

Physics at LHC

lecture six

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Plan

- Large extra dimensions
 - hierarchy problem
 - theory
 - phenomenology
- Literature
 - *Phenomenology, astrophysics and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity*
Arkani-Hamed, Dimopoulos, Dvali [[hep-ph/9807344](#)]
 - *A Large mass hierarchy from a small extra dimension*
Randall, Sundrum [[hep-ph/9905221](#)]
- Review
 - *TASI 2004 Lectures on the phenomenology of Extra Dimensions*
Kribs [[hep-ph/0605325](#)]
 - *Dark Matter and Collider Phenomenology of Universal Extra Dimensions*
Hooper, Profumo [[hep-ph/0701197](#)]

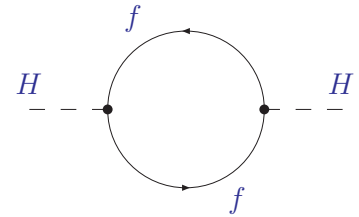
Motivation

History

- Unification of gravity with electromagnetism in 5-dim. space-time Kaluza 1919
- Compactification of fifth dimension (circle of small radius) Klein 1926
 - particles moving through extra dimension appear very massive
 - excited states of ordinary particles known as Kaluza-Klein modes
- String theory cannot be formulated in four dimensional space-time
 - bosonic string in $D = 26$
 - fermionic string ($\mathcal{N} = 1$ supersymmetry) in $D = 10$
- Hierarchy problem
 - Radiative corrections to Higgs mass with quadratic divergences induce large hierarchies $\mathcal{O}(M_{\text{Planck}}^2/M_{\text{electro-weak}}^2) \sim 10^{-32}$

Hierarchy problem

- Higgs mass receives radiative corrections from fermion loops
 - the size of corrections is proportional to the UV cutoff Λ squared



$$\delta M_H^2 = \frac{\lambda_f^2}{4\pi^2} (\Lambda^2 + m_f^2) + \dots$$

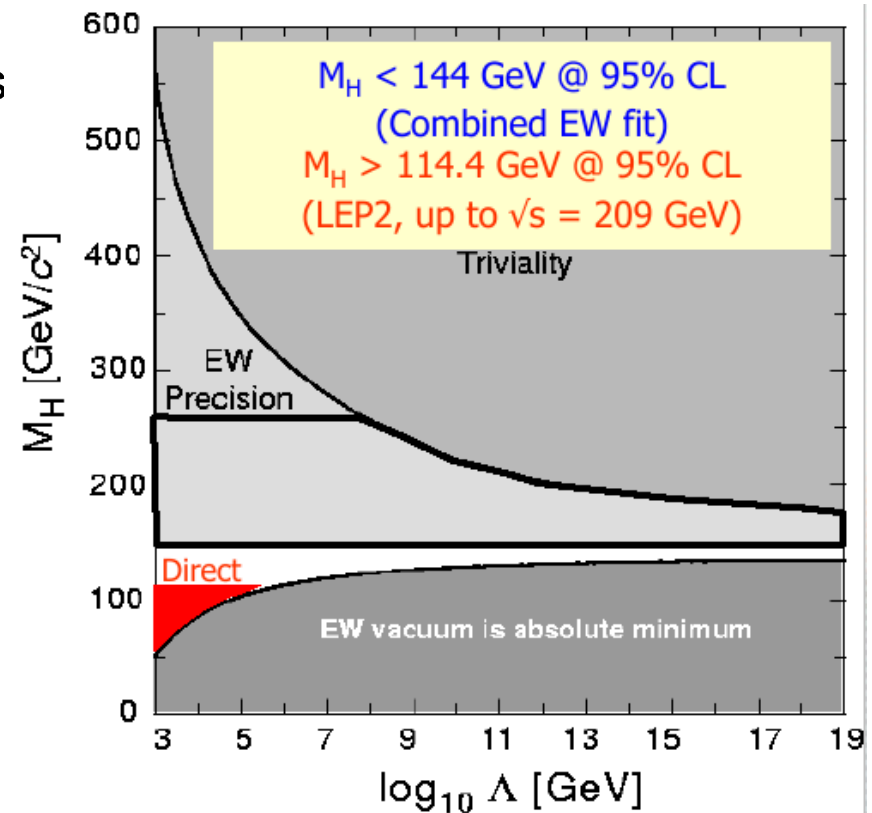
- Cancellation of quadratic divergences to Higgs mass δM_H^2 under renormalization requires fine tuning
 - if UV cutoff $\Lambda \sim \mathcal{O}(M_{\text{Planck}})$ then a precision $\mathcal{O}(M_{\text{Planck}}^2/M_{\text{electro-weak}}^2) \sim 10^{-32}$ is required
- Standard Model as effective theory should have ratio of scales $\sim \mathcal{O}(1)$
 - high degree of fine tuning considered to be unnatural

Large extra dimensions

- M_{Planck} in 4 dimensions is not a fundamental scale

Higgs mass and triviality

- Higgs mass cannot be too light or the Higgs potential will not have a Mexican hat shape and will turn negative at large values
 - for Standard model to be valid up to Planck scale $M_H > 135 \text{ GeV}$
- Triviality
 - If Higgs mass is too large the Higgs self-coupling drives mass to infinity above certain scale
- Standard model holds all the way up to Planck scale if $135 \text{ GeV} < M_H < 175 \text{ GeV}$



Gravity

Gravity in a nut-shell

- Einstein-Hilbert action in 4-dimensions

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} M_\star^2 R$$

- Ricci tensor $R_{\mu\nu}$ and Ricci scalar $R = g^{\mu\nu} R_{\mu\nu}$
- mass scale M_\star related to Newton's constant G_N :

$$M_\star = \left(\frac{1}{8\pi G_N} \right)^{1/2}$$

- Variational calculus δS leads to Einstein equation
 - Metric tensor $g_{\mu\nu}$
 - Energy-momentum tensor $T_{\mu\nu}$
 - Cosmological constant Λ

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Kaluza-Klein theory

- Gravity and electromagnetism in 5-dim. space-time with one dim. compactified on circle of circumference L Kaluza 1919; Klein 1926

- Einstein-Hilbert action $S = \int d^5x \sqrt{-g^{(5)}} M_\star^3 R^{(5)}$

- metric $g_{MN}^{(5)}$ on $R_4 \otimes S^1$ expanded $g_{MN} = \eta_{MN} + h_{MN}/(2M_\star^{3/2})$

- five-dimensional graviton h_{MN} contains five physical components

- Massless fields

- 4-dimensional $(4-d)$ graviton $h_{\mu\nu}$, massless vector field $A_{\mu 5}$ and massless scalar field ϕ

$$\begin{pmatrix} h_{\mu\nu} & A_{\mu 5} \\ A_{5\nu} & \phi \end{pmatrix}$$

- Action comprises 4-dimensional gravity plus a gauge field

- coupling strength $g^2 = (M_\star L)^{-1}$ (ϕ with gravitational couplings only)

$$\int d^4x (M_\star^3 L) R^{(4)} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$$

Kaluza-Klein theory

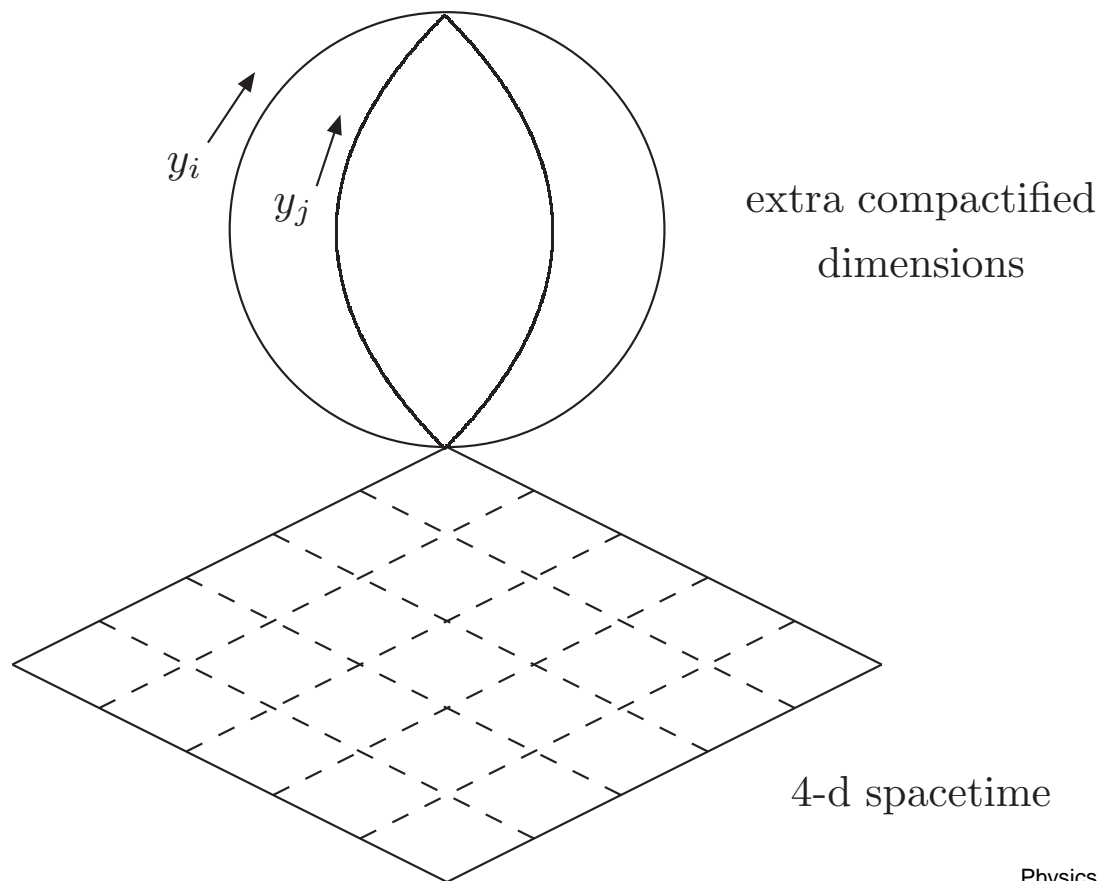
- Original Kaluza-Klein proposal suffers from three problems
 - gravitationally coupled scalar field ϕ postulated
 - gauge field strength of order one only for $L^{-1} \sim M_*$
 - no chiral fermions exist in 5-dim. space-time (“doubling” of massless fermions)

