Resonant Cavities as Beam Position Monitor

Michele Viti*

Deutsches Elektronen-Synchrotron DESY

15738 Zeuthen, Germany

May 2009

Abstract

A brief introduction to the work principles for resonant cavities as Beam Position Monitor (BPM) including some description of signal processig

*michele.viti@desy.de

1 Resonant Cavity and Beam Coupling

A charged particle traveling within a beam pipe induces a mirror charge in the pipe itself. For a perfect conducting pipe, this charge is traveling together with the beam without loss of energy.

A resonant cavity represents a discontinuity along the beam pipe where stationary waves are induced by the passage of a charged particle. In this case, some energy is stored which oscillates between pure electric and magnetic energy. The total stored energy, W_s, is given as $W_s =< W_e > + < W_m >= 2 < W_e >$, where $< W_e >$ and $< W_m >$ are the mean electric and magnetic energy respectively, averaged over one period. Thus, a cavity can be schematically represented by an LC circuit with a frequency $\omega = 1/\sqrt{(LC)}$ [1]. In other words, a particle passing a discontinuity induces an infinite number of stationary waves n, each of them can be represented by an LC circuit with its own frequency ω_n.

Considering the electric and magnet fields within a cylindrical cavity, we are interested on the fields with pure transverse magnetic oscillations, i.e. on magnetic fields with no longitudinal component ($H_z = 0$). Such fields are denoted as TM modes. They are defined by the geometry of the cavity (length and radius) and by three integer numbers m, n and p. It is common to identify a mode with the notation TM_{mnp} and its frequency is given by

$$\omega_{mnp} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{j_{mn}}{R}\right)^2 + \left(\frac{p\pi}{l}\right)^2} , \tag{1}$$

where R is the radius of the cavity, l its length and j_{mn} the n-th zero of the m-th Bessel function. For particles near the center of the cavity, the TM_{010} or monopole mode has the strongest excitation. The explicit expressions for the fields of this mode are [2]

$$\begin{aligned} E_{z,010} &= C_{010} J_0 \left(\frac{j_{01} r}{R}\right) e^{i\omega_{010} t} && (2)\\ H_{r,010} &= 0 && (3)\\ H_{\phi,010} &= -iC_{010} \frac{\omega_{010}\epsilon_0 R}{j_{01}} J_0' \left(\frac{j_{01} r}{R}\right) e^{i\omega_{010} t} \ . && (4) \end{aligned}$$

The electric field $E_{z,010}$ has a weak symmetric dependence on the distance r from the center. On the other hand, the mode TM_{110} or dipole mode is antisymmetric and its amplitude has a strong dependence on the r. The explicit expressions for the dipole fields are [2]

$$\begin{aligned} E_{z,110} &= C_{110} J_1 \left(\frac{j_{11} r}{R}\right) \cos\phi e^{i\omega_{010} t} && (5)\\ H_{r,110} &= -iC_{110} \frac{\omega_{110}\epsilon_0 R^2}{j_{11}^2 r} J_1 \left(\frac{j_{11} r}{R}\right) \sin\phi e^{i\omega_{010} t} && (6)\\ H_{\phi,110} &= -iC_{110} \frac{\omega_{110}\epsilon_0 R}{j_{11}} J_1' \left(\frac{j_{11} r}{R}\right) \cos\phi e^{i\omega_{010} t} \ . && (7) \end{aligned}$$

A schematic representation of both modes TM_{010} and TM_{110} is shown in Fig. 1.

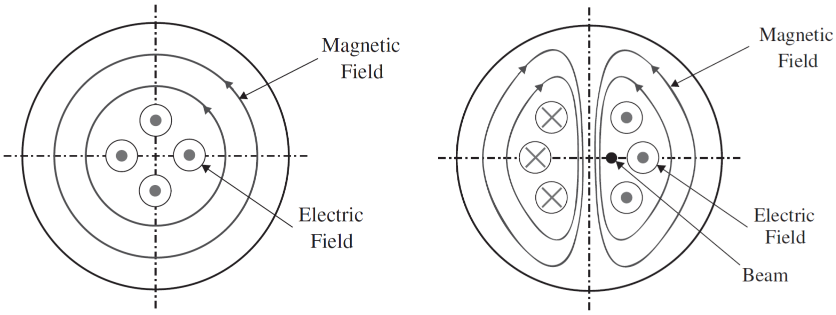

Figure 1: Left: Transverse view of the fields of the TM_{010} mode. Right: Transverse view of the fields of the TM_{110} mode. Here, the electric field has a strong dependence on the beam offset from the center of the BPM.

