matrix to a vector. As an example how the principle works we simulated with GUINEA PIG bunches of width $\sigma_x = 600$ nm and length $\sigma_z = 250$ μm. The TESLA standard parameters are $\sigma_x = 553$ nm and $\sigma_z = 300$ μm. With the technique described above we get $\sigma_x = (596.69 \pm 16.32)$ nm and $\sigma_z = (241.49 \pm 7.94)$ μm where the errors are statistical only.

4 Observables

To use this technique in practice we define a new set of observables.

1. Average Radius $\bar{r}$

   $\bar{r} = \sum_i r_i E_i$

   where the sum runs over all bins of the histogram (for example figure 1) with $r_i \geq 12$ mm

2. Maximum Radius r_{max}

   radius that contains 99.9% of the energy

3. Angle α

   $\min_{\alpha \in [0,180]} \left(\sum_i (\alpha - \alpha_i) E_i \right)$

   where the sum runs over all bins of the histogram (for example figure 1) with $r_i \geq 12$ mm

4. Total Energy E_{total}

   total energy that is deposit in the calorimeter

5. Sine Amplitude A

   from the fit (for example figure 7)

6. up, right, down, left u, r, d, l

   u= energy deposition between 10° and 50°

   r= energy deposition between -40° and -80°

   d= energy deposition between -130° and -170°

   l= energy deposition between 100° and 140°

7. Ratio r_1

   $r_1 = \frac{(l+r)}{(u+d)}$

8. Ratio r_2

   $r_2 = \frac{u}{d}$

   The definition of the ratios r_1 and r_2 was proposed by Tauchi and Yokoya [9].

9. Up, Right, Down, Left U, R, D, L

   U= energy deposition between $\alpha - 90°$ and $\alpha + 90°$

   R= energy deposition between $\alpha - 180°$ and α

   D= energy deposition between $\alpha - 270°$ and $\alpha - 90°$

   L= energy deposition between $\alpha°$ and $\alpha + 180°$

All observables but the first two are defined twice, first for all measurable particles, i.e. for particles with an radius $r \geq 12$ mm and second for particles with $r \geq 50$ mm.

5 Numerical Results

For e^+e^- bunches with the same bunch sizes, i.e. $\sigma_x = 600$ nm, $\sigma_y = 8$ nm and $\sigma_z = 250$ μm we obtained the following values $\sigma_x = 633.0$ nm, $\sigma_y = 5.7$ nm and $\sigma_z = 231.5$ μm.

In practice it is probable that the bunches have different sizes. For this reason we applied the technique to that case.

	nom	*in*	*out*
$\sigma_{x_1} [nm]$	553	600	781
$\sigma_{x_2} [nm]$	553	500	574
$\sigma_{y_1} [nm]$	5.0	12.0	14.6
$\sigma_{y_2} [nm]$	5.0	8.0	-3.4
$\sigma_{z_1} [\mu m]$	300	280	303
$\sigma_{z_2} [\mu m]$	300	250	221

The nominal beam sizes are listed in the first column. In the second column you find the input parameter for the simulation program. The last column shows the result obtained using the matrix equation $x = A^+b$. Due to statistical fluctuations the calculated beamsizes are not exactly right, only three of the six parameters show the right tendency. There are improvements necessary (see chapter 6). Then it should be possible to use this method for a feedback system.

In a third example we also changed the angles, first for both bunches in the same direction.

	nom	*in*	*out*
$\sigma_x[nm]$	553	600	614
$\sigma_y[nm]$	5.0	8.0	5.2
$\sigma_z[\mu m]$	300	250	227
$angle_x[\mu rad]$	0	-700	1088
$angle_y[\mu rad]$	0	10.0	24.7
$angle_\phi[°]$	0	-5.0	-2.85

Up to now it is only possible to calculate the absolute value of the angle because a positive and a negative angle would lead to very similar observables. But it should be possible to get the sign from the ratio $r_2 = \frac{u}{d}$.