Large extra dimensions

- Large extra dimensions at the millimeter scale (ADD model) [Arkani-Hamed, Dimopoulos, Dvali '98](#)
 - all SM fields confined to 4-dim. brane
 - only gravity propagates in space of extra dimensions (the bulk)
 - fundamental Planck scale as perceived in bulk similar in magnitude to electroweak scale (cf. hierarchy problem)
- Fundamental Planck scale M_{Planck} lowered to electroweak scale by additional small space dim. (RS model) [Randall, Sundrum '98](#)
 - large degree of spatial curvature
 - exponential “warp factor” in metric of RS model makes M_{Planck} (near the electroweak scale) appear to 4-dim. observer on our brane to be much higher (cf. hierarchy problem)
- Universal Extra Dimensions (UED) [Appelquist, Cheng, Dobrescu '01](#)
 - all SM fields propagate through all of the dimensions of space (no confinement to a particular brane)

ADD cartoon

- The world according to ADD
- $R_4 \times M_n$ spacetime with n -dim. compact manifold M_n
 - 3-brane for Standard Model $(3 + 1)$ -dimensional spacetime
 - extra compact spatial dimensions of radius $\sim r$



ADD models

- Action for brane and bulk separated $S = S_{\text{bulk}} + S_{\text{brane}}$
- Bulk
 - $4 + n$ -dimensional Einstein-Hilbert action

$$S_{\text{bulk}} = -\frac{1}{2} \int d^{4+n}x \sqrt{-g^{(4+n)}} M_{\star}^{n+2} R^{(4+n)}$$

- line element $ds^2 = g_{MN}^{(4+n)} dx^M dx^N$
- Extra dimensions in higher dimensional action “integrated out”
 - expansion of g_{MN} about flat spacetime with fluctuations $h_{\mu\nu}$
- Line element $ds^2 = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu - r^2 d\Omega_{(n)}$
 - metric $d\Omega_{(n)}$ on n -dimensional torus
- Factorization of metric
 - effective replacements of $(4 + n)$ -dim. metric and Ricci scalar

$$\begin{aligned} \sqrt{-g^{(4+n)}} &\rightarrow \sqrt{-g^{(4)}} \\ R^{(4+n)} &\rightarrow R^{(4)} \end{aligned}$$

ADD models

- Effective 4-dim. action for bulk

$$\begin{aligned} S_{\text{bulk}} &= -\frac{1}{2} M_{\star}^{n+2} \int d^{4+n}x \sqrt{-g^{(4+n)}} R^{(4+n)} \\ &= -\frac{1}{2} M_{\star}^{n+2} \int d^4x \int d\Omega_{(n)} r^n \sqrt{-g^{(4)}} R^{(4)} \\ &= -\frac{1}{2} M_{\star}^{n+2} (2\pi r)^n \int d^4x \sqrt{-g^{(4)}} R^{(4)} \end{aligned}$$

- Famous result for 4-dim. Planck scale

$$M_{\text{Pl}}^2 = M_{\star}^{n+2} (2\pi r)^n$$

- M_{Pl} depended on number n and radius r of extra dimensions

Deviations from Newtonian Gravity

- Gauss's law in $(4 + n)$ dimensions
 - two masses m_1, m_2 feel gravitational potential
 - $r' \ll r$

$$V(r') = -G_N^{(4+n)} \frac{m_1 m_2}{r'^{1+n}}, \quad (r' \ll r)$$

- $r' \gg r$ (gravitational flux cannot penetrate in extra dimensions)
- usual $1/r'$ potential is obtained

$$V(r') = -G_N \frac{m_1 m_2}{r'}, \quad (r' \gg r)$$

- D -dim. Newton's constant G_N^D related to D -dim. Planck scale

- $M_\star = \left(\frac{1}{8\pi G_N^D} \right)^{1/(D-2)}$
- effective 4-dimensional M_{Planck}

$$M_{\text{Pl}}^2 \sim \left(M_{\text{Pl}(4+n)} \right)^{n+2} r^n$$

Limits on numbers of dimension

- Assume equal-sized extra dimensions

$$r = \frac{1}{2\pi} \left(\frac{M_{\text{Pl}}^2}{M_\star^{n+2}} \right)^{1/n}$$

- Require M_\star close to weak scale for solving the hierarchy problem
 - for $M_\star = 1 \text{ TeV}$ expect $\mathcal{O}(1)$ deviations from Newtonian gravity

number of extra dimensions	r
$n = 1$	$\sim 10^{12} \text{ m}$
$n = 2$	$\sim 10^{-3} \text{ m}$
$n = 3$	$\sim 10^{-8} \text{ m}$
\vdots	
$n = 6$	$\sim 10^{-11} \text{ m}$

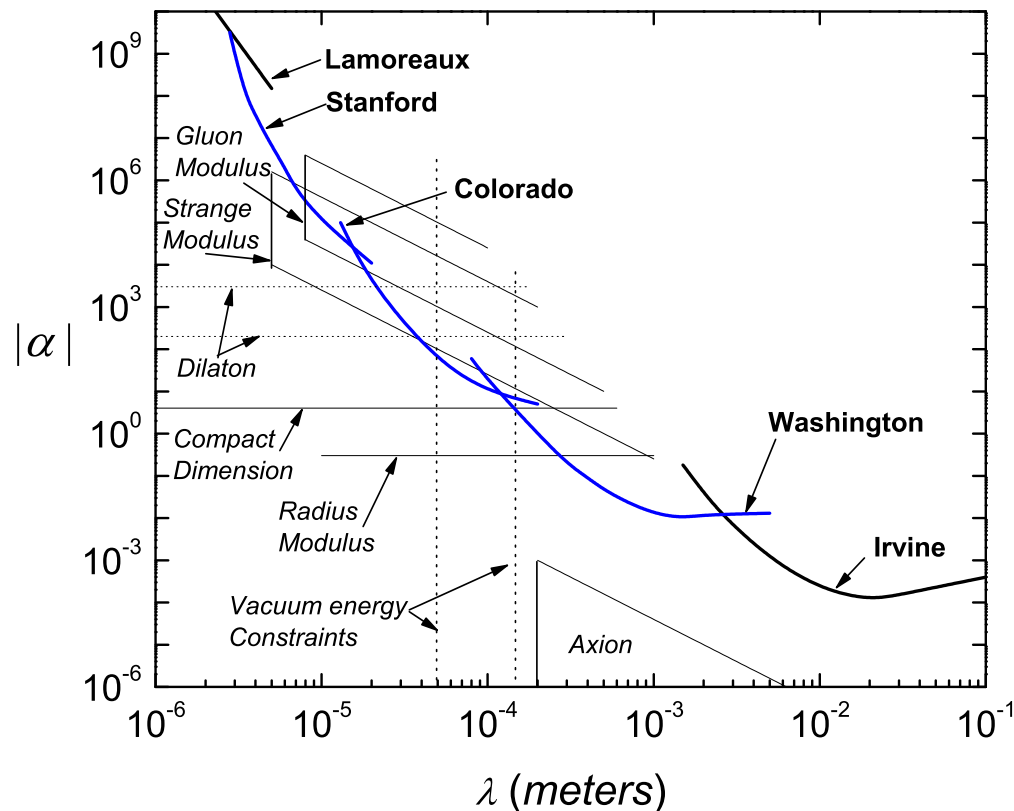
- Upshot
 - gravitational force comparable to gauge forces at weak scale
 - gravity is brought down to weak scale
 - 4-dim. Planck scale is not fundamental scale

Gravity at short distances

- Search for deviations of gravity potential in submillimeter range

$$V = -\frac{G_N m_1 m_2}{r_{12}} \left[1 + \alpha \exp(-r_{12}/\lambda) \right]$$

- restrictions on ADD model with 2 extra dimensions (best sensitivity: Univ. of Washington experiment Adelberger et al. '01-'08)
- compilation of experimental limits Long, Price '03



Universal extra dimensions

Motivation

- UED models are remarkably simple
 - flat metric with one or more compact extra dimensions
- UED models allow for stable proton (at times $\tau_p \geq 10^{33}$ yrs) (unlike RS and ADD models)
 - global symmetries in UED limit higher dimensional operators for proton decay
- Dark matter candidate in UED models
 - LKP (lightest Kaluza-Klein particle) is neutral and absolutely stable
- UED models avoid constraints from electroweak precision tests of SM
 - e.g. absence of FCNC, lepton number violating interactions, decays $B \rightarrow X_s \gamma, \dots$
- UED testable at terascale collider experiments

Orbifolds

- Compactifications of manifolds
 - e.g. $\mathbf{R}_1 \rightarrow \mathbf{S}_1$ (identification of end-points in \mathbf{R}_1)
- Orbifolds are quotient spaces of manifold \mathbf{M} and discrete group \mathbf{G}
 - there exist additional fixed points on \mathbf{M} invariant under \mathbf{G}
 - example: $\mathbf{R}_1 \rightarrow \mathbf{S}_1/\mathbf{Z}_2$
- String theory relies on compactification (e.g. toroidal orbifold constructions $\mathbf{T}_2/\mathbf{Z}_3$)

Orbifolds for UED models

- Complication: fermions in $D > 4$ are non-chiral
- Orbifold compactification for chiral fermions to exist
- Phenomenology in UED models typically restricted to 5-dimensions
 - 5-dim. case with $\mathbf{S}_1/\mathbf{Z}_2$ orbifold removes unwanted fermionic degrees of freedom