As mentioned above, a charged particle traveling through a cavity interacts with the modes and releases some energy. To understand the coupling between the particle and the modes we use the so-called *fundamental theorem of the beam loading*: the voltage induced by a charge traveling through a cavity is twice the effective voltage "seen" by the charge itself [3, 4]. Hence, the energy stored in the cavity by the dipole mode can be calculated as the volume integral of the modulus square of the electric field

$$W_{110} \;=\; \frac{1}{2}\epsilon_0 \int_V |E_{z,110}|^2 dV = \frac{\pi}{4}\epsilon_0 C_{110}^2 J_0^2(j_{11}) R^2 l \; . \tag{8}$$

On the other hand, according to the fundamental theorem of the beam loading, the variation of the energy stored in the cavity ΔW_{110} can be written as

$$\begin{aligned} \Delta W_{110} &= q \cdot V = \frac{q}{2}\int_{-\infty}^{+\infty} E_{z,110}\vec{v}dt = \\ &= \frac{q}{2}\int_{-l/2}^{l/2} C_{110} J_1\left(\frac{j_{11}\delta x}{R}\right) e^{ik_{110}z} dz = \\ &= \frac{q}{2} C_{110} J_1\left(\frac{j_{11}\delta x}{R}\right) Tr^{110} l \end{aligned} \tag{9}$$

for a charge q traveling parallel to the cavity axis with an offset δx, $\phi = 0$ and velocity $\vec{v}$ close to the light speed. Here, the integral is calculated along the path of the particle, q its charge, k_{110} is the wave-number and Tr^{110} the transit time factor

$$Tr^{110} = \frac{\left(\int_{-l/2}^{l/2} E_z \cdot e^{ikz} dz\right)}{\left(\int_{-l/2}^{l/2} E_z dz\right)} = \frac{\sin k_{110} l/2}{k_{110} l/2} \; . \tag{10}$$

It is important to note that the phase of the particle with respect to the field induced by the particle itself is chosen such that the field maximally opposes the motion of the particle [4].

Considering the cavity initially empty, Eq. (9) is equal to the energy stored (8) and approximating the Bessel function $J_1(x)$, for small arguments, by $x/2$, the amplitude C_{110} can be written as

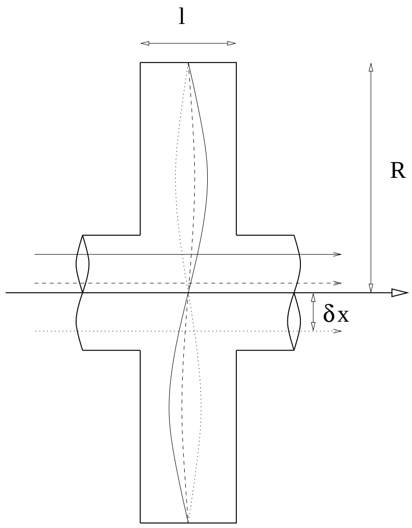

Figure 2: Schematic representation of the induced dipole mode in a cylindrical cavity. A beam with small offset (dashed line) induces a smaller signal than a beam with bigger offset (continuous line). Beams with the same offset but opposite in X induce the same signal but with opposite phases (continuous and dotted lines).

$$
\begin{aligned}
C_{110} &= \frac{2qTr^{110}J_1\left(\frac{j_{11}\delta x}{R}\right)}{\pi\epsilon_0 J_0^2(j_{11})R^2} \approx \\
&\approx \frac{qTr^{110}j_{11}\delta x}{\pi\epsilon_0 J_0^2(j_{11})R^3}
\end{aligned} \tag{11}
$$

and, in a similar way, C_{010} of the monopole mode

$$
C_{010} \approx \frac{qTr^{010}}{\pi\epsilon_0 J_1^2(j_{01})R^2} \ . \tag{12}
$$

Both modes depend linearly on the particle charge q. However, only the dipole mode has a linear dependence on the beam offset. It can be easily shown that two beams with the same δx opposite in X ($\Delta\phi = \pi$) induce the same voltage with opposite phase. A schematic representation of this behavior is given in Fig. 2.

