Exercises "Physics at LHC" (sheet 2)

Event shapes:

In the e^+e^- rest frame thrust and sphericity are given by

$$T = \max_{\overrightarrow{n}_T} \frac{\sum_i |\overrightarrow{p}_i \cdot \overrightarrow{n}_T|}{\sum_i |\overrightarrow{p}_i|} \qquad S = \frac{3}{2} \min_{\overrightarrow{n}_S} \frac{\sum_i |\overrightarrow{p}_i \times \overrightarrow{n}_S|^2}{\sum_i |\overrightarrow{p}_i|^2}$$

where \overrightarrow{n}_T and \overrightarrow{n}_S are unit vectors

- 1. Show that $S = \frac{3}{2}(\lambda_2 + \lambda_3)$ where $\lambda_1 \ge \lambda_2 \ge \lambda_3$ are the eigenvalues of the matrix $S^{\alpha\beta} = \frac{\sum_i p_i^{\alpha} p_i^{\beta}}{\sum_i |\overrightarrow{p}_i|^2}$.
- 2. Are T,S infrared and collinear safe?
- 3. Show that T = 1, S = 0 for $e^+e^- \rightarrow q\bar{q}$, $T = \frac{2}{3}$, $S = \frac{3}{4}$ for $e^+e^- \rightarrow q\bar{q}g$ with $|p_q| = |p_{\bar{q}}| = |p_g|$, $T = \frac{1}{2}$, S = 1 for isotropic $e^+e^- \rightarrow q\bar{q}gg...$

Impact parameter:



The impact parameter is defined as the distance of the (extrapolated) track parameter to the primary vertex.

- 1. A particle has a mass m, an energy E and a lifetime τ . What is its mean decay length?
- 2. Show that under the assumption $\beta = 1$ the impact parameter is of order $\delta \sim c\tau$ independent of the particle energy. (Assume e.g. that the particle undergoes a symmetric decay into two massless particles)