6 Possible Improvements of this Method

1. Use additional obserables.
   For example time, average energy deposition per calorimeter cell, ratios $R_1 = \frac{(L+R)}{(U+D)}$ and $R_2 = \frac{U}{D}$
2. Average over several bunches.
   To minimize the errors it is better to use the average over a few bunches.
3. Use second order approximations for equations (6) to (8):

$$\begin{aligned}\bar{E}(\sigma_x,\sigma_z) = \bar{E}(\sigma_{x_0},\sigma_{z_0}) & \\ +\frac{\partial\bar{E}}{\partial\sigma_x}(\sigma_x-\sigma_{x_0}) + \frac{\partial\bar{E}}{\partial\sigma_z}(\sigma_z-\sigma_{z_0}) & \\ +\frac{1}{2}\frac{\partial^2\bar{E}}{\partial\sigma_x^2}(\sigma_x-\sigma_{x_0})^2 & \\ +\frac{\partial^2\bar{E}}{\partial\sigma_x\partial\sigma_z}(\sigma_x-\sigma_{x_0})(\sigma_z-\sigma_{z_0}) & \\ +\frac{1}{2}\frac{\partial^2\bar{E}}{\partial\sigma_z^2}(\sigma_z-\sigma_{z_0})^2 & \\ +\ldots, & \qquad (11)\end{aligned}$$

In matrix notation this reads

$$\begin{pmatrix}\bar{E}-\bar{E}_0\\ \bar{t}-\bar{t}_0\\ \bar{r}-\bar{r}_0\end{pmatrix} = \begin{pmatrix}\frac{\partial\bar{E}}{\partial\sigma_x} & \frac{\partial\bar{E}}{\partial\sigma_z}\\ \frac{\partial\bar{t}}{\partial\sigma_x} & \frac{\partial\bar{t}}{\partial\sigma_z}\\ \frac{\partial\bar{r}}{\partial\sigma_x} & \frac{\partial\bar{r}}{\partial\sigma_z}\end{pmatrix}\begin{pmatrix}\sigma_x-\sigma_{x_0}\\ \sigma_z-\sigma_{z_0}\end{pmatrix}$$
$$+\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}\sigma_x-\sigma_{x_0}\\ \sigma_z-\sigma_{z_0}\end{pmatrix}^T\begin{pmatrix}\frac{\partial^2\bar{E}}{\partial\sigma_x^2} & \frac{\partial^2\bar{E}}{\partial\sigma_x\partial\sigma_z}\\ \frac{\partial^2\bar{E}}{\partial\sigma_x\partial\sigma_z} & \frac{\partial^2\bar{E}}{\partial\sigma_z^2}\end{pmatrix}\begin{pmatrix}\sigma_x-\sigma_{x_0}\\ \sigma_z-\sigma_{z_0}\end{pmatrix}$$
$$+\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}\sigma_x-\sigma_{x_0}\\ \sigma_z-\sigma_{z_0}\end{pmatrix}^T\begin{pmatrix}\frac{\partial^2\bar{t}}{\partial\sigma_x^2} & \frac{\partial^2\bar{t}}{\partial\sigma_x\partial\sigma_z}\\ \frac{\partial^2\bar{t}}{\partial\sigma_x\partial\sigma_z} & \frac{\partial^2\bar{t}}{\partial\sigma_z^2}\end{pmatrix}\begin{pmatrix}\sigma_x-\sigma_{x_0}\\ \sigma_z-\sigma_{z_0}\end{pmatrix}$$
$$+\begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix}\sigma_x-\sigma_{x_0}\\ \sigma_z-\sigma_{z_0}\end{pmatrix}^T\begin{pmatrix}\frac{\partial^2\bar{r}}{\partial\sigma_x^2} & \frac{\partial^2\bar{r}}{\partial\sigma_x\partial\sigma_z}\\ \frac{\partial^2\bar{r}}{\partial\sigma_x\partial\sigma_z} & \frac{\partial^2\bar{r}}{\partial\sigma_z^2}\end{pmatrix}\begin{pmatrix}\sigma_x-\sigma_{x_0}\\ \sigma_z-\sigma_{z_0}\end{pmatrix}$$