In general, the motion of a particle is not exactly parallel to the Z-axis of the cavity but has an inclination or slope x'. In the linear regime of small offsets, any trajectory can be represented as a sum of a trajectory with only an offset and a trajectory with only an inclination (see Fig. 3), and for the voltage seen by a particle with pure inclination and no offset we have

$$
\begin{aligned}
\Delta W_{110,x'} &= \frac{q}{2}\int_{-l/2}^{l/2} C_{110}J_1\left(\frac{j_{11}z\tan x'}{R}\right)e^{ik_{110}z}dz \\
&\approx i\frac{q}{2}C_{110}\frac{j_{11}x'}{k_{110}^2R}\left(\sin\frac{k_{110}l}{2} - \frac{k_{110}l}{2}\cos\frac{k_{110}l}{2}\right) .
\end{aligned} \tag{13}
$$

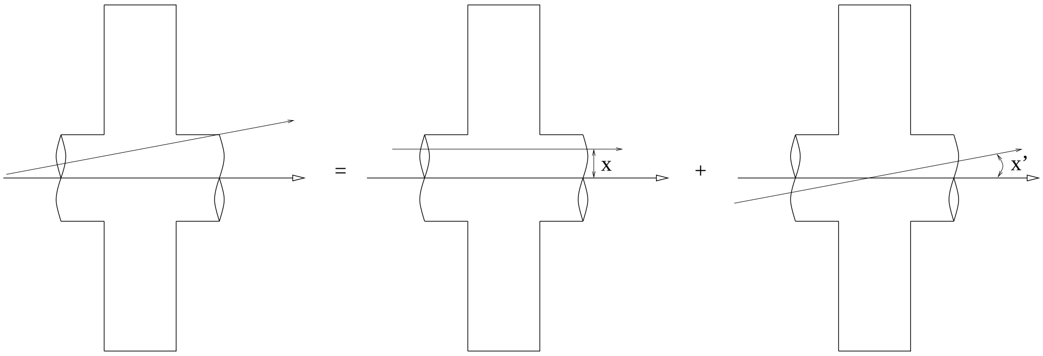

Figure 3: A generic trajectory through a cavity interpreted as a superposition of a trajectory with pure offset and a trajectory with pure inclination.

Thus, the signal is proportional to x'. Comparing this results with Eq. (8), the amplitude

$$C_{110} \approx i\frac{2qj_{11}x'}{\pi\epsilon_0 J_0^2(j_{11})k_{110}^2 R^3 l}\left(\sin\frac{k_{110}l}{2} - \frac{k_{110}l}{2}\cos\frac{k_{110}l}{2}\right) \quad (14)$$

reveals that a phase of 90° exists between the field induced by a particle with only an offset and the field induced by a particle with only an inclination. In the next section it will be shown how the two signals can be disentangled.

So far, only cavities with perfect conducting walls were considered. In practice, however, some dissipation of energy in the wall will happen so that the cavity behaves like a RLC circuit with a decay constant τ. For this reason, as a Fourier analysis shows, a broad spectrum of frequencies for each mode occurs. In particular, the monopole mode substantially overlaps the dipole mode as indicated in Fig. 4(right).

To extract the relevant dipole signal from the cavity, a mode selection is necessary. The mode selection is based on the fact that the boundary conditions for the dipole and monopole modes are different on the wall of the cavity. In fact, the dipole mode generates a field transverse to the Z-axis which might have a strong coupling to an opportune modeled waveguide. Thus, it is expected that inside the waveguide the dipole mode is dominant [2, 5] and its amplitude is proportional to the beam offset (see Fig. 4(left)).

Finally, a generic dipole mode can be interpreted as a superposition of two orthogonal polarizations of the mode itself. Hence, one cavity can provide X- as well as Y-position readings at the same time (see Fig. 5).

Figure 6 (left) shows a photograph of a cylindrical cavity BPM. It is BPM 7 installed in the mid-chicane of the experiment. The cavity, as can be seen, is connected with four waveguides, two in vertical direction to extract the X-position signal and two in horizontal direction for the Y-position signal.