The Lagrangian

- Standard Model in $4 + D$ space-time dim.
 - all SM fields can propagate in the extra (universal) dimensions
 - compactification at a scale $1/R$

$$\begin{aligned}
 \mathcal{L}(x^\mu) = & \int d^D y \left\{ - \sum_{i=1}^3 \frac{1}{2\hat{g}_i^2} \text{Tr} \left[F_i^{AB}(x^\mu, y^a) F_{iAB}(x^\mu, y^a) \right] + \right. \\
 & + |(D_\mu + D_{3+a})H(x^\mu, y^a)|^2 + \mu^2 H^*(x^\mu, y^a) H(x^\mu, y^a) - \lambda [H^*(x^\mu, y^a) H(x^\mu, y^a)]^2 + \\
 & + i (\bar{Q}, \bar{u}, \bar{d}, \bar{L}, \bar{e})(x^\mu, y^a) (\Gamma^\mu D_\mu + \Gamma^{3+a} D_{3+a})(Q, u, d, L, e)(x^\mu, y^a) + \\
 & \left. \left[\bar{Q}(x^\mu, y^a) \left(\hat{\lambda}_u u(x^\mu, y^a) i\sigma_2 H^*(x^\mu, y^a) + \hat{\lambda}_d d(x^\mu, y^a) H(x^\mu, y^a) \right) + \text{H.c.} \right] + \right. \\
 & \left. \left[\bar{L}(x^\mu, y^a) \hat{\lambda}_e e(x^\mu, y^a) H(x^\mu, y^a) + \text{H.c.} \right]. \right. \quad (3)
 \end{aligned}$$

In Eq. (3), the summation over fermion generations has been suppressed, and we indicate with F_i^{AB} the $(4+D)$ -dimensional gauge field strengths associated with the $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge group. $D_\mu = \partial/\partial x^\mu - \mathcal{A}_\mu$ and $D_{3+a} = \partial/\partial y^a - \mathcal{A}_{3+a}$ are the covariant derivatives, with $\mathcal{A}_A = -i \sum_{i=1}^3 \hat{g}_i \mathcal{A}_{Ai}^r T_i^r$ being the $(4+D)$ -dimensional gauge fields, and \hat{g}_i the $(4+D)$ -dimensional gauge couplings. The latter, as well as the Yukawa matrices, $\hat{\lambda}_{u,d,e}$, have dimension $(\text{mass})^{-D/2}$. The symbols Q, u, d, L, e describe the $(4+D)$ -dimensional fermions, whose zero modes correspond to the SM fermions. Capitalized letters indicate $SU(2)_W$ doublets, while lower case letters indicate $SU(2)_W$ singlets. The $(4+D)$ -dimensional gamma matrices, Γ^A , are anticommuting $2^{K+2} \times 2^{K+2}$ matrices, where $D = 2K$ if D is even, and $D = 2K + 1$ for odd D , satisfying the $(4+d)$ -dimensional Clifford algebra $\{\Gamma^A, \Gamma^B\} = 2g^{AB}$. In particular, for the case of $D = 1$, one can set $\Gamma_\mu = \gamma_\mu$, and $\Gamma_4 = i\gamma_5$.

KK modes

- decomposition of gauge/scalar fields in KK modes for $5D$ model

$$(H, \mathcal{A}_\mu)(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \left[(H_0, \mathcal{A}_{\mu,0})(x_\mu) + \sqrt{2} \sum_{n=1}^{\infty} (H_n, \mathcal{A}_{\mu,n})(x_\mu) \cos\left(\frac{ny}{R}\right) \right]$$
$$\mathcal{A}_5 = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \mathcal{A}_{5,n}(x_\mu) \sin\left(\frac{ny}{R}\right)$$

- Two $5D$ fermion fields $\psi_{L,R}$ (left and right handed spinors)
- Construct SM fields from chiral projection of zero modes

$$\psi^{\text{SM}} = P_L \psi_{L,0} + P_R \psi_{R,0}$$

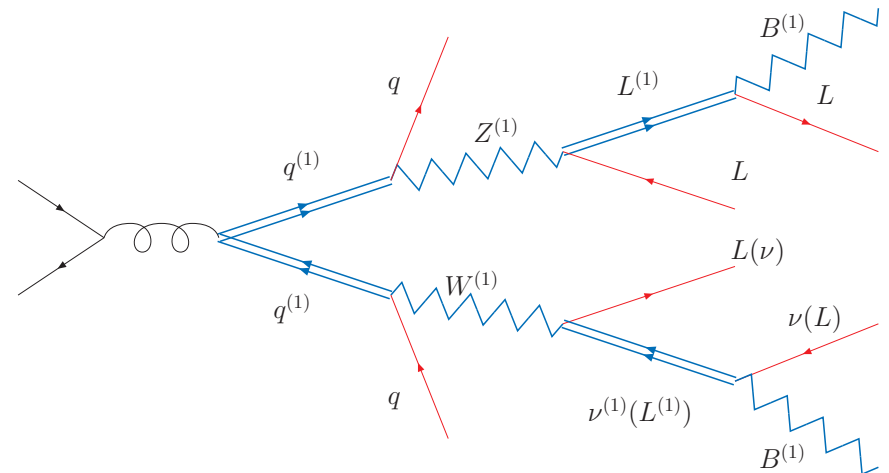
$$\psi(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \left[\psi^{\text{SM}}(x^\mu) + \sqrt{2} \sum_{n=1}^{\infty} P_L \psi_{L,n}(x^\mu) \cos\left(\frac{ny}{R}\right) + P_R \psi_{R,n}(x^\mu) \sin\left(\frac{ny}{R}\right) \right]$$

KK parity

- KK level of particle measures its momentum in extra dimension
- Orbifold compactification breaks translational invariance in extra dimension
 - KK-number is not conserved quantity
- KK parity (subgroup of KK-number conservation)
 - conservation of *evenness* or *oddness* of KK number in interaction
e.g. 5-dim. case with S_1/Z_2 leaves KK parity as Z_2 symmetry
(odd KK numbers charged)

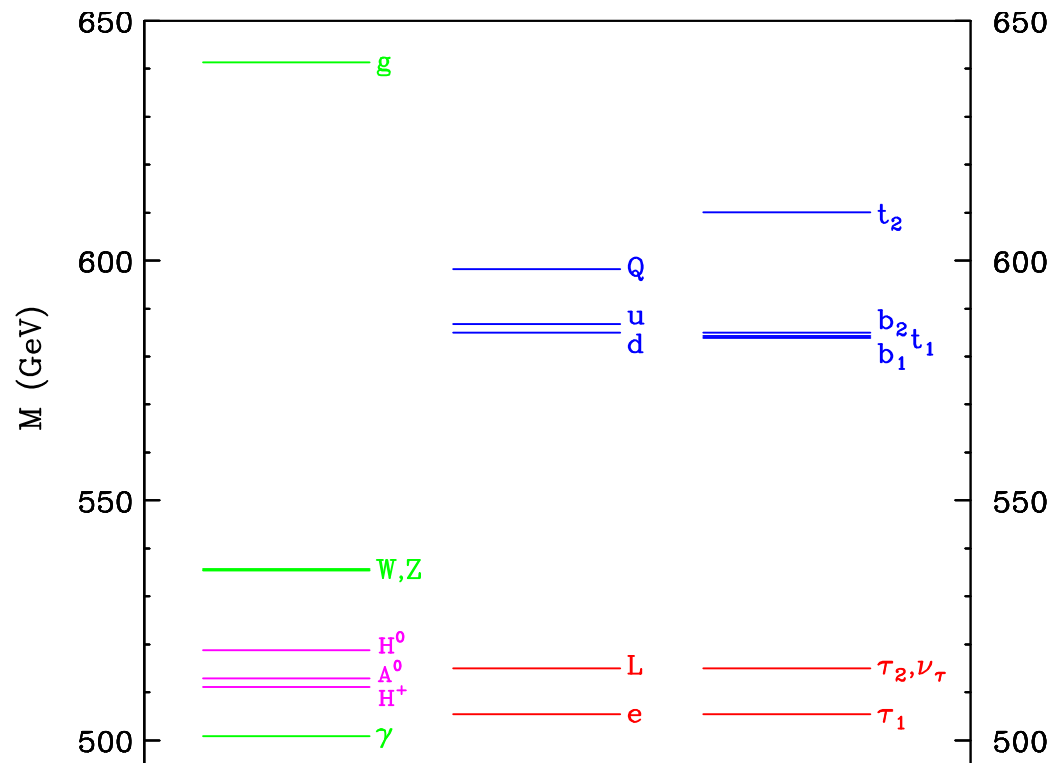
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 - conservation of *evenness* or *oddness* of KK number in interaction e.g. 5-dim. case with S_1/Z_2 leaves KK parity as Z_2 symmetry (odd KK numbers charged)
- Phenomenological consequences
 - pair-production of excited KK-modes in interactions
 - LKP is stable
 - similarity to R-parity in SUSY



