On the kinematic reconstruction of multiparticle events

A.D.Bukin
Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia

Kinematic reconstruction of multiparticle events with special parametrization of likelihood function is considered. For those cases when approximate form of angular part of likelihood function is acceptable, the analytical solution for optimal general rotation was found, that allows to decrease the total number of free parameters by three rotation angles and strongly simplifies the likelihood function profile.

Key words: data processing; event reconstruction; kinematic fit

1 Introduction

Data processing in high energy physics often contains the procedure of improving the accuracy of experimental measurements of particle characteristics, using some constraints, typically conservation laws and known values of unstable particle masses. Most of all it is done by numerical minimization of the log-likelihood function depending on some number of parameters. The choice of parameters for the events in center-of-mass system can be done so that some subset of parameters defines the rigid construction of outgoing particles while the last three parameters define the rotation of this construction as a whole. Then the task of analytical minimization of the aim function over these three rotation parameters becomes very attractive because of: 1) decrease of CPU time necessary to minimize the function and 2) simplifying the function profile, that decreases the probability of finding some false minimum.

2 Sample of parametrization of the events $e^+e^- \to \eta\gamma \to \pi^+\pi^-\pi^0\gamma$

For conveniency let us consider the particular process $e^+e^- \to \phi \to \eta\gamma \to \pi^+\pi^-\pi^0\gamma \to \pi^+\pi^-\gamma\gamma\gamma$. The final state contains five particles and is described by 15 parameters: 5 momenta and 10 angles (θ and φ angles in the polar system). But these parameters are not independent. There are 5 constraints: 4 energy-momentum conservation laws and a known value of the π^0 mass. So there are only 10 free parameters. In general, if a detector measures the angles of all 5 particles, one can reconstruct momenta of all particles. Of course the accuracy of angular measuments will determine the accuracy of the reconstructed η-meson mass. If there are some additional measured parameters, then the accuracy of η meson mass will increase and the minimum value of the log-likelihood function can be used for rejection of background events.

The possible set of 10 free parameters is the following. The first parameter is an invariant mass M_η of the η-meson. For an e^+e^- collider with equal energies of the initial electron and positron E_0, the total momentum equals zero and the total energy equals $2E_0$. Using this information one can easily derive the energy of the first photon and momentum of the η-meson. Let η-meson move along the Z-axis in some system of reference and π^0 move in the XZ-plane in this system. In the system of η-meson we can define two more free parameters: momentum p_0 of π_0 and angle θ_0 between this momentum and the Z-axis. Now in the system of π^0-meson we can define the angle θ_γ between one of the photons and π^0 momentum and axial angle φ_γ, which determines the rotation of $\pi^0 \to \gamma\gamma$ decay plane with respect to the XZ-plane. The corresponding angles θ_+ and φ_+ define the π^+ momentum vector in the center-of-mass system of $\pi^+\pi^-$. At last the rotation angles ψ_1, ψ_2, ψ_3 allow an arbitrary rotation of this 5-particles construction as a whole.

Preprint submitted to Elsevier Preprint *28 February 1997*

Let us represent the log-likelihood function L as a sum $L = L_E + L_A$, where L_E takes into account the deviations of the particles energies from the measured values (for those particles, whose energies are measured) and L_A takes into account the deviations of particle directions from the measured ones:

$$L_A = \sum_i \frac{\Delta\alpha_i^2}{2\sigma_i^2} \tag{1}$$

Here $\Delta\alpha_i$ is the angle between the direction of the i-th particle momentum in our model and experimentally measured one, σ_i is an estimation of experimental angular accuracy.

For the further illustration let us use the simulated events for the detector SND [1] (Spherical Neutral Detector) at the energy $E_0 = 510$ MeV. This detector has no magnetic field, so the energies of the charged pions are not measured. Hence the energy part of the likelihood function L_E takes into account only the deviations of the measured energies of the three photons from the "theoretical" energies.

Obviously the energies of particles in our model do not depend on the angles ψ_1, ψ_2, ψ_3, hence the energy part of likelihood function L_E does not depend on these angles. There is no profit to minimize separately L_A by numerical methods over $\psi_{1,2,3}$ for every set of 7 parameters M_η, p_0, θ_0, θ_γ, φ_γ, θ_+, φ_+. On opposite there will be great increase of CPU time consumption. In this paper the analytic solution for minimization of the approximate form of L_A over $\psi_{1,2,3}$ is presented.