or

$$b = Ax + \begin{pmatrix}1\\0\\0\end{pmatrix}x^T H_{\bar{E}}x + \begin{pmatrix}0\\1\\0\end{pmatrix}x^T H_{\bar{t}}x + \begin{pmatrix}0\\0\\1\end{pmatrix}x^T H_{\bar{r}}x$$

or

$$b = Ax + \begin{pmatrix}x^T H_{\bar{E}}x\\ x^T H_{\bar{t}}x\\ x^T H_{\bar{r}}x\end{pmatrix}$$

or

$$b = (A + x^T H)x \qquad (12)$$

where

$$x^T H = \begin{pmatrix}x^T H_{\bar{E}}\\ x^T H_{\bar{t}}\\ x^T H_{\bar{r}}\end{pmatrix} \qquad (13)$$

contains the three Hessian matrices (see [10]). The problem is the inversion of equation (12). It is possible to solve numerically $x_n = (A + x_{n-1}^T H)^+ b$ but this is not very fast.

Acknowledgments

I would like to thank my supervisor Achim Stahl for all his help, Jens Dittrich for first aid in mathematical questions, Wolfgang Lohmann and Eric Switzer for correcting the English spelling and grammar - and last not least the TESLA group Zeuthen.

References

[1] TESLA Technical Design Report, p II-12ff, http://tesla.desy.de/new_pages/TDR_CD/PartII/accel.html

[2] JLC-I, KEK Report 92-16, 1992, p 126, http://www-jlc.kek.jp/JLC.proposal-e.html

[3] NLC Zeroth-Order Design Report, p 785, http://www.slac.stanford.edu/accel/nlc/zdr/

[4] Giovanni Bonvicini *et al.*, First Observation of Beamstrahlung, Phys. Rev. Lett. **62** (1989) 2381-2384

[5] Giovanni Bonvicini and Jim Welch, Large Angle Beamstrahlung as a Beam-beam Monitoring Tool, arXiv:physics/9812023, http://motor1.physics.wayne.edu/∼cinabro/beamstrahlung/beamstrahlung.html

[6] TESLA Technical Design Report, p IV-89f, http://tesla.desy.de/new_pages/TDR_CD/PartIV/detect.html

[7] Daniel Schulte, GUINEA PIG, http://www-sldnt.slac.stanford.edu/nlc/programs/guinea_pig/Original%20GP%20Home.html

[8] Eric W. Weisstein, CRC Consise Encyclopedia of Mathematics, p 1197, or Eric Weisstein's World of Mathematics http://mathworld.wolfram.com/Moore-PenroseMatrixInverse.html

[9] Toshiaki Tauchi and Kaoru Yokoya, Nanometer Beam-Size Measurement during Collisions at Linear Colliders, Phys. Rev. E, **51** (1995) 6119-6126, http://www-jlc.kek.jp/subg/ir/lib/DOC/Publication/HTML/Nanometer_beam_epsf/
Manabu Saigo, Pair Monitor for Beam Profile, Talk at LCWS 2002, http://lcws2002.korea.ac.kr/

[10] Eric W. Weisstein, CRC Consise Encyclopedia of Mathematics, p 834, or Eric Weisstein's World of Mathematics http://mathworld.wolfram.com/HessianDeterminant.html