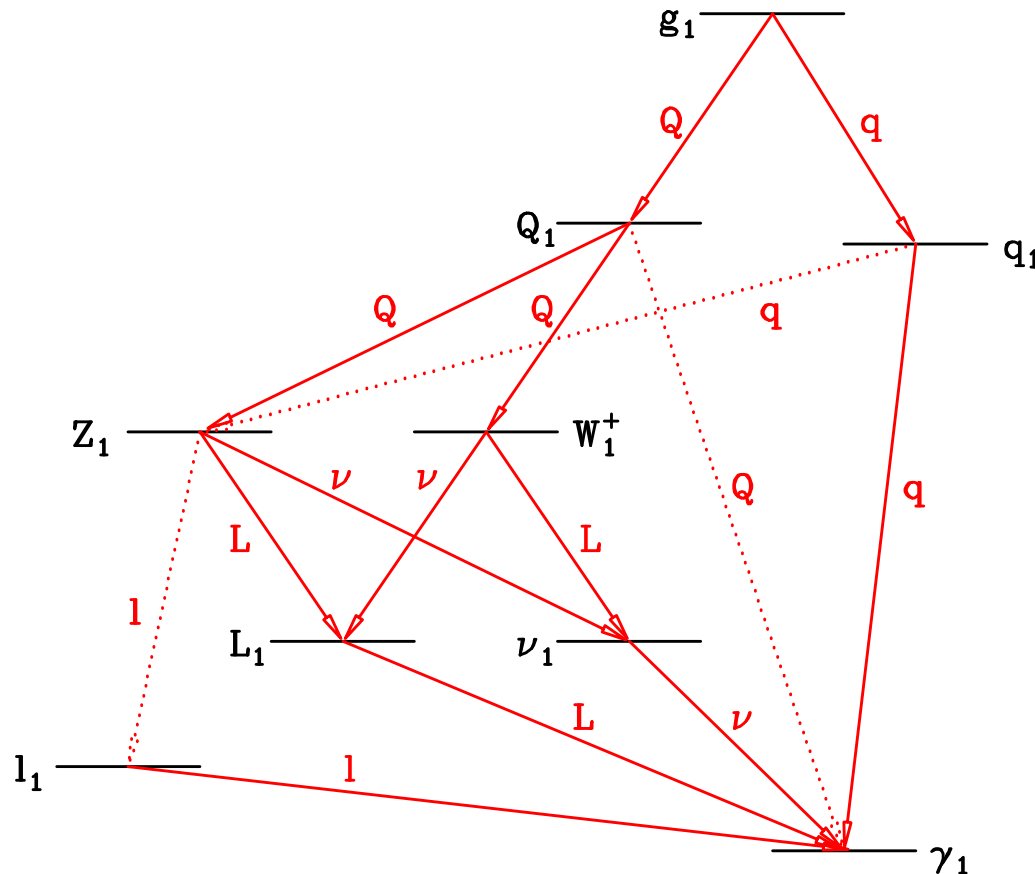
Particle spectrum

- Spectrum of first level Kaluza-Klein states
- effective mass $m_{X^{(n)}}^2 = \frac{n^2}{R^2} + m_{X^{(0)}}^2$
- compactification radius $R^{-1} = 500\text{GeV}$



Particle decays

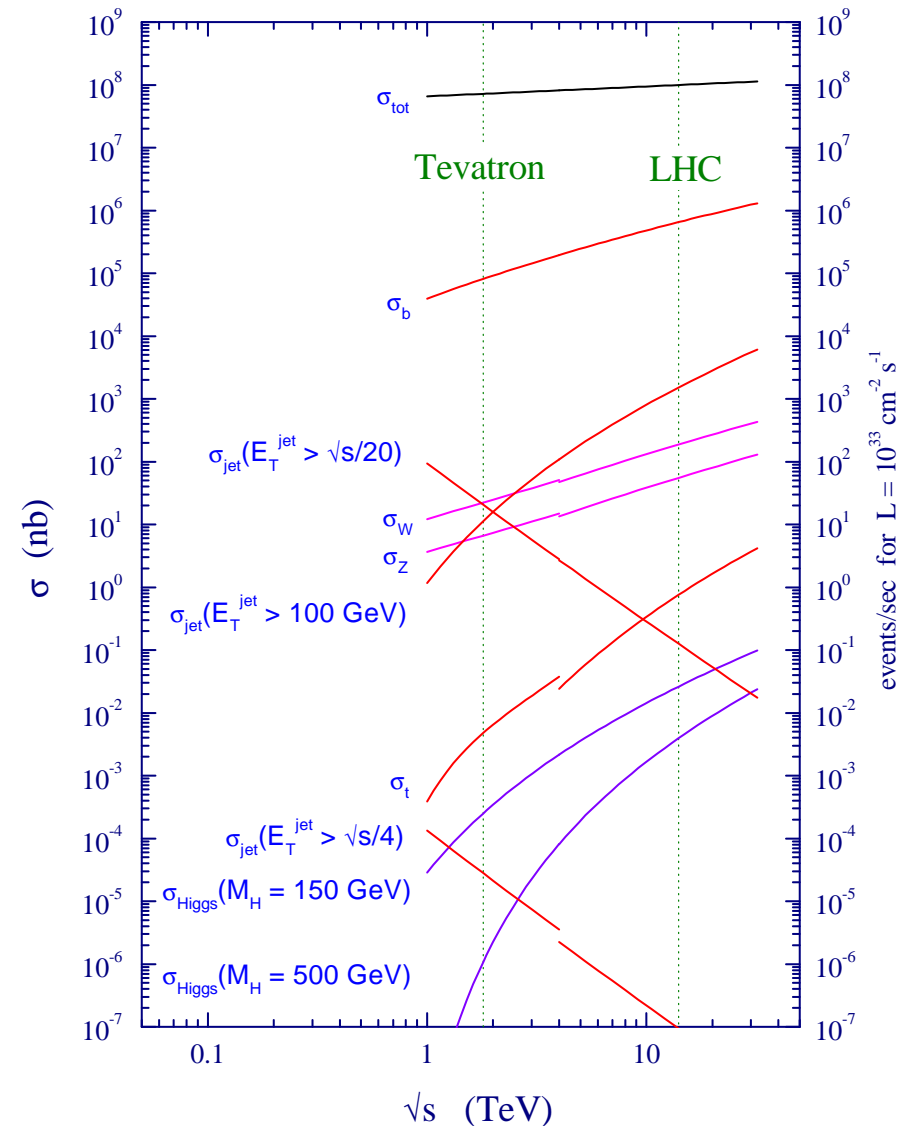
- KK decay chain for $n = 1$ with dominant (solid) and sub-dominant (dotted) transitions
- heavier $n = 1$ states cascade decay into stable LKP $B^{(1)}$ under emission of soft SM particles
- LKP escapes detection, leading to missing energy signature



LHC phenomenology

- Expected production rates strongly depend on compactification scale R^{-1}
 - assume minimal UED with $\Lambda R = 20$ (boundary couplings flavor-conserving, boundary terms vanish at cutoff Λ)
 - radiative corrections lift KK mass degeneracy
 - decay cascades terminate with stable LKP
 - generic missing energy signatures

proton - (anti)proton cross sections

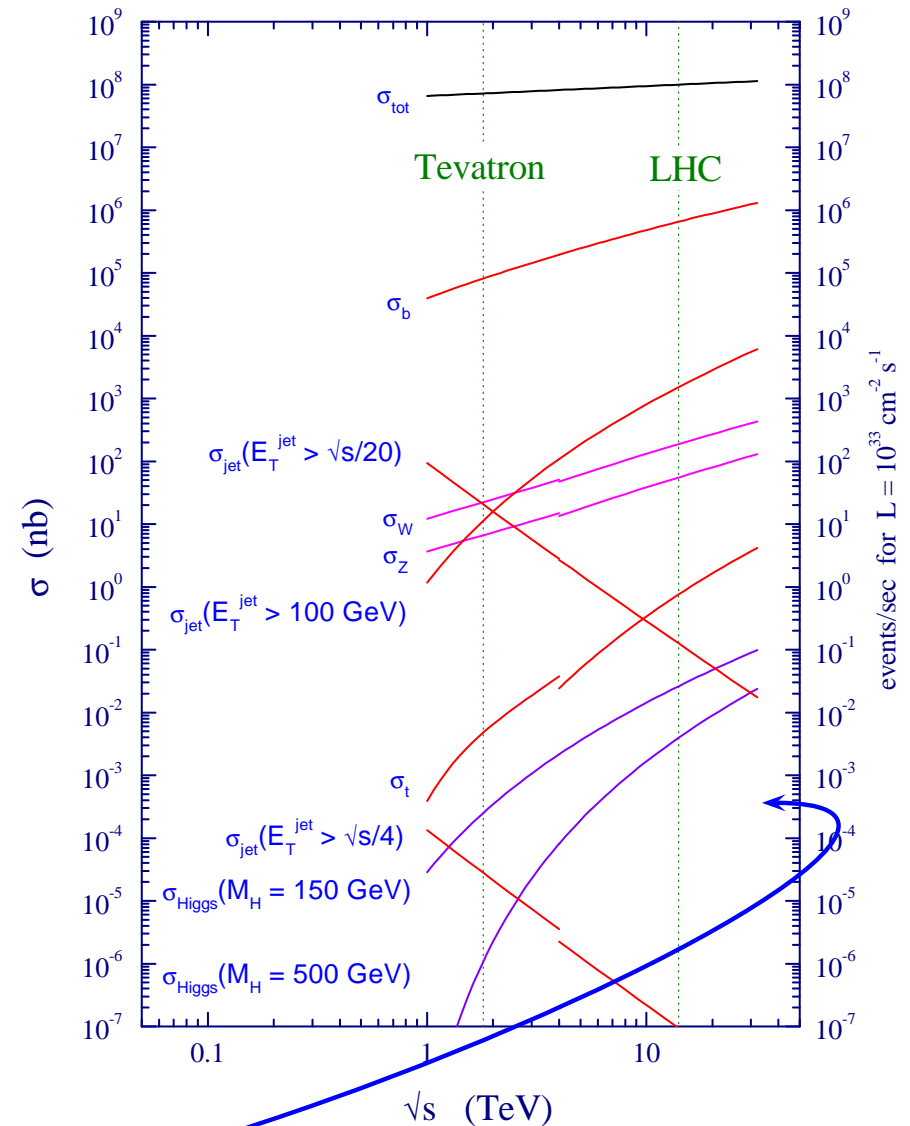


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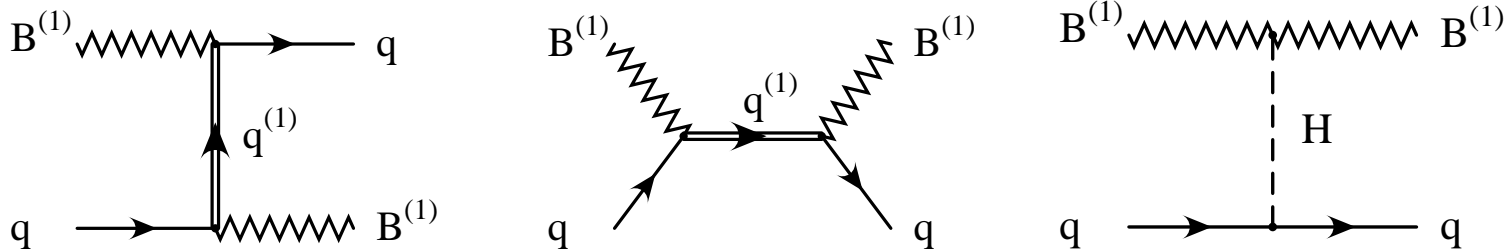
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proton - (anti)proton cross sections