Similar conclusions can be drawn for rectangular cavities. An example of such a cavity, BPM 9 is shown in Fig. 6 (right). The main difference to the cylindrical cavity is that X- and Y-position reading is performed by two distinct cavities, the two rectangular cavities in the picture.

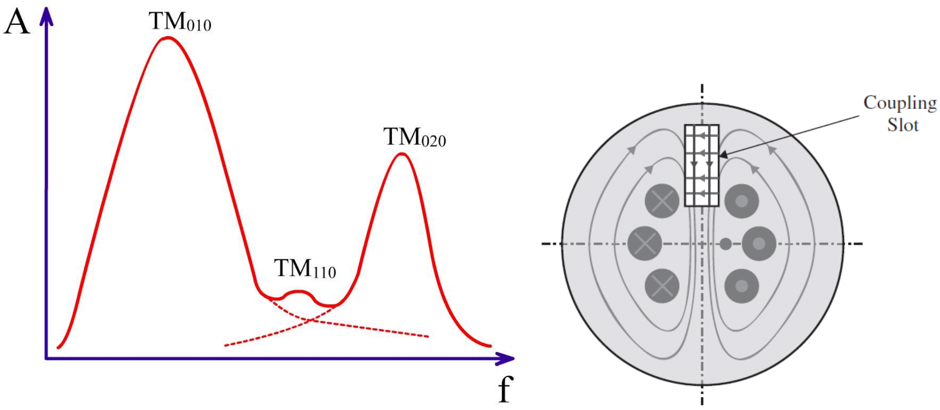

Figure 4: Left: Amplitude vs. frequency of the first two monopole modes and the first dipole mode of a cylindrical cavity with non-zero resistivity. The monopoles TM_{010} and TM_{020} surround and overlap the dipole mode TM_{110} [5]. Right: The dipole mode is selectively coupled out by means of a narrow radial slot on one face of the cavity [2].

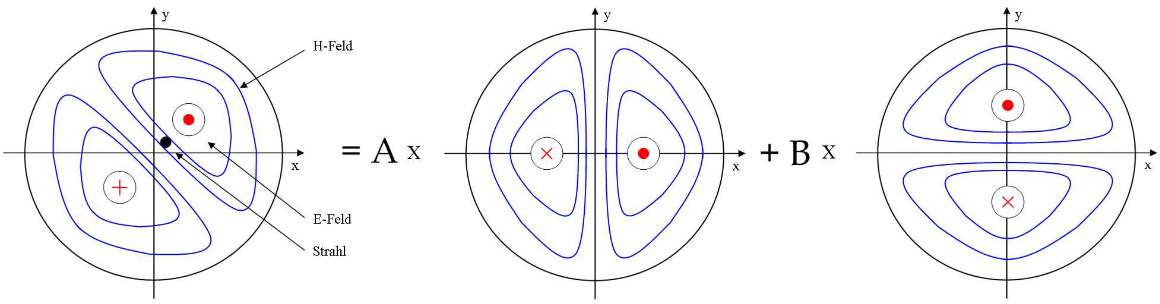

Figure 5: A generic dipole mode can be represented as a superposition of two orthogonal polarizations of the mode.

2 Signal Processing

It is common to express the output voltage of the dipole mode as a function of the shunt impedance and quality factor. The shunt impedance is defined as

$$R_{110} = \frac{V_{110}}{P_{110,\ loss}} \quad , \tag{15}$$

where $P_{110,loss}$ is the power dissipated in the cavity walls. The internal quality factor is defined as

$$Q_{110}^{int} = \frac{w_{110} W_{110}}{P_{110,\ loss}} \quad . \tag{16}$$

Thus the energy stored in the cavity can be written as

$$W_{110} = \frac{\omega_{110}}{4} \left(\frac{R}{Q}\right)_{110} q^2 \quad , \tag{17}$$