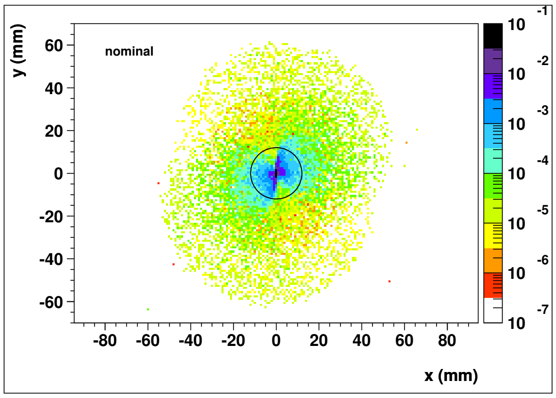

Fig. 1: *Energy distribution at LCAL for standard TESLA setup:* σ_x = 553.0 *nm,* σ_y = 5.0 *nm and* $\sigma_z = 300$ μm.

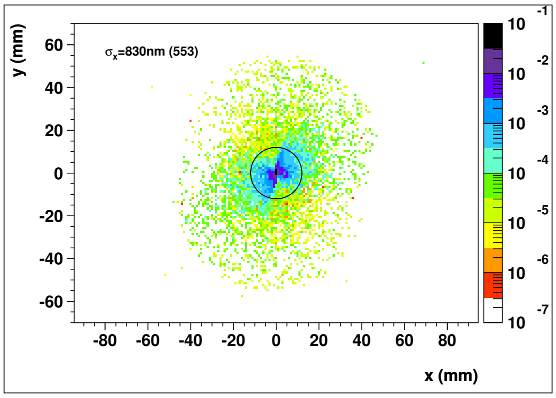

Fig. 2: *Energy distribution at LCAL for a bunch of width* σ_x = 829.5 *nm corresponding to 150% of the standard TESLA setup.*

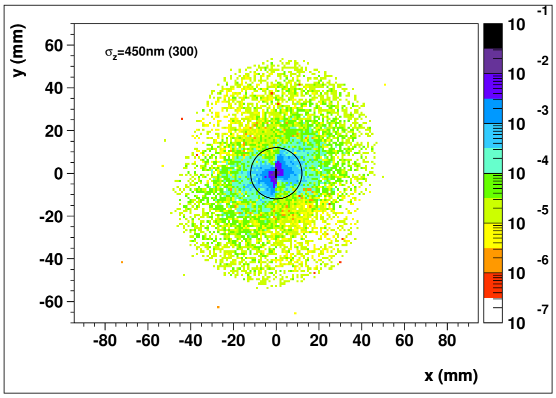

Fig. 3: *Energy distribution at LCAL for a bunch of length* σ_z = 450 μm *corresponding to 150% of the standard TESLA setup.*

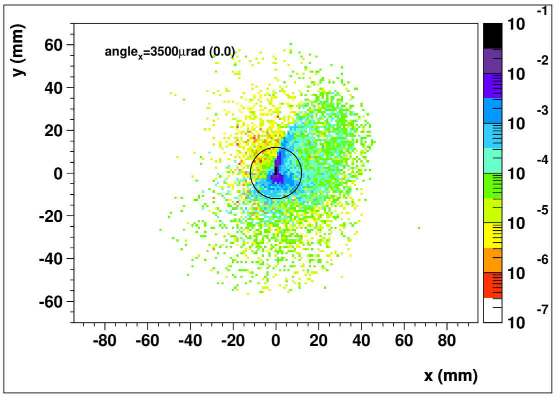

Fig. 4: *Energy distribution at LCAL for a bunch with an angle of 0.2° in the x-z plane. TESLA standard is 0.0°.*

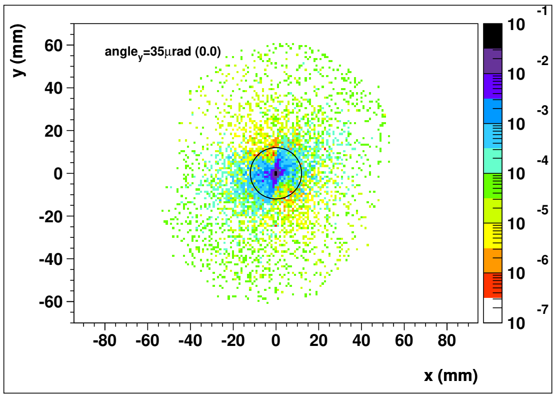

Fig. 5: *Energy distribution at LCAL for a bunch with an angle of 0.002° in the y-z plane. TESLA standard is 0.0°.*