Collider signatures

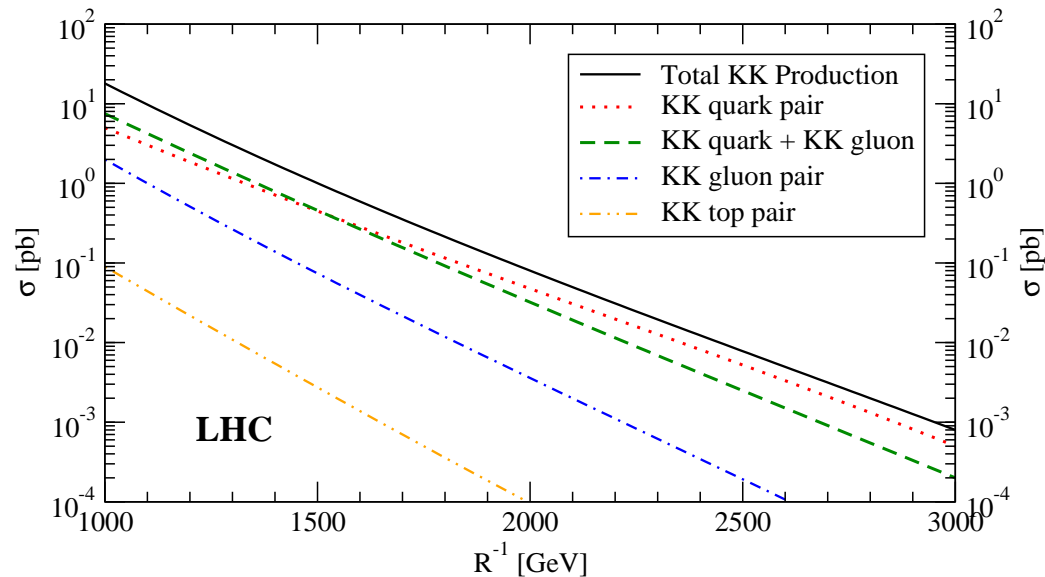
- Feynman diagrams for LKP production
 - $B^{(1)}$ -quark scattering
 - $B^{(1)}$ -gluon scattering (through quark/KK-quark loop)



- LHC: predominantly production of colored KK-particles
 - pair-production of KK quarks, KK gluons and KK top-quarks

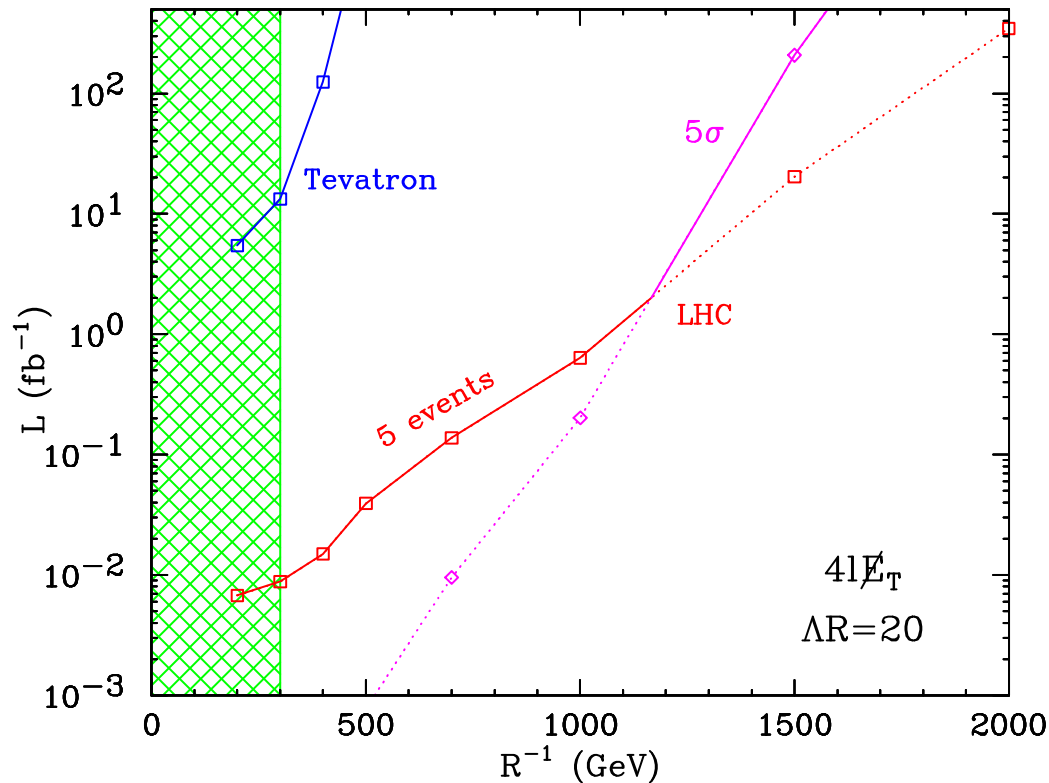
LHC cross section

- Production cross section of KK quarks and KK gluons at LHC as function of compactification radius R^{-1}
 - separate contributions [Macesanu, McMullen, Nandi '02](#)
 - KK quark pairs
 - KK quark + KK gluon
 - KK gluon pairs
 - KK top pairs



LHC reach

- Discovery reach for minimal UED scenario in $4l + \cancel{E}_T$ channel as function of luminosity (fb^{-1}) and compactification radius R^{-1}
 - Tevatron (blue) and at the LHC (red) for $\Lambda R = 20$
Cheng, Matchev, Schmaltz '02
- Reach contours correspond to 5σ excess (or observation of 5 signal events)

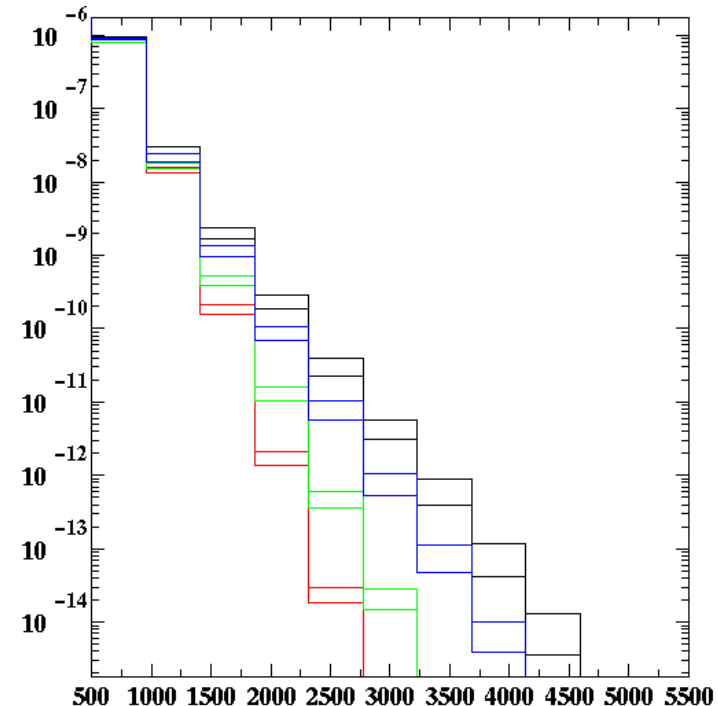
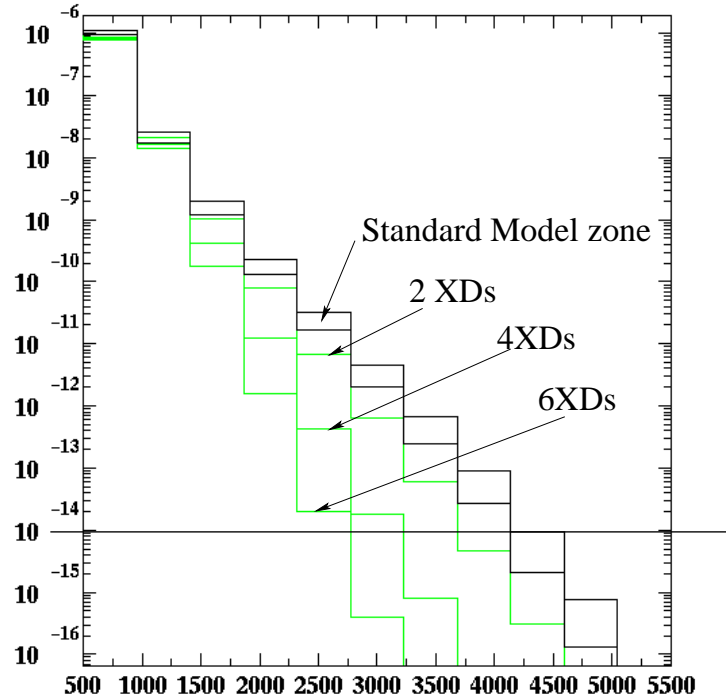


Sensitivity to PDFs

- Sensitivity of LHC dijet cross section to large extra dimensions Ferrag '04
 - large extra dimensions accelerate running of α_s as compactification scale M_c is approached
- PDF uncertainties
 - potential sensitivity to M_c reduced from 6 TeV to 2 TeV