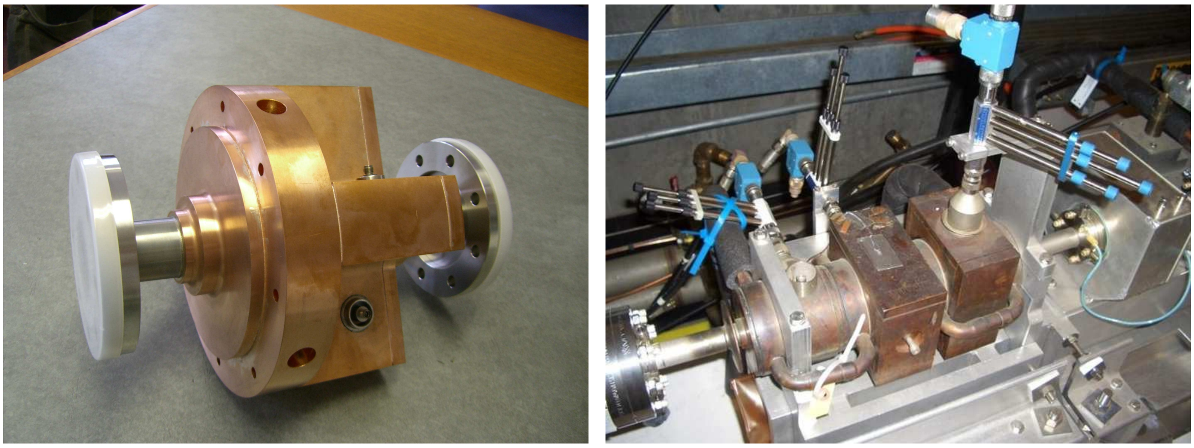

Figure 6: Left: Cylindrical cavity BPM as designed by UCL. Right: BPM 9 in End Station A. The two rectangular cavities for X- and Y-position reading and the cylindrical reference cavity are clearly seen.

where $\left(\frac{R}{Q}\right)_{110}$ is the normalized shunt impedance. This quantity is independent on the material of the cavity and depends only on its geometry. Moreover, it has a finite value also when the cavity has a wall with zero resistivity (which corresponds to an infinite value of the internal quality factor).

According to Eqs. (8) and (11), we have $\left(\frac{R}{Q}\right)_{110} \propto (\delta x)^2$. Defining the external quality factor as

$$Q_{110}^{ext} = \frac{w_{110} W_{110}}{P_{110,\ out}} \ , \tag{18}$$

the dipole output power is then given by

$$P_{110,\ out} = \frac{\omega_{110}}{4Q_{110}^{ext}} \left(\frac{R}{Q}\right)_{110} q^2 \quad . \tag{19}$$

Finally, the readout electronics with impedance Z provides an output voltage of

$$V_{out} = \sqrt{Z P_{110,\ out}} = \frac{q\omega_{110}}{2} \sqrt{\frac{Z}{Q_{110}^{ext}} \left(\frac{R}{Q}\right)_{110}} \quad . \tag{20}$$

As can be seen, this voltage has a linear dependence on the charge and the offset of the particle, denoted as x in the following. In order to measure a signal which is only proportional to the offset, normalization to the charge is needed. For this reason the monopole signal is simultaneously extracted from a reference cavity, which is tuned such that its monopole mode has the same frequency as the dipole mode of the beam position cavity, i.e. $\omega_{010}^{ref} = \omega_{110}^{BPM}$. Moreover, the reference cavity provides the arrival time of the beam, allowing to determine the phase of the signal.

The voltage signal at the front-end of the analogue electronics has thus the form

$$\begin{aligned} V(t) &= e^{-\Gamma t}[A_x x \sin \omega t + A_{x'} x' \cos \omega t] \\ &= a e^{-\Gamma t} \sin (\omega t + \phi) \ , \end{aligned} \tag{21}$$

where $a = \sqrt{(A_x x)^2 + (A_{x'} x')^2}$ and $\phi = \arctan (A_{x'} x' / A_x x)$, with amplitudes A_x and $A_{x'} \propto q$. In a similar way the reference cavity provides

$$V(t) = a_{ref} e^{-\Gamma t} \sin (\omega t + \phi_{ref}) \ , \tag{22}$$

where $a_{ref} \propto q$.