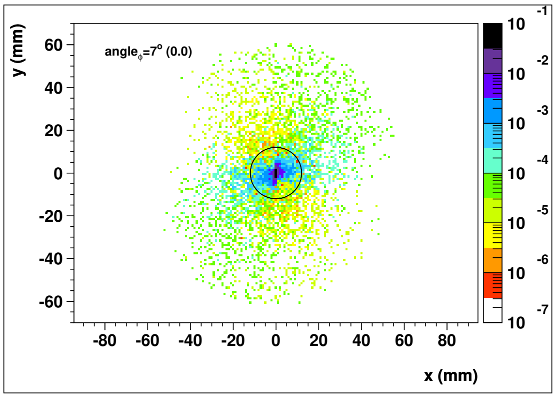

Fig. 6: *Energy distribution at LCAL for a bunch with a tranverse tilt angle of 7.0 °. TESLA standard is 0.0°.*

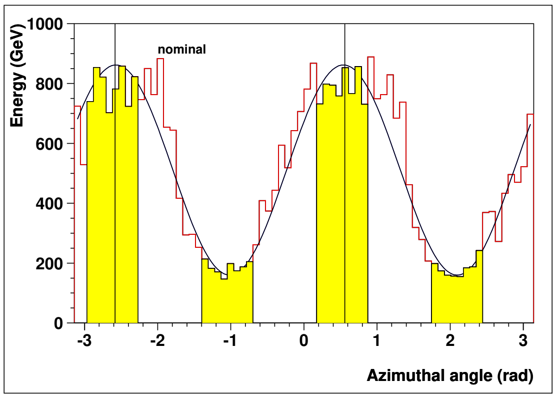

Fig. 7: *Angular distribution at LCAL for standard TESLA setup:* $\sigma_x = 553.0$ *nm,* $\sigma_y = 5.0$ *nm and* $\sigma_z = 300$ *µm.*

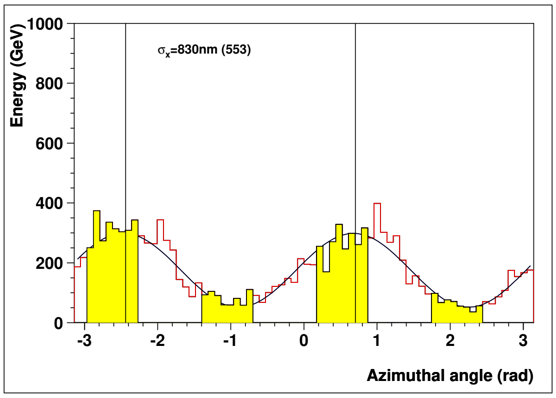

Fig. 8: *Angular distribution at LCAL for a bunch of width* $\sigma_x = 829.5$ *nm corresponding to 150% of the standard TESLA setup.*

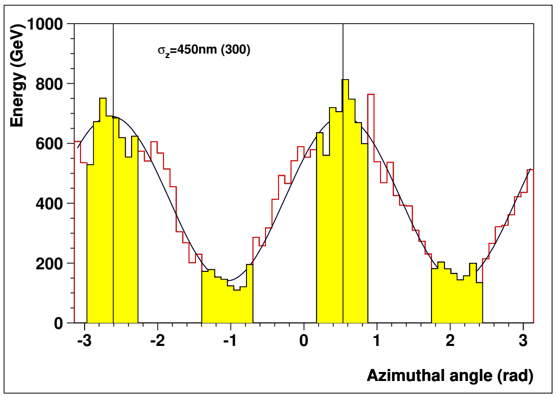

Fig. 9: *Angular distribution at LCAL for a bunch of length* $\sigma_z = 450$ *µm corresponding to 150% of the standard TESLA setup.*