$M_c = 2 \text{ TeV}$ no PDF error

$M_c = 2 \text{ TeV}$ with PDF error



UED vs. SUSY

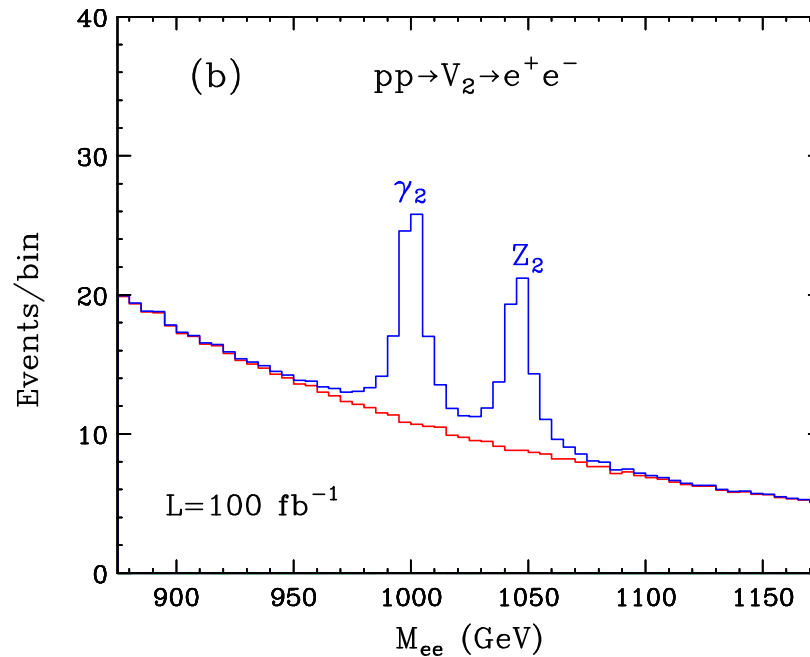
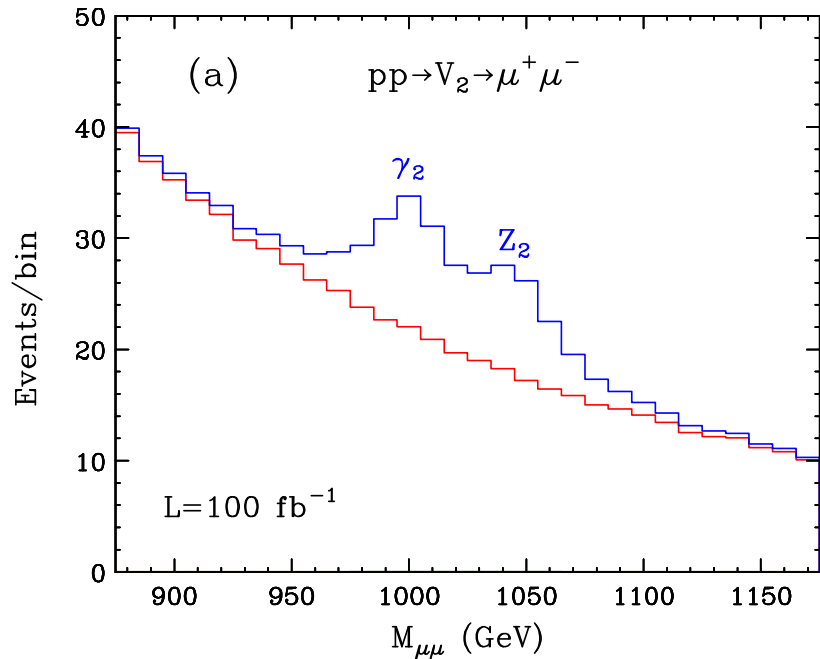
- *Bosonic supersymmetry? Getting fooled at LHC*
Cheng, Matchev, Schmaltz '02
- Common features of extra dimensions and supersymmetry
 - lightest new state is neutral and stable
 - collider signatures with missing transverse momentum (energy) plus jets/leptons
 - same couplings for the SM particles and their heavier counterparts

Differences

- KK first level states in UED have same spin as SM counterparts
 - SUSY partners have opposite spin
- Higgs sector of UED has different KK parity assignment than heavy Higgs bosons in MSSM (H , A , H^\pm)
 - Higgs sectors in SUSY and UED share same gauge quantum numbers
 - UED Higgses more similar to SUSY *higgsinos*,
- UED feature higher level KK modes (unlike supersymmetry)

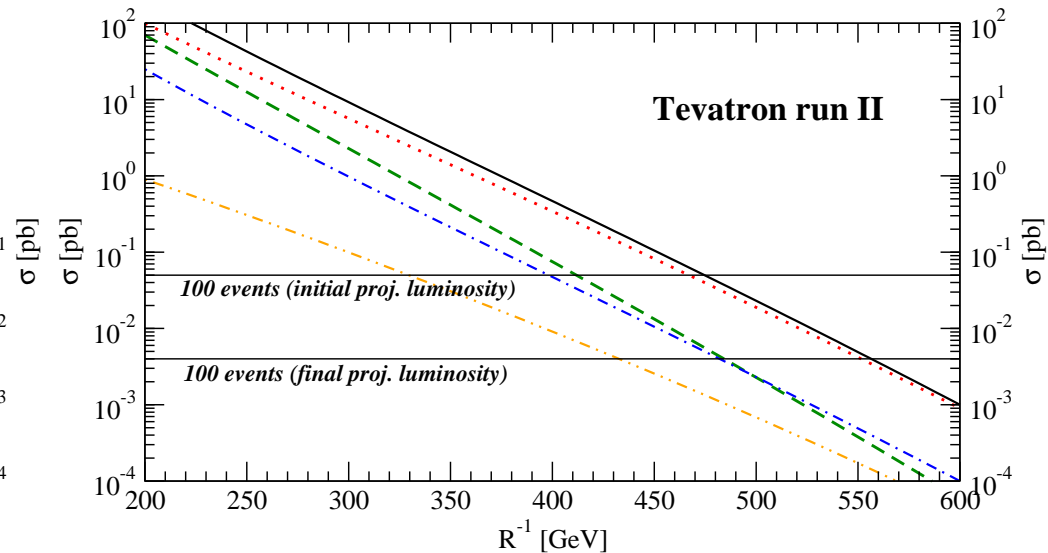
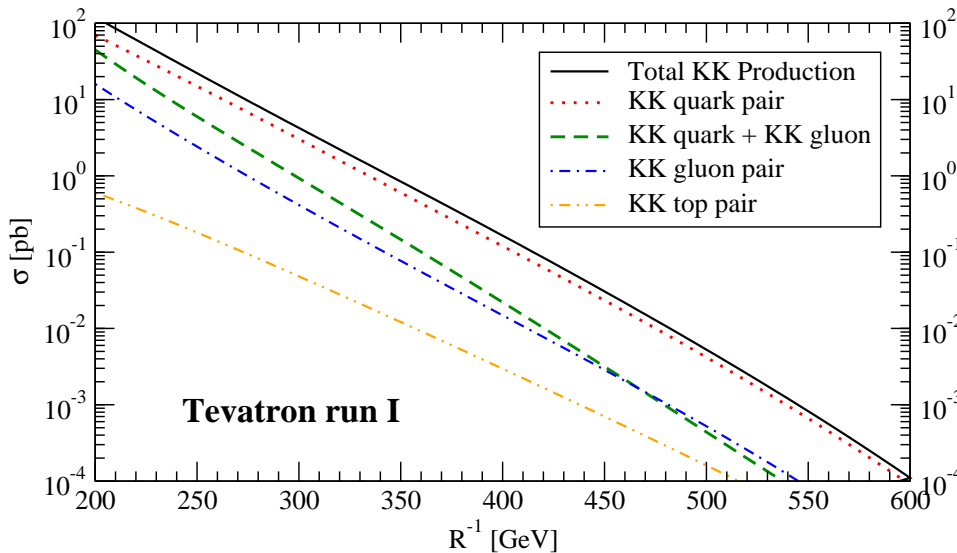
Spin correlations

- second level KK states
- $B^{(2)} - Z^{(2)}$ di-resonance structure with $R^{-1} = 500 \text{ GeV}$ at LHC with $L = 100 \text{ fb}^{-1}$
 - Datta, Kong, Matchev '05
 - di-muon channel (left)
 - di-electron channel (right)
 - SM background (red)



Tevatron cross section

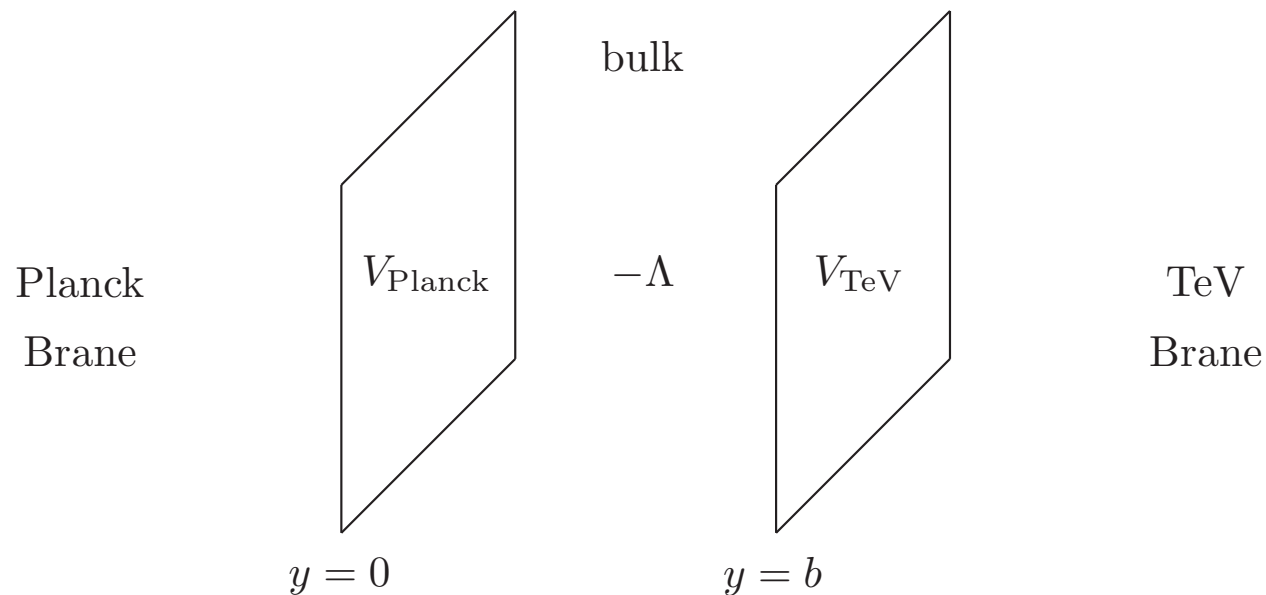
- Production cross section of KK quarks and KK gluons at Tevatron as function of compactification radius R^{-1}
 - run I $\sqrt{s} = 1.8$ TeV, run II $\sqrt{s} = 1.96$ TeV
 - channels: [Macesanu, McMullen, Nandi '02](#)
 KK quark pairs, KK quark + KK gluon, KK gluon pairs, KK top pairs
- Tevatron run II sensitivity limits