After filtering and digitization, both signals are multiplied by a complex local oscillator (LO) of the same frequency as the signal. This process, called digital down-conversion (DDC), results in a signal which describes the envelope of the initial waveform

$$\begin{aligned} V_{DDC}(t) &= a e^{-\Gamma t} \sin (\omega t + \phi) \cdot e^{i \omega t} \\ &= \frac{a}{2} e^{-\Gamma t} \left[-e^{i(\phi + \frac{\pi}{2})} + e^{i(2\omega t + \phi + \frac{\pi}{2})} \right] \ . \end{aligned} \tag{23}$$

The mixing process also generates an unwanted wave with a frequency 2ω, which has to be eliminated by an additional filter. Customarily, the quantities I and Q denote the real, respectively, imaginary part of the normalized down-converted signal

$$I = \frac{a}{a_{ref}} \cos(\phi - \phi_{ref}) \tag{24}$$

$$Q = \frac{a}{a_{ref}} \sin(\phi - \phi_{ref}) \ . \tag{25}$$

An example of an I-Q plot is shown in Fig. 7 for a beam particle with a generic offset and tilt generated at the point P. If the tilt (or slope) is kept constant while the position is changed, P moves along the continuous line, which has, in general, a non-zero slope Θ (IQ-phase). When the beam crosses the center of the BPM, P changes sign (point P'). If on the other hand, the beam position is kept unaltered while the beam tilt is changed, the point P moves along the dashed line, perpendicular to the continuous line (point P").

To extract position and tilt of the particle, the IQ-phase Θ and the scale factors S and S' have to be determined by an appropriate calibration procedure. Finally, the position and tilt are obtained after rotation of the plane by angle Θ and multiplication with the scale factors:

$$x = S \cdot Re \left[\frac{a}{a_{ref}} e^{i(\phi - \phi_{ref} - \Theta)} \right] \tag{26}$$

$$x' = S' \cdot Im \left[\frac{a}{a_{ref}} e^{i(\phi - \phi_{ref} - \Theta)} \right] \ . \tag{27}$$

In the experiment T474/T491, calibration was performed by generating a well-known beam offset using the corrector magnets or the Helmholtz coils of the A-line (see [6]). For BPMs 4 and 7, also the mover systems have been utilized for calibration purposes. In ESA, only the offset and not the tilt could be calibrated.

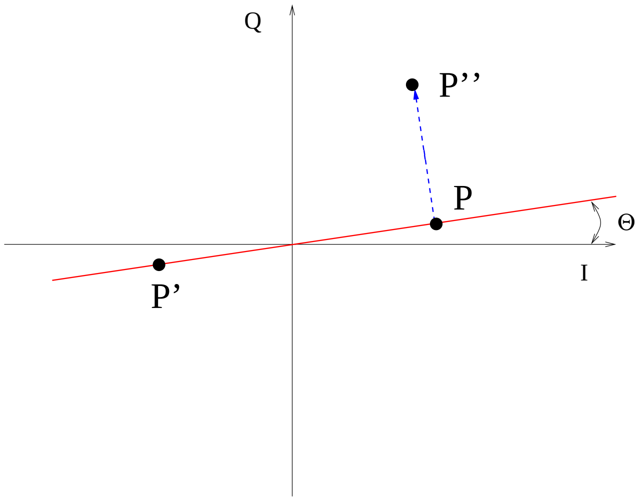

Figure 7: I-Q plot with an IQ-phase Θ. Changing the position of the beam, the point P moves along the continuous line and changes sign when it crosses the cavity center. Changing the tilt, P moves along the dashed line, perpendicular to the continuous line.

References

[1] R. Lorenz. Cavity beam position monitors. Contributed to 8th Beam Instrumentation Workshop (BIW 98), Stanford, CA, May 4-7 1998.

[2] S. Walston et al. Performance of a high resolution cavity beam position monitor system. *Nucl. Instrum. Meth.*, A578:1–22, 2007.

[3] P. B. Wilson. Introduction to wake fields and wake potentials. *AIP Conf. Proc.*, 184:525–564, 1989.

[4] P. B. Wilson. High-energy electron linacs: applications to storage ring rf systems and linear colliders. *AIP Conf. Proc.*, 87:450–555, 1982.

[5] A. Lyapin. Strahllagemonitor für das TESLA-Energiespektrometer, Dissertation. TU-Berlin, Berlin, 2003.

[6] M. Slater et al. Cavity BPM system tests for the ILC energy spectrometer. *Nucl. Instrum. Meth.*, A592:201–217, 2008.