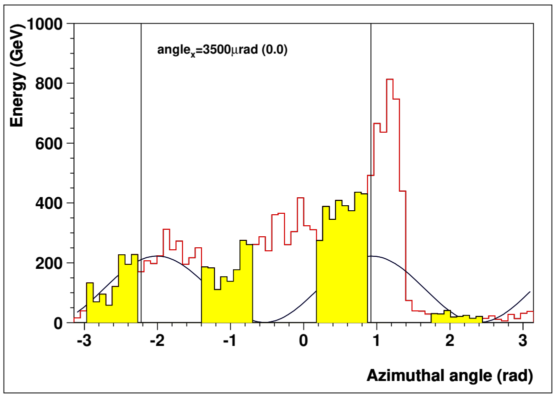

Fig. 10: *Angular distribution at LCAL for a bunch with an angle of 0.2° in the x-z plane. TESLA standard is 0.0°.*

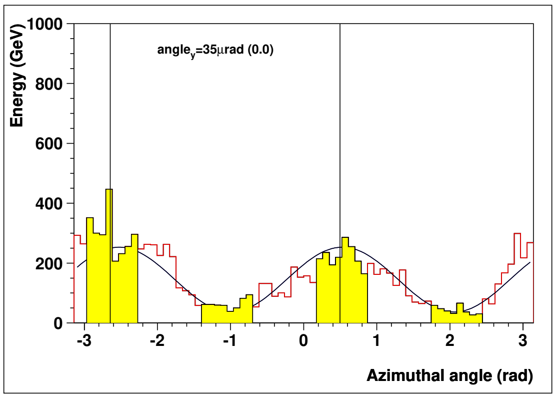

Fig. 11: *Angular distribution at LCAL for a bunch with an angle of 0.002° in the y-z plane. TESLA standard is 0.0°.*

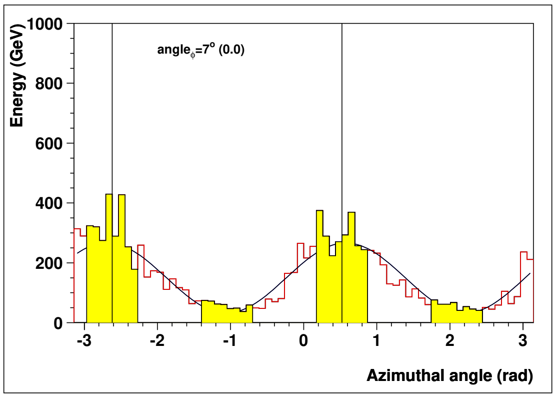

Fig. 12: *Angular distribution at LCAL for a bunch with a tranverse tilt angle of 7.0 °. TESLA standard is 0.0°.*