Randall-Sundrum models

RS cartoon

- The world according to RS
 - only gravity exists in the warped extra dimension
 - Standard Model confined to 3-brane
 - all dimensionful SM parameters scaled to the TeV scale



Randall-Sundrum models

- One extra spatial dimension (space S_1/Z_2) and “warped” five-dimensional geometry
 - “branes” extend infinitely in usual three spatial dimensions (sufficiently thin in warped direction)
- Metric non-factorizable

$$ds^2 = e^{-2kr_c\phi} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2$$

- “warp” factor in four-dimensional metric (rapidly changing function of additional dimension)
 - coordinates x^μ in familiar four dimensions
 - $0 \leq \phi \leq \pi$ parameter of extra dimension of radius r_c ($S_1/Z_2 \longrightarrow$ identification of (x, ϕ) with $(x, -\phi)$)
 - $k \sim \mathcal{O}(M_{\text{Planck}})$
- Upshot
 - four-dimensional mass scales related to five-dimensional input mass parameters and warp factor $e^{-2kr_c\phi}$

Action for RS models

- Orbifold fixed points at $\phi = 0, \pi$ support two 3-branes (boundaries of the five-dimensional spacetime)
 - 3-branes support $(3 + 1)$ -dimensional field theories
 - classical action $S = S_{gravity} + S_{vis} + S_{hid}$

- Matter field Lagrangian

- example a fundamental Higgs field with mass parameter v_0

$$S_{vis} \supset \int d^4x \sqrt{-g_{vis}} \{ g_{vis}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda (|H|^2 - v_0^2)^2 \}$$

- Substitution of metric and rescaling of Higgs field $H \rightarrow e^{kr_c\pi} H$

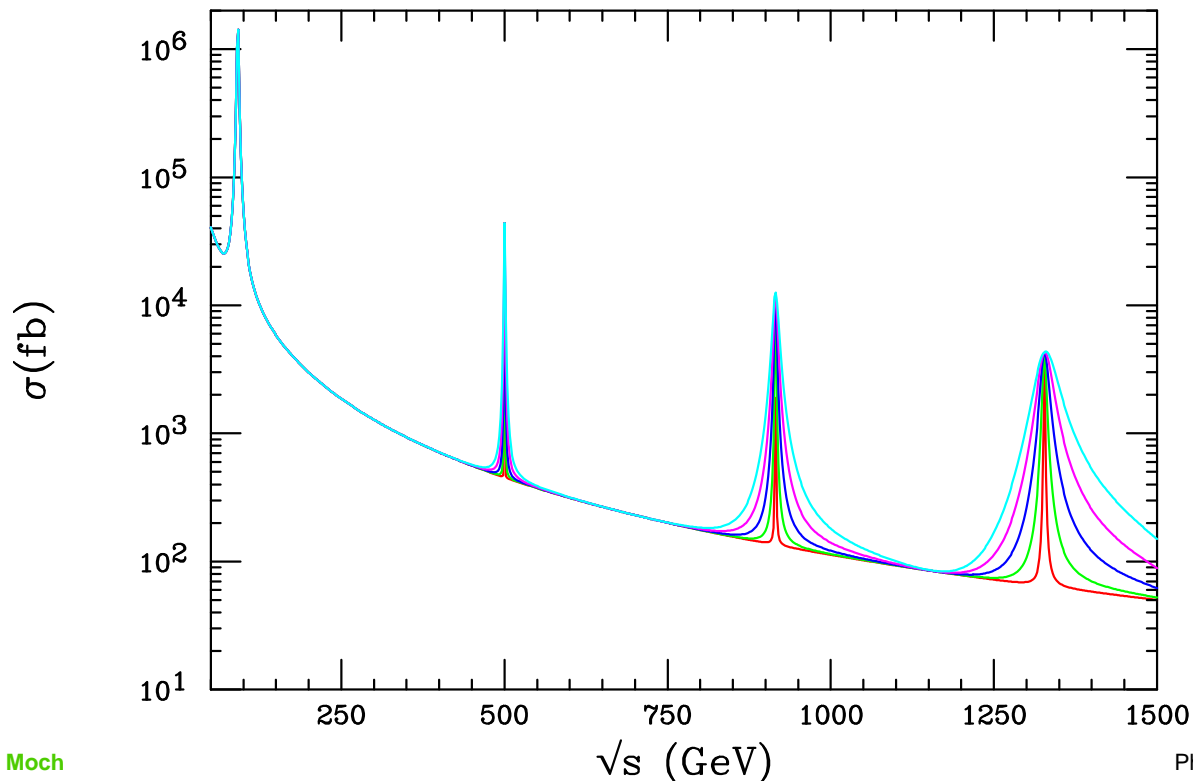
$$S_{eff} \supset \int d^4x \sqrt{-\bar{g}} \{ \bar{g}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda (|H|^2 - e^{-2kr_c\pi} v_0^2)^2 \}$$

- Upshot

- physical scale set by symmetry-breaking scale $v \equiv e^{-kr_c\pi} v_0$
- any mass parameter m_0 on visible 3-brane in fundamental higher-dim. theory corresponds physical mass $m \equiv e^{-kr_c\pi} m_0$

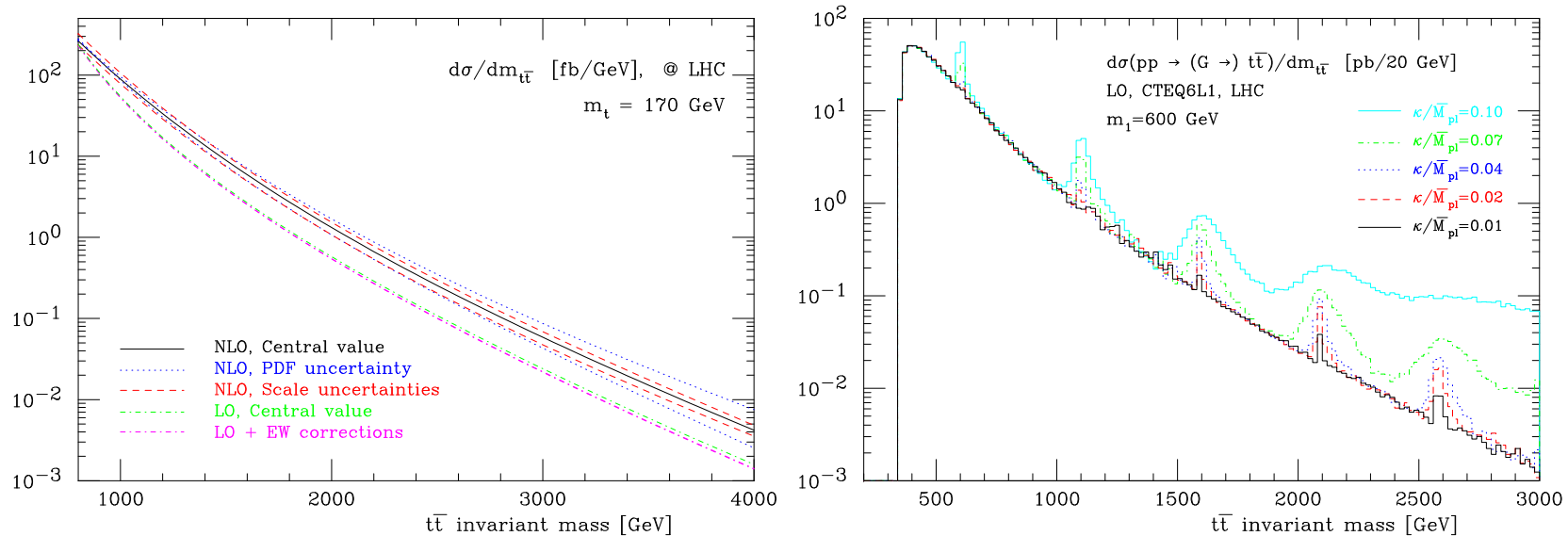
Resonances

- RS-I scenario predicts di-fermion or di-bosons resonances at LHC from KK gravitons
 - in RS-I coupling of each KK graviton mode is only suppressed by $\mathcal{O}(\text{TeV})$ (unlike KK gravitons of ADD)
 - width of these resonances is controlled by ratio $c = k/M$
- Example: cross section for $e^+e^- \rightarrow \mu^+\mu^-$ including KK graviton exchange [Hewett, Spiropulu '02](#)



Top-quark invariant mass distribution

- Invariant mass distribution of top-quark pair invariant $M_{t\bar{t}}$



- Left: the $t\bar{t}$ invariant mass spectrum at LHC with NLO electroweak corrections
- Right: s -channel graviton exchange in $t\bar{t}$ invariant mass spectrum at LHC Frederix, Maltoni '07
 - Kaluza-Klein resonances in RS model

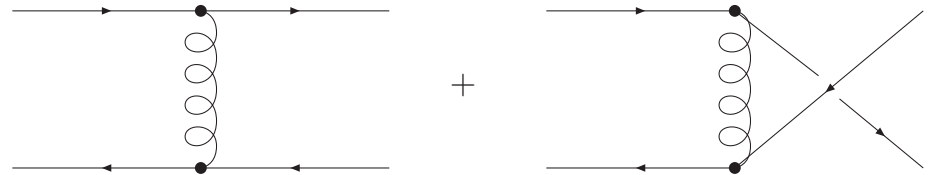
Hadronic di-jets

- Di-jet differential cross section for scattering
 $\text{parton}_i(k_1) + \text{parton}_j(k_2) \rightarrow \text{parton}_k(k_3) + \text{parton}_l(k_4)$

$$\frac{d^3\sigma}{dy_3 dy_4 dp_t^2} = \frac{1}{16\pi s^2} \sum_{i,j,k,l=q,\bar{q},g} \frac{f_i(x_1)}{x_1} \frac{f_j(x_2)}{x_2} \overline{\sum} \frac{1}{1 + \delta_{kl}} |\mathcal{A}(ij \rightarrow kl)|^2$$