3 Approximate form of likelihood function

For small deviations $\Delta\alpha_i$ in (1) likelihood function can be approximated by

$$L_A \approx \sum_i \frac{1-\cos\Delta\alpha_i}{\sigma_i^2} = \sum_i \frac{1-\mathrm{e}_i\cdot\mathrm{n}_i}{\sigma_i^2} = \sum_i \frac{1}{\sigma_i^2} - \sum_i \frac{\mathrm{e}_i\cdot\mathrm{n}_i}{\sigma_i^2} \tag{2}$$

Here vector e_i is the unit vector, defining the measured direction of i-th particle, vector n_i is the theoretical unit vector for i-th particle, defined by 10 variable parameters in our model.

First sum in (2) is constant and can be omitted. Likelihood function L_A reaches its minimum value, when the second sum in (2)

$$L_M = \sum_i \frac{\mathrm{e}_i\cdot\mathrm{n}_i}{\sigma_i^2} \tag{3}$$

reaches its maximum value. Vectors n_i can be presented as unit vectors s_i, depending in our model only on the first 7 parameters, transformed with a rotation matrix T, depending only on the angles $\psi_{1,2,3}$: $\mathrm{n}_i = \mathrm{T}\cdot\mathrm{s}_i$. Now we can repeat the task in another way: it is necessary to find the rotation matrix T such that

$$L_M = \sum_i \frac{\mathrm{e}_i^T \mathrm{T}\mathrm{s}_i}{\sigma_i^2} = Tr\left(\mathrm{T}\sum_i \frac{\mathrm{s}_i\mathrm{e}_i^T}{\sigma_i^2}\right) = Tr\,(\mathrm{T}\ \mathrm{V}) \tag{4}$$

reaches its maximum value, where superscript "b^T" means transposed matrix b.

The analytic solution of this problem was found by the well-known method of Lagrange coefficients (for example, in [2]). All auxilliary transformations were made with REDUCE code [3]. Detailed de-

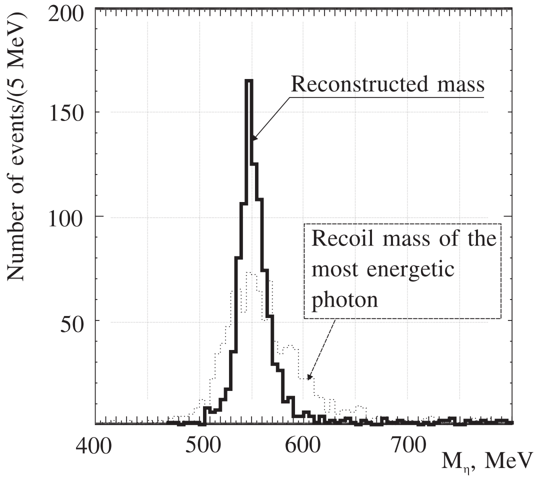

Fig. 1. Distribution of simulated events over parameter M_η ($e^+e^- \to \eta\gamma \to \pi^+\pi^-\pi^0\gamma$). Dashed line shows the distribution of these events over the recoil mass of the most energetic photon

scription of the solution and corresponding subroutine BUROTAT, implemented in Fortran 77, is sent to "Computer Physics Communications".

Fig. 1 demonstrates the effect of kinematic reconstruction of the non-magnetic detector SND [1] events of the considered process.

Measurements of the directions of all 5 final particles and energies of the 3 final photons are fitted. Events are simulated by UNIMOD2 code [4] and reconstructed by SND software package.

4 Conclusion

One of the possible procedures for the kinematic reconstruction of multiparticle events is considered. For special parametrization of an event and suggested approximate angular part of the likelihood function the optimal rotation procedure is derived and implemented in Fortran-77.

This procedure allows to decrease the number of free optimized parameters by three, and hence decrease the CPU consumption by more than two times for the sample process $e^+e^- \to \eta\gamma \to \pi^+\pi^-\gamma\gamma\gamma$ (decrease from 10 to 7 optimized parameters). Even more important there is an increase of detection efficiency for this process by several percents, that characterises the simplification of the likelihood function profile and hence the less probability of finding the false minimum (minimization was performed by MINUIT code).

References

[1] *V.M.Aulchenko et al.* SND — detector for VEPP-2M and ϕ–factory, in: Proc. Workshop on Physics and Detectors for DAΦNE (Frascati, 1991) 1991, pp. 605 – 613.

[2] *G.A.Korn and T.M.Korn.* Mathematical handbook for scientists and engineers. McGraw-Hill, 1968.

[3] *Anthony C. Hearn.* REDUCE user's manual. Rand, 1987.

[4] *A.D.Bukin et al.,* Proc.: Workshop on Detector and Event simulation in High Energy Physics, Amsterdam, April 1991, NIKHEF, 1991, p.79.
Budker INP, 94-20, Novosibirsk, 1994.