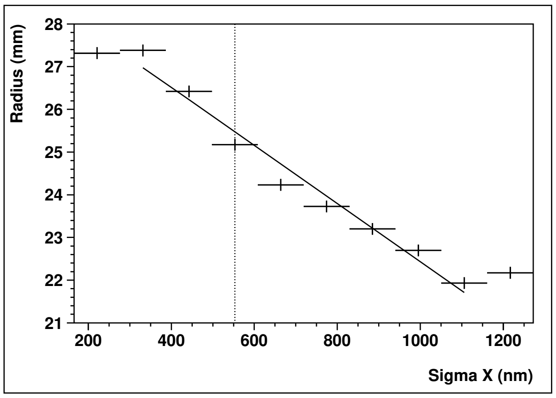

Fig. 13: *The radius versus the bunch width* σ_x

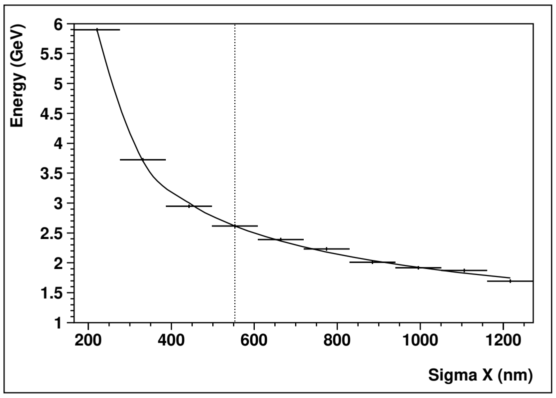

Fig. 14: *The energy versus the bunch width* σ_x

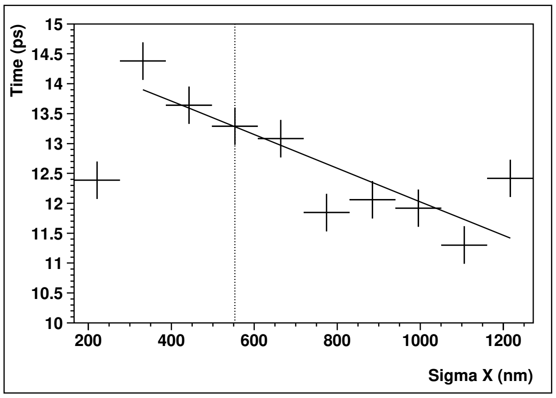

Fig. 15: *The arrival time versus the bunch width* σ_x

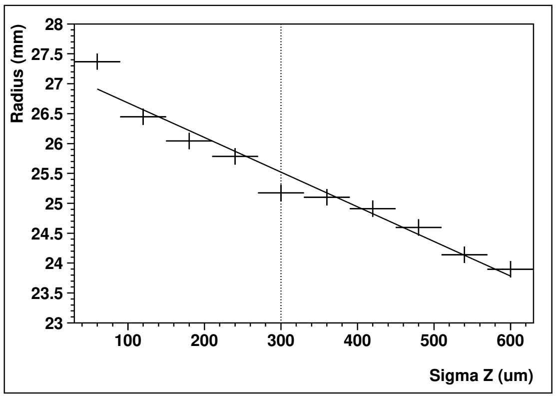

Fig. 16: *The radius versus the bunch length* σ_z

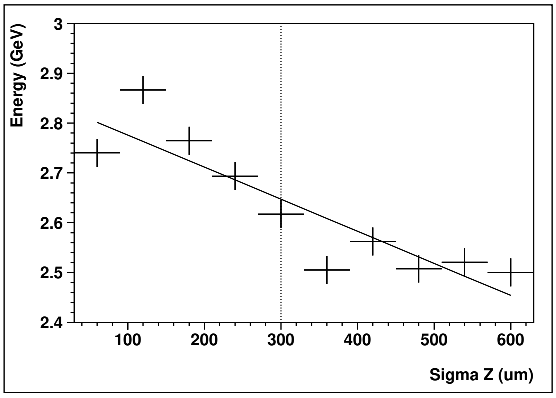

Fig. 17: *The energy versus the bunch length* σ_z

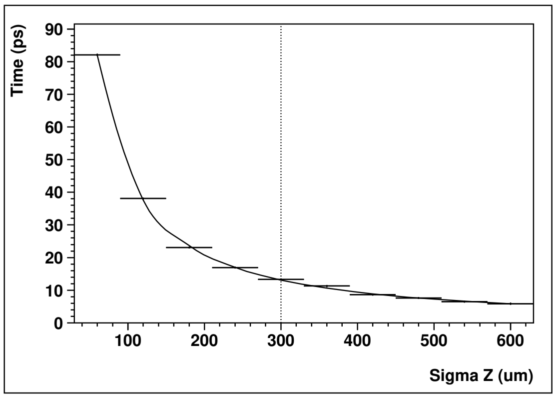

Fig. 18: *The arrival time versus bunch length* σ_z