- Example: $\hat{\sigma}^{ud}$ with

$$\overline{\sum} |\mathcal{A}|^2 = (4\pi\alpha_s)^2 \frac{4}{9} \frac{s^2 + u^2}{t^2}$$



- Kinematics in di-jet cms

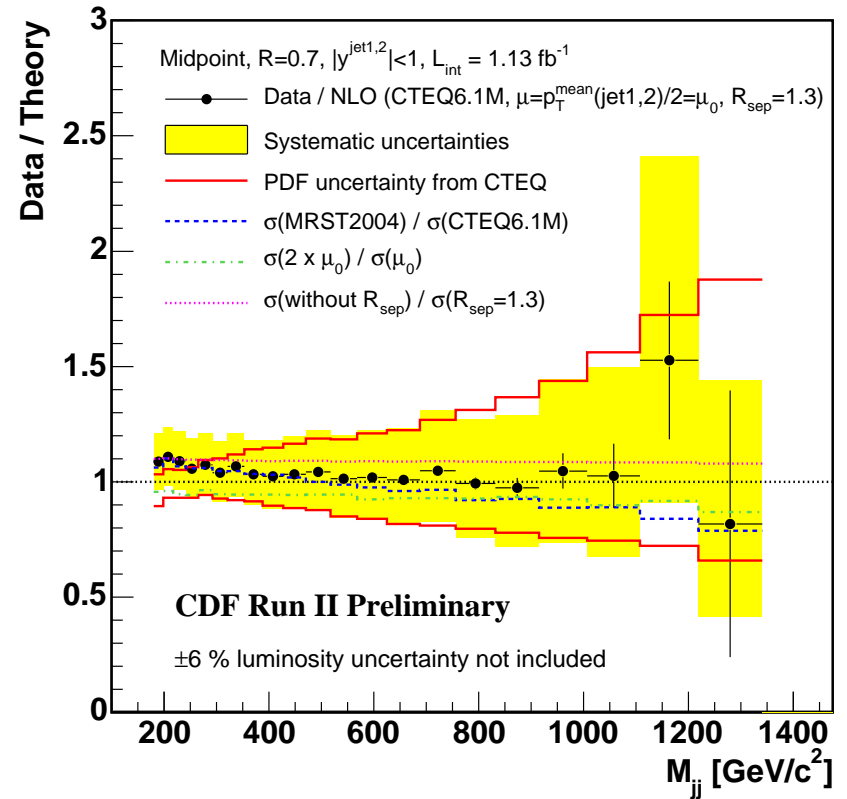
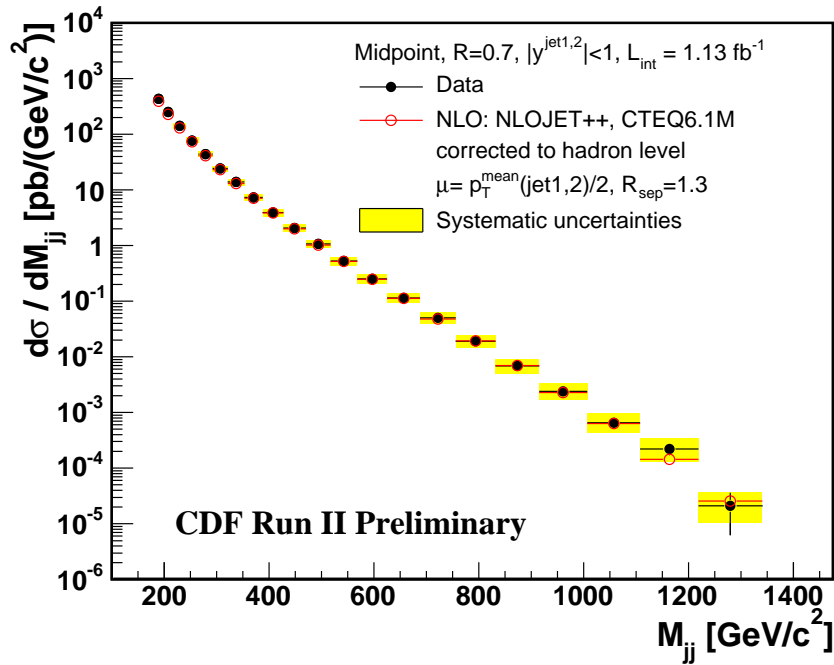
- di-jet rapidity $y^* = \frac{y_3 - y_4}{2}$ determines cms scattering angle

$$\cos \theta^* = \frac{p_z^*}{E^*} = \frac{\sinh y^*}{\cosh y^*} = \tanh \left(\frac{y_3 - y_4}{2} \right)$$

- di-jet invariant mass M_{JJ}^2

$$dy_3 dy_4 dp_t^2 = \frac{1}{2} dx_1 dx_2 d \cos \theta^*$$

Jets at Tevatron



- Di-jet invariant mass distribution
 - agreement with perturbative QCD over eight orders of magnitude
 - larger uncertainties for high M_{jj}^2

- Cross section σ^{ud} in di-jet cms kinematics

$$\frac{d\hat{\sigma}^{ud}}{d\cos\theta^*} = \frac{\pi\alpha_s^2}{2M_{JJ}^2} \frac{4}{9} \left[\frac{4 + (1 + \cos\theta^*)^2}{(1 - \cos\theta^*)^2} + \frac{4 + (1 - \cos\theta^*)^2}{(1 + \cos\theta^*)^2} \right]$$

- Small angles $\frac{d\hat{\sigma}^{ud}}{d\cos\theta^*} \sim \frac{1}{\sin^4(\theta^*/2)}$ (Rutherford)

- transform to $\chi = \frac{1 + \cos\theta^*}{1 - \cos\theta^*}$

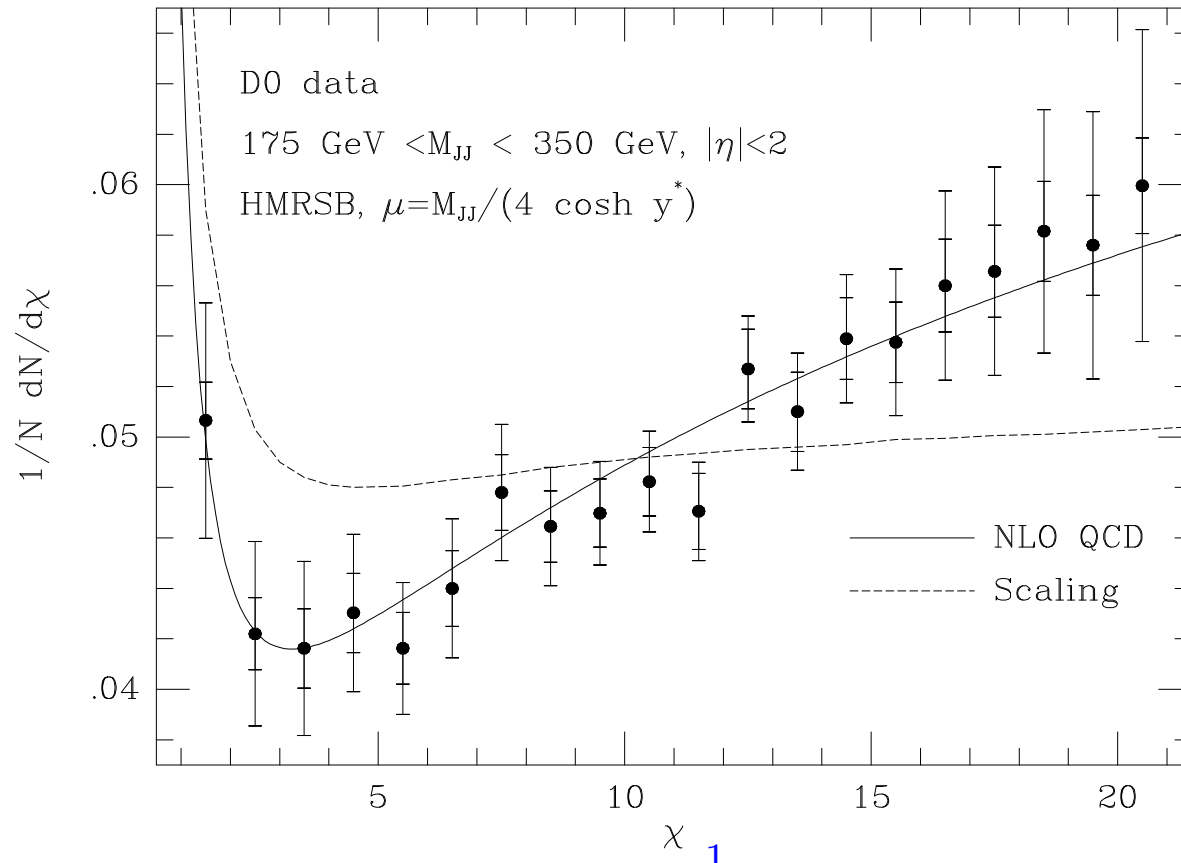
- $\frac{d\hat{\sigma}^{ud}}{d\chi} \sim \text{const}$

- Scalar colored particle (e.g. scalar gluon)

- $\frac{d\hat{\sigma}^{ud}}{d\cos\theta^*} \sim \text{const}$ transforms to $\frac{d\hat{\sigma}^{ud}}{d\chi} \sim \frac{1}{(1 + \chi)^2}$

Quark substructure

- Searches for quark sub-structure in di-jet angular correlations



$$\frac{d^2\sigma}{dM_{JJ}^2 d\cos\theta^*} = \sum_{i,j=q,\bar{q},g} \int_0^1 f_i(x_1) f_j(x_2) \delta(x_1 x_2 s - M_{JJ}^2) \frac{d\hat{\sigma}^{ij}}{d\cos\theta^*}$$

Summary table

- “Metric” for models with extra dimensions

	SM fields	gravity
ADD	confined to 3-brane	gravity acts on 3-brane and in bulk
RS	confined to 3-brane	gravity acts on 3-brane and in bulk (extra dimension with large curvature)
UED	propagate through all spacetime (not confined to particular brane)	propagates through all